Chapter – 9
Some Applications of Trigonometry

Q. 1 A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see Fig. 9.11)

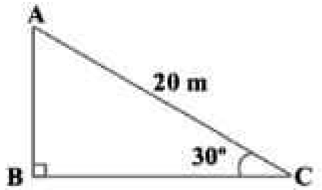
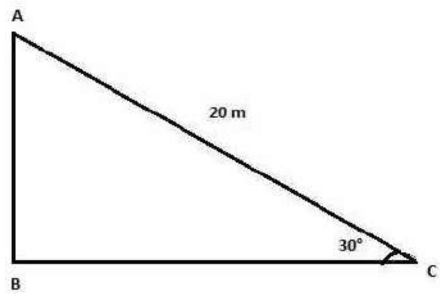


Fig. 9.11

Answer:



In this figure; AC is the rope which is 20 m long and we have to find AB (height of the pole).

In Δ ABC;

We know,

$\sin\theta = \frac{p}{h}$

where p = perpendicular and h = hypotenuse

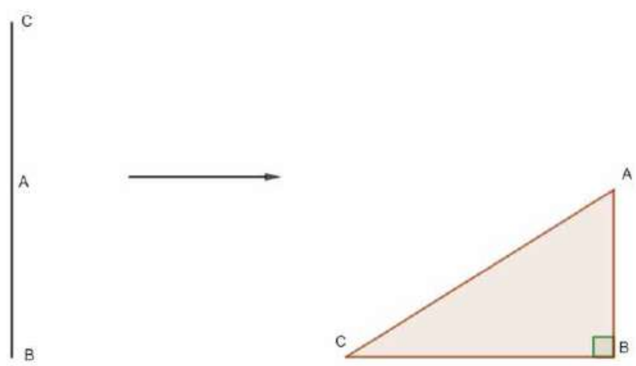
$\sin 30° = \frac{AB}{AC}$

$\frac{1}{2} = \frac{AB}{20}$

Hence, Height of the pole, AB = 10 m

Q. 2 A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree

Answer: Let BAC be the tree.



Now, due to storm AC is broken and leans to make a shape as right-angled triangle shown in figure.

AC is the broken part of the tree and AB is the part which is still upright.

Hence,

Distance from tree foot to where top touches, BC = 8 m

And,

$\angle$ BCA = 30°

Clearly, Sum of AB and AC will give the height of the tree.

In Δ ABC:

$$\cos\theta = \frac{Base}{Hypotenuse}$$

$$\cos 30° = \frac{BC}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$AC = \frac{16}{\sqrt{3}}$$

Hence,

$\tan\theta = \frac{Perpendicular}{Base}$

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{8}$

$AB = \frac{8}{\sqrt{3}}$

Now,

Height of tree= AB + AC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$

$= \frac{24}{\sqrt{3}}$

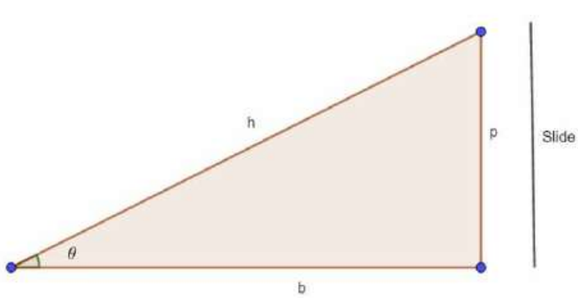
rationalizing, we get

= $8\sqrt{3}$ m

Hence, height of the tree is $8\sqrt{3}$ m.

