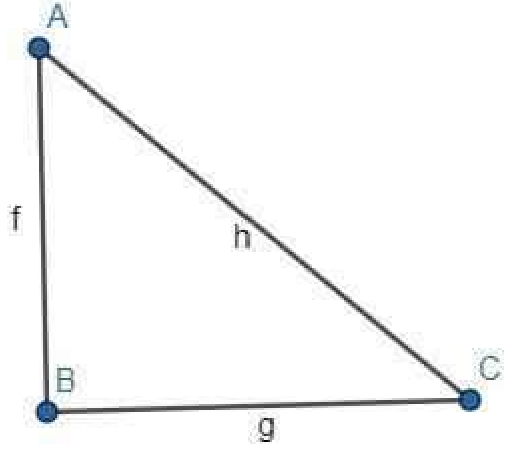
Chapter – 8
Introduction to Trigonometry

Q. 1 In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) Sin A, Cos A
(ii) Sin C, Cos C

Answer:



Given: AB = 24 cm, BC = 7 cm

Pythagoras theorem: the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. Applying Pythagoras theorem for ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

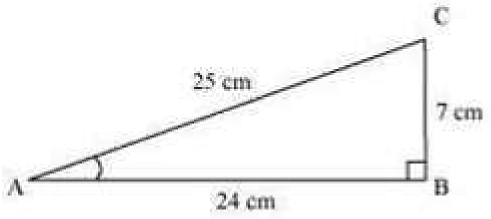
$= (24)^2 + (7)^2$

$= (576 + 49)$

$= 625 \text{ cm}^2$

$\therefore AC = \sqrt{625}$

$AC = 25$ cm



$$\sin\theta = \frac{Perpendicular}{Hypotenuse}$$

$$\cos\theta = \frac{Base}{Hypotenuse}$$

$$\tan\theta = \frac{Perpendicular}{Base}$$

To determine when a line is perpendicular and when its a base for a particular angle, first look for the angle for which you are calculating the trigonometric ratio. The side opposite to which you are calculating angle is called perpendicular, side opposite to right angle is hypotenuse and side left is called base. For example if you have to calculate sin A, then your side opposite to A will be perpendicular and side left other than hypotenuse is base.

(i) $Sin A = \frac{BC}{AC}$

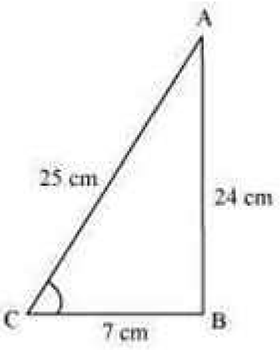
$Sin A = \frac{7}{25}$

$Cos\ A = \frac{AB}{AC}$

$Cos\ A = \frac{24}{25}$

(ii) $Sin C = \frac{AB}{AC}$

$= \frac{24}{25}$



$$Cos\ C = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

Q. 2 In Fig. 8.13, find tan P – cot R.

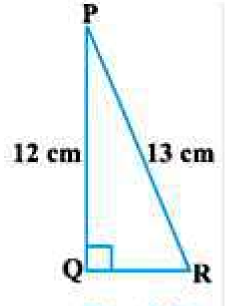
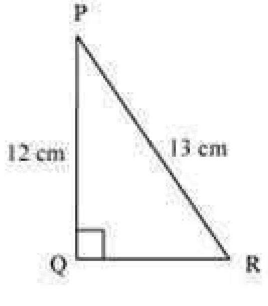


Fig. 8.13

Answer: Applying Pythagoras theorem for ΔPQR, we obtain



Pythagoras Theorem: the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

$PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 \text{ cm}^2 = 144 \text{ cm}^2 + QR^2$

$25 \text{ cm}^2 = QR^2$

$QR = 5 \text{ cm}$

$\tan\theta = \frac{Perpendicular}{Base}$

$\tan P = \frac{QR}{PQ}$

$= \frac{5}{12}$

$\cot\theta = \frac{Base}{Perpendicular}$

$\cot R = \frac{QR}{PQ}$

$= \frac{5}{12}$

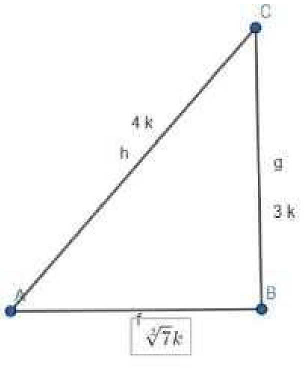
$\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$