Q. 3 A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Answer:



In the first case:

Height of slide= 1.5 m and angle of elevation = 30°

Now,

$\sin\theta = \frac{p}{h}$

where p = perpendicular, i.e. height of the slide and h = hypotenuse, i.e. length of the slide and θ is the angle of elevation

$\sin 30° = \frac{1.5}{h}$

$\frac{1}{2} = \frac{1.5}{h}$

Hence, h = 3 m

In the second case:

Height of slide, = 3 m, angle of elevation = 60°

$\sin\theta = \frac{p}{h}$

$\sin 60° = \frac{3}{h}$

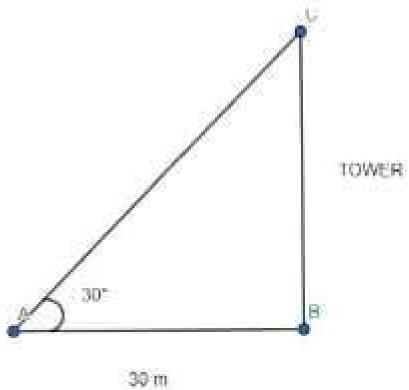
$\frac{\sqrt{3}}{2} = \frac{3}{h}$

Hence, h = $2\sqrt{3}$ m

Therefore, the length of the slide in the first case and the second case are 3 m and $2\sqrt{3}$ m respectively.

Q. 4 The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Answer:



In this question:

b = 30 m,

Angle of elevation = 30°

And

p = height of tower =?

$\tan\theta = \frac{p}{b}$ $\quad\quad [p = perpendi\ cul\ arb = base]$

$\tan 30° = \frac{p}{30}$

$\frac{1}{\sqrt{3}} = \frac{p}{30}$

$p = \frac{30}{\sqrt{3}}$

Rationalizing the value of p, we get

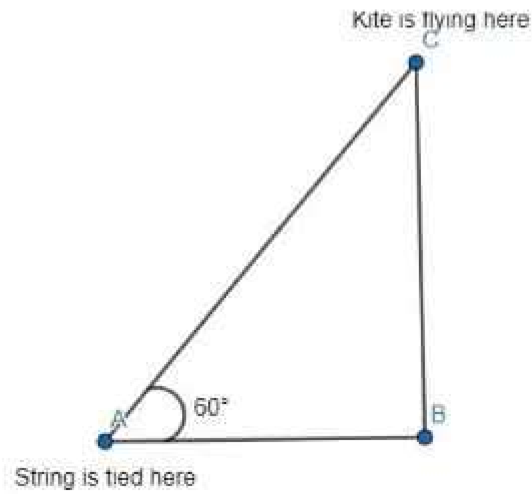
$p = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$p = 10\sqrt{3}$

Hence the height of the tower is $10\sqrt{3}$ m

Q. 5 A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer:



To find: Length of String, AC

Given:

Height of kite from the ground = BC = 60 m, angle of elevation = 60° and length of string = AC =?

Let the length of the string be h
Now we know that,

$\sin\theta = \frac{Perpendicular}{Hypotenuse}$

So in the given triangle, $\sin 60° = \frac{BC}{AC}$

$\sin 60° = \frac{60}{h}$

$\frac{\sqrt{3}}{2} = \frac{60}{h}$

Cross multiplying we get,

$h = \frac{120}{\sqrt{3}}$

rationalizing, we get

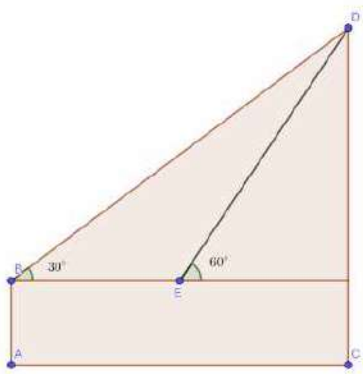
$h = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$h = \frac{120\sqrt{3}}{3}$

$h = 40\sqrt{3}$ m

Hence, length of the string is $40\sqrt{3}$ m

Q. 6 A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building



Let AB be the boy, and CD be the building and angles becomes 60°, as the boy reaches E.

The angle of elevation in first case = 30°

And

The angle of elevation, in the second case = 60°.

Difference between bases in the first case and second case will give the distance covered by the boy.

1st case:

here p = height of the building - the height of the boy = 30 - 1.5 = 28.5 m

$$\tan\theta = \frac{p}{b}$$

$$\tan 30° = \frac{28.5}{b}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{b}$$

$b = 28.5\sqrt{3}$

2^{nd} case:

$\tan\theta = \frac{p}{b}$

$\tan 60° = \frac{28.5}{b}$

$\sqrt{3} = \frac{28.5}{b}$

Hence, the distance covered by the boy:

$= 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$

$= \frac{28.5 \times 3 - 28.5}{\sqrt{3}}$

$= \frac{57}{\sqrt{3}}$

$= 19\sqrt{3}$ m

Q. 7 From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

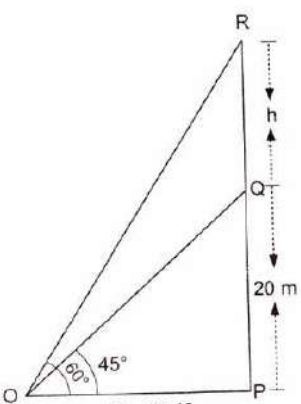


Fig. 12.48

Since the building is vertical. ∠QPO = 90°

In a right-angled triangle, we know,

$\tan\theta = \frac{perpendicular}{base}$

$In\,\Delta OPQ$

$\tan 45° = \frac{QP}{OP}$

$1 = \frac{20}{OP}$

OP = 20 ….. (1)

Now in ΔOPR

$\tan 60° = \frac{PR}{OP}$

$\sqrt{3} = \frac{RQ+QP}{OP}$

$\sqrt{3} = \frac{h+20}{20}$

$20\sqrt{3} = h+20$

$h = 20\sqrt{3}-20$

$h = 20(\sqrt{3}-1)$ m.

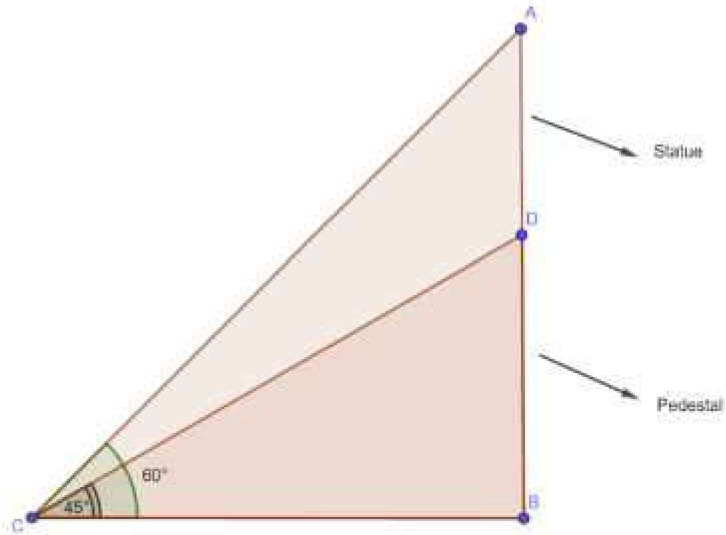
Therefore the height of transmission tower is 20(√3-1) m.

Q. 8 A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Answer: Let AD be the height of the statue which is 1.6 m,

And DB be the height of the pedestal.

And C be the point where the observer is present.



To find: height of pedestal, BD

Now,

In Δ ABC,

$\tan\theta = \frac{p}{b}$

where p = perpendicular and b = base

$\tan 60° = \frac{p}{b}$

$\sqrt{3} = \frac{DB+1.6}{b}$

$b = \frac{DB+1.6}{\sqrt{3}}$ (i)

In Δ DBC,

$\tan\theta = \frac{p}{b}$

$\tan 45° = \frac{p}{b}$

$1 = \frac{DB}{b}$

b = DB (ii)