$= 0$

$\tan P - \cot R = 0$

Q. 3 If $\sin A = \frac{3}{4}$ calculate Cos A and tan A

Answer: Let ΔABC be a right-angled triangle, right-angled at point B



Given that:

$\sin A = \frac{3}{4}$

We know that $\sin A = \frac{Perpendicular}{Hypotenuse}$

$\frac{BC}{AC} = \frac{3}{4}$

As we know trigonometrical ratios give ratio between sides of right angled triangle instead of their actual values. So from sin A we get the ratio of Perpendicular and Hypotenuse of the triangle. Now as we don't know the exact value let Perpendicular = 3 k and Hypotenuse = 4 k

According to Pythagoras theorem: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

$AB = \sqrt{7}\ k$

As we know $\cos A = \frac{Base}{Hypotenuse}$. So,

$Cos\ A = \frac{AB}{AC}$

$= \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

And we know that

$\tan A = \frac{Perpendicular}{Base}$

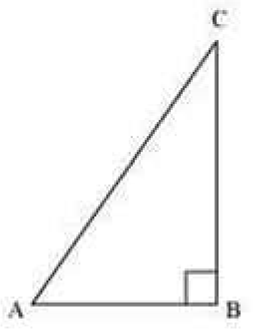
$Tan\ A = \frac{BC}{AB}$

$= \frac{3k}{\sqrt{7}k} = \frac{3k}{\sqrt{7}}$

So the ratios are cos A = √7/4 and tan A = 3/√7

Q. 4 Given 15 cot A = 8, find sin A and sec A

Consider a right-angled triangle, right-angled at B



Given: 15 cot A = 8

To find: sin A and sec A

$\cot A = \frac{Base}{Perpendicular}$

$Cot\ A = \frac{AB}{BC}$

It is given that,

$Cot\ A = \frac{8}{15}$

Let AB be $8k$. Therefore, BC will be $15k$, where k is a positive integer.

Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

$AC = 17k$

Now we know that,

$\sin A = \frac{Perpendicular}{Hypotenuse}$

$Si\,nA = \frac{BC}{AC}$

$= \frac{15\,k}{17k} = \frac{15}{17}$

And also,

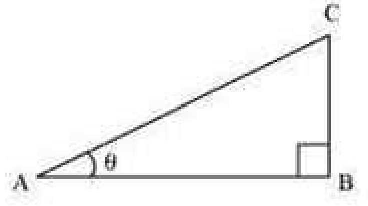
$\sec A = \frac{Hypotenuse}{Base}$

$Sec\ A = \frac{AC}{AB}$

$= \frac{17}{8}$

Q. 5 Given $\sec\theta = \frac{13}{12}$calculate all other trigonometric ratios

Answer: Consider a right-angle triangle ΔABC, right-angled at point B



$sec\ \theta = \frac{Hypotenuse}{Base}$

$\sec\theta = \frac{AC}{AB}$

$\frac{13}{12} = \frac{AC}{AB}$

If AC is $13k$, AB will be $12k$, where k is a positive integer.

Now according to Pythagoras theorem,
The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

Applying Pythagoras theorem in ΔABC, we obtain

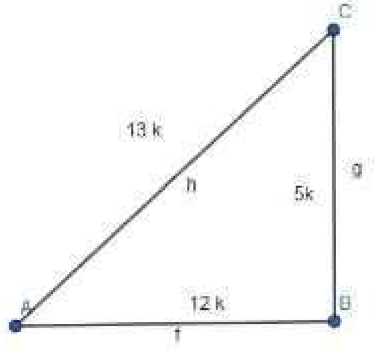
$(AC)^2 = (AB)^2 + (BC)^2$

$(13\ k)^2 = (12\ k)^2 + (BC)^2$

$169\ k^2 = 144\ k^2 + BC^2$

$25\ k^2 = BC^2$

$BC = 5\ k$



$$\sin\theta = \frac{Perpendicular}{Hypotenuse}$$

$$\text{Sin}\ \theta = \frac{BC}{AC}$$

$$= \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{Base}{Hypotenuse}$$

$$\cos\theta = \frac{AB}{AC}$$

$$\frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{Perpendicular}{Base}$$