On comparing (i) and (ii),

$DB = \frac{DB+1.6}{\sqrt{3}}$

$DB\sqrt{3} = DB + 1.6$

$DB(\sqrt{3} - 1) = 1.6$

$DB = \frac{1.6}{\sqrt{3}-1}$

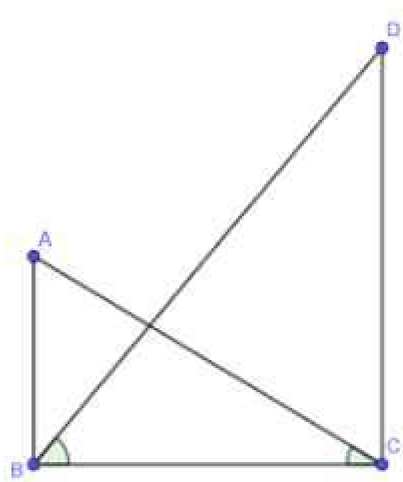
$DB = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$DB = \frac{1.6\times(\sqrt{3}+1)}{3-1}$

$DB = 0.8(\sqrt{3} + 1)$

Hence, height of pedestal is $0.8(\sqrt{3} + 1)m.$

Q. 9 The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.



Let us take AB as building and Let us take DC as tower = 50 m,

∠ACB = 30°

And,

∠ DBC = 60°;

AB = ?

In Δ DCB;

$\tan\theta = \frac{p}{b}$

where, p = perpendicular and b = base

$\tan 60° = \frac{50}{b}$

$\sqrt{3} = \frac{50}{b}$

$b = \frac{50}{\sqrt{3}}$

In Δ ACB

$\tan\theta = \frac{p}{b}$

where p = perpendicular and b = base

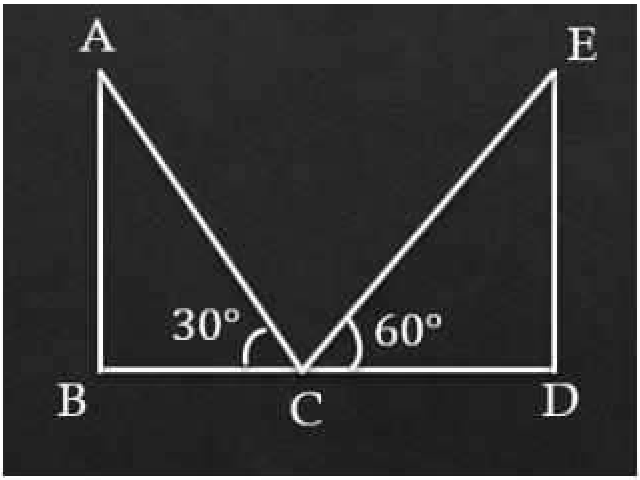
$\tan 30° = \frac{AB}{b}$

$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{50}{\sqrt{3}}}$

$AB = \frac{50}{\sqrt{3}\times\sqrt{3}}$

AB= 16.67 m

Q. 10 Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles



Let AB and DE be the two poles, and C be the point of observation. Given, width of road, BD = 80 m Angle of elevation to AB, ∠ACB = 30°Angle of elevation to DE, ∠ ECD = 60°**To find:** Height of buildings AB and DE.

In Δ ACB

$\tan\theta = \frac{p}{b}$

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BC}$

$AB = \frac{BC}{\sqrt{3}}$ (i)

In ΔEDC

$\tan\theta = \frac{p}{b}$

$\tan 60° = \frac{ED}{CD}$

$\sqrt{3} = \frac{ED}{80-BC}$

$ED = \sqrt{3}(80 - BC)$ (ii)

We know that AB = ED as the poles are of same height.

Hence, from (i) and (ii),

$\frac{BC}{\sqrt{3}} = \sqrt{3}(80 - BC)$

cross multiplying, we get

BC = 3 (80 - BC)

BC = 240 – 3BC

4 BC = 240

BC = 60m

Now using the value of BC in (i),

AB = $20\sqrt{3}\ m$

tan 60° $= \frac{ED}{CD}$

Now from triangle EDC, $\sqrt{3} = \frac{20\sqrt{3}}{CD}$

CD = 20 m

Therefore, Height of Pole = 20√3 m. Distances of poles from observing point = 60 m and 20 m

Q. 11 A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see Fig. 9.12). Find the height of the tower and the width of the canal

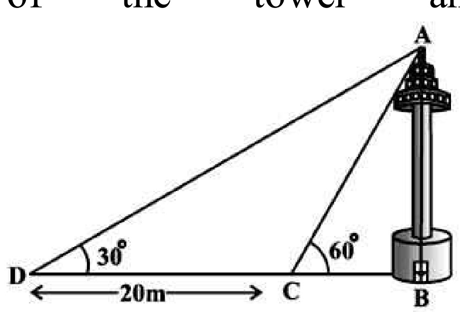
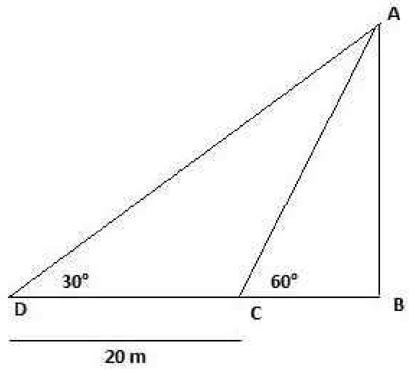


Fig. 9.12

Answer:



According to this figure:

CD = 20 m,

$\angle ACB = 60°$,

$\angle ADB = 30°$,

AB =? and BD =?

In Δ ABC,

$\tan\theta = \frac{p}{b}$

where p = perpendicular and b = base of triangle

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC}$

$AB = \sqrt{3}BC$ (i)

In Δ ABD,

$\tan\theta = \frac{p}{b}$

$\tan 30° = \frac{AB}{BC+20}$

$\frac{BC+20}{\sqrt{3}} = AB$ (ii)

From (i) and (ii),

$\sqrt{3}(BC) = \frac{BC+20}{\sqrt{3}}$

3BC = BC + 20

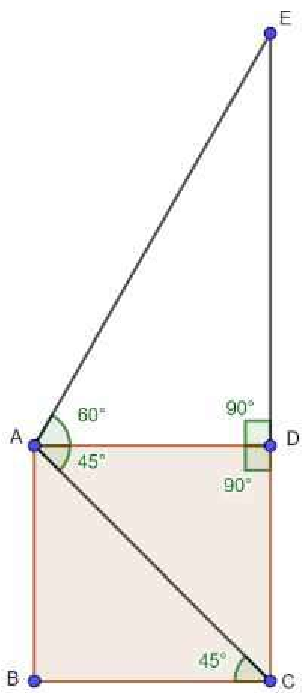
BC = 10 m

Using the value of BC in (i) we get;

AB = 10√3 m which is the height of the tower

Width of canal = 10 m

Q. 12 From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.



Let us take AB as a building, with AB = 7 mand CE as a cable tower. Angle of elevation from top of tree to top of cable tower∠EAD = 60°,and angle of depression from the top of the building to the bottom of the tower is, ∠CAD = 45°Also, ∠CAD = ∠ACB [Alternate angles]

Clearly,
AB = DC and EC = ED + DC?

In Δ ABC:

$\tan\theta = \frac{p}{b}$

where p = perpendicular and b = base, therefore $\tan 45° = \frac{AB}{BC}$

AB = BC = 7 m

In Δ EDA,

$\tan\theta = \frac{p}{q}$

$\tan 60° = \frac{ED}{7}$

$\sqrt{3} = \frac{ED}{7}$

$ED = 7\sqrt{3}m$

However,

The height of tower can be calculated as:

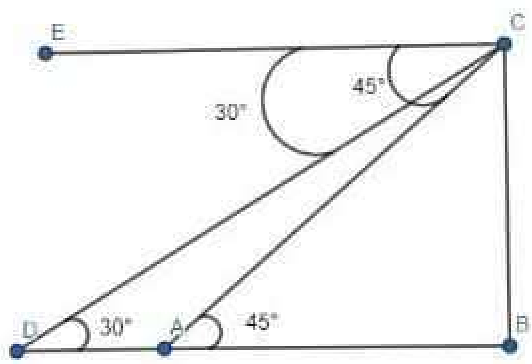
EC = ED + DC

$= 7\sqrt{3} + 7$

$= 7(\sqrt{3} + 1)$ m

Q. 13 As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer:



A and D are the two ships and the distance between the ships is AD. BC is the lighthouse and is the height of the lighthouse. observer is at point C

Give: BC = height of lighthouse = 75m

∠CAB = 45°,

∠CDB = 30°

To Find: DA

In Δ ABC,

$\tan\theta = \frac{p}{b}$

[p = perpendicular and b = base of the right angled triangle]

$\tan(45°) = \frac{75}{AB}$

AB = 75 m $\quad [\tan 45° = 1]$

In Δ CDB,

$\tan\theta = \frac{p}{b}$

$\tan 30° = \frac{75}{BD}$

$\frac{1}{\sqrt{3}} = \frac{75}{BD}$

$BD = 75\sqrt{3}m$

$AD + AB = 75\sqrt{3}$ m

$DA = 75\sqrt{3} - 75$

$DA = 75\left(\sqrt{3} - 1\right)m$

Hence the distance between the ships is $75\left(\sqrt{3} - 1\right)m$ m

Q. 14 A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°.

After some time, the angle of elevation reduces to 30° (see Fig. 9.13).Find the distance travelled by the balloon during the interval.

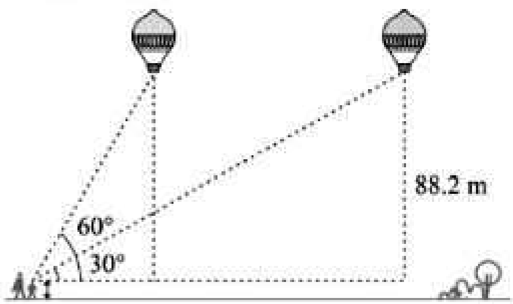


Fig. 9.13

Answer:

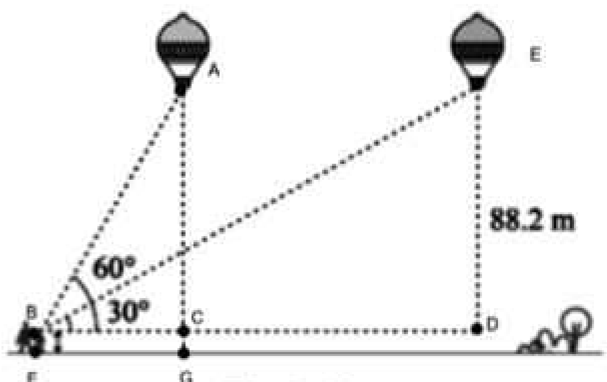


Fig. 9.13

From this figure:

BF = height of girl = 1.2 m,

AG = height of balloon from ground initially = 88.2 m,

AC = AG – GC

= 88.2 – 1.2

= 87 m

∠EBD = 30° and ∠ABC = 60°,

CD =?

In Δ ABC;

$\tan\theta = \frac{p}{b}$

where p = perpendicular and b is base

$\tan 60° = \frac{87}{BC}$

$\sqrt{3} = \frac{87}{BC}$

$BC = \frac{87}{\sqrt{3}}$

In ΔABC

$\tan \theta = \frac{p}{b}$

$\tan 30° = \frac{87}{BD}$

$\frac{1}{\sqrt{3}} = \frac{87}{BD}$

$BD = 87\sqrt{3}$

Now, the distance covered by the balloon will be:

CD = BD – BC

$CD = 87\sqrt{3} - \frac{87}{\sqrt{3}}$

$CD = \frac{87 \times 3 - 87}{\sqrt{3}}$

$CD = \frac{174}{\sqrt{3}}$

Now for rationalizing, multiply and divide by $\sqrt{3}$

$CD = \frac{174}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

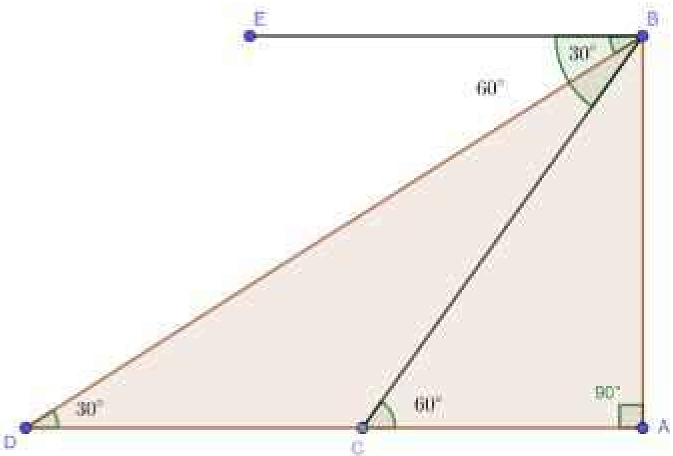
$CD = \frac{174}{3} \times \sqrt{3}$

$CD = 58\sqrt{3}$

$\boldsymbol{CD = 58\sqrt{3}}$ **m**

Q. 15 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer: The diagram is:



Let AB is the tower and AD is the highway.

Now from triangle ADB,

$\tan 30° = AB / AD$

$\Rightarrow 1/\sqrt{3} = AB/AD$

$\Rightarrow AB = AD/\sqrt{3}$ ....(1)

Again from triangle ACB

$\tan 60° = AB/AC$

$\Rightarrow \sqrt{3} = AB/AC$

$\Rightarrow AB = AC\sqrt{3}$ ....(2)

from equation 1 and 2

$AD/\sqrt{3} = AC\sqrt{3}$

$\Rightarrow (DC + CA)/\sqrt{3} = AC\sqrt{3}$

$\Rightarrow DC + CA = AC\sqrt{3}\times\sqrt{3}$

$\Rightarrow DC + CA = 3AC$

$\Rightarrow 3AC - Ac = CD$

$\Rightarrow$ 2AC = CD

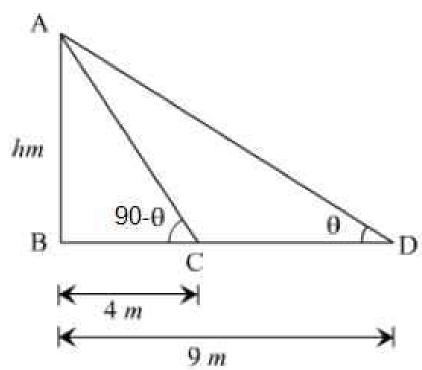
$\Rightarrow$ AC = CD/2

Since time taken by car to cover CD = 6 Second

So time taken by car to cover AC = 6/2 = 3 seconds.

Q. 16 The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m

Answer:



Let ∠ADB = θ and ∠ACB = 90°-θ [As angles are complementary, their sum will be equal to 90°, and if one is θ other will be 90°-θ] In Δ ABD;

$$\tan\theta = \frac{AB}{DB}$$

$$\tan\theta = \frac{AB}{9}$$

In ΔABC

$$tan\ (90-\theta) = \frac{AB}{BC}$$

$$cot\ \theta = \frac{AB}{4}$$

But we know that,

$$\cot\theta = \frac{1}{\tan\theta}$$

And also by complementary angle formula that $\tan(90°-\theta) = \cot\theta$
$\tan(\theta)\tan(90°-\theta) = 1$

$$\tan(90° - \theta) = \frac{1}{\tan\theta}$$

Therefore,

$$\frac{AB}{4} = \frac{9}{AB}$$

$AB^2 = 36$

$AB = 6$ m

Hence proved.