$$Tan\ \theta = \frac{BC}{AB}$$

$$= \frac{5k}{12k} = \frac{5}{12}$$

$\cot\theta = \frac{Base}{Perpendicular}$

$\text{Cot}\,\theta = \frac{AB}{BC}$

$= \frac{12k}{5k} = \frac{12}{5}$

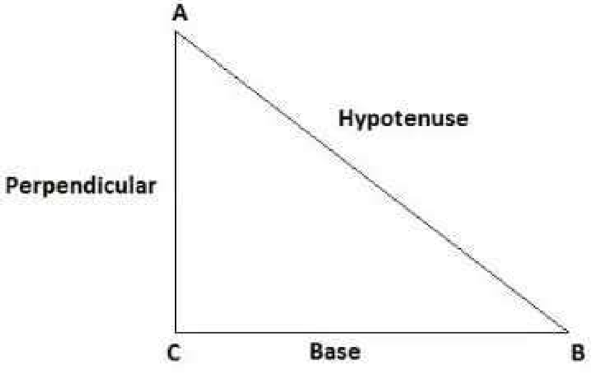
$cosec\,\theta = \frac{Hypotenuse}{Perpendicular}$

$Cosec\ \theta = \frac{AC}{BC}$

$= \frac{13k}{5k} = \frac{13}{5}$

Q. 6 If ∠ A and ∠ B are acute angles such that Cos A = Cos B, then show that ∠A = ∠B.

Answer:



To Prove: ∠ A = ∠ B

Given: cos A = cos B

Let there be a right angled triangle ABC, right angled at C.Now we know that the side opposite to the angle of which we are taking trigonometric ratio is perpendicular, side opposite to right angle is hypotenuse and the side left is base. So, now let cos of A and B We

Know that, $\cos\theta = \frac{Base}{Hypotenuse}$

$\cos A = \frac{AC}{AB}$

Since for angle A, BC is perpendicular and AC is the base. Now,

$$Cos\ B = \frac{BC}{AB}$$

Since for angle B, BC is base and AC is perpendicular.

Given: cos A = cos B Therefore, AC = BC. Now, the angles opposite to the equal sides are also equal.

Therefore, ∠A = ∠B

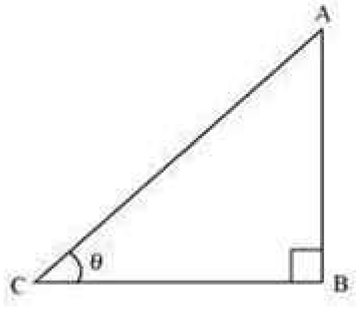
Hence, Proved.

Q. 7 If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $cot^2\theta$

Answer: Let us consider a right triangle ABC, right-angled at point B



Given:

$$\cot\theta = \frac{Base}{Perpendicular}$$

$$\cot\theta = \frac{7}{8}$$

As, a trigonometric Ratio shows the ratio between different sides, cot also shows the ratio of Base and Perpendicular. As we don't know the absolute value of Base and Perpendicular, let the base and perpendicular be multiplied by a common term, let it be k Therefore, Base = 7 k Perpendicular = 8kAccording to pythagoras theorem,$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$AC = \sqrt{113}\,k$

Now we have all the three sides of the triangle,
Base = 7 k Perpendicular = 8 k Hypotenuse = $\sqrt{113}$ k

Now applying other trigonometric angle.

Formulas $\sin\theta = \frac{Perpendicular}{Hypotenuse}$

$\text{Sin}\,\theta = \frac{AB}{AC}$

$= \frac{8k}{\sqrt{113}k}$

$= \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{Base}{Hypotenuse}$

$\text{Cos}\,\theta = \frac{BC}{AC}$

$= \frac{7k}{\sqrt{113}k}$

$= \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Putting the obtained trigonometric ratios into the expression we get, $= (1 - \sin^2\theta)/(1 - \cos^2\theta)$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = cot^2\theta$$

$$[1 - sin^2\theta = cos^2\theta, 1 - cos^2\theta = sin^2\theta]$$

= 49/64

ii) $Cot^2\ \theta = (\cot\theta)^2$

$$cot^2\theta = \left(\frac{7}{8}\right)^2$$

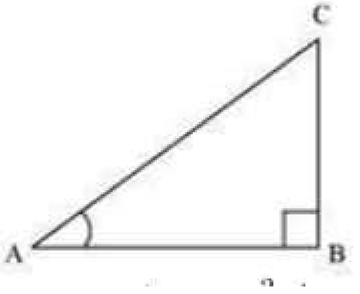
$$cot^2\theta = \frac{49}{64}$$

Q. 8 If 3 cot A = 4, check whether $\frac{1-tan^2A}{1+tan^2A} = cos^2A - sin^2A$ or not

Answer: It is given that 3 cot A = 4

$$cot\,A = \frac{4}{3}$$

Consider a right triangle ABC, right-angled at point B



To find: $\frac{1-tan^2A}{1+tan^2A} = Cos^2A - Sin^2A$ or not

$$\cot A = \frac{Base}{Perpendicular}$$

$$Cot\ A = \frac{AB}{BC}$$

$$= \frac{4}{3}$$

This fraction shows that if the length of base will be 4 then the altitude will be 3. And base and altitude will both increase with this proportion only. Let AB be 4k then BC will be 3k

In ΔABC,
Pythagoras theorem : It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. By pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$AC = 5k$

Now,

$\cos A = \frac{base}{hypotenuse}$

$Cos\ A = \frac{AB}{AC}$

$= \frac{4k}{5k}$

$= \frac{4}{5}$

And also we know that $\sin A = \frac{Perpendi\ cular}{Hypotenuse}$

$Si\ nA = \frac{BC}{AC}$

$= \frac{3k}{5k}$

$= \frac{3}{5}$

And,

$\tan A = \frac{Perpendi\ cular}{Base}$

$Tan\ A = \frac{BC}{AB}$

$= \frac{3k}{4k}$

$= \frac{3}{4}$

Now putting the values of so obtained we get,

$$\frac{1-tan^2A}{1+tan^2A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}}$$

$$Cos^2A - Sin^2A = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

Therefore,

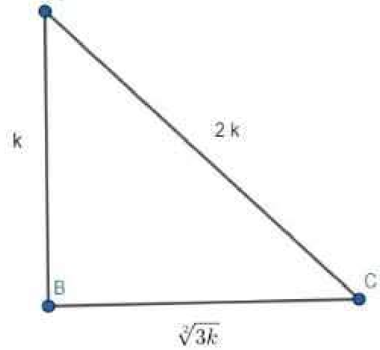
$$\frac{1-tan^2A}{1+tan^2A} = Cos^2A - \ Sin^2A$$

Hence, the expression is correct.

Q.9 In triangle ABC, right-angled at B, if $tan\ A = \frac{1}{\sqrt{3}}$, find the value of:

(i) Sin A Cos C + Cos A Sin C
(ii) Cos A Cos C – Sin A Sin C

Answer:



For finding value of sin A cos C + cos A sin C , we need to find values of sin A, cos C, cos A, sin C

$\tan\theta = \frac{Perpendicular}{Base}$

$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

If BC is k, then AB will be, $\sqrt{3}k$ where k is a positive integer

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= (\sqrt{3}k)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

$AC = 2k$

For finding the perpendicular and base for an angle of a right angled triangle, always take the side opposite to the angle as perpendicular and the other side as base

We know that, $\sin\theta = \frac{Perpendicular}{Hypotenuse}$

Now, for angle A, Perpendicular will be side opposite to angle A that is BC and Hypotenuse will be AC

$Si\,nA = \frac{BC}{AC}$

$= \frac{k}{2k} = \frac{1}{2}$

And also by cosine formula, $\cos\theta = \frac{base}{Hypotenuse}$

For angle A, base will be AB and hypotenuse will be AC

$\text{Cos}\,A = \frac{AB}{AC}$

$= \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{Perpendicular}{Hypotenuse}$

For angle C, Perpendicular will be AB and hypotenuse will be AC

$\sin C = \frac{AB}{AC}$

$= \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{base}{Hypotenuse}$

For angle C, base will be equal to BC and hypotenuse equal to AC

$Cos\ C = \frac{BC}{AC}$

$= \frac{k}{2k} = \frac{1}{2}$

(i) $si\,nA\,c\,os\,C + c\,os\,A\,si\,nC$

$= \left(\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4} = 1$

(ii) $c\,os\,A\,c\,os\,C - si\,nA\,si\,nC$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

= 0

Q. 10 In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of Sin P, Cos P and tan P**.**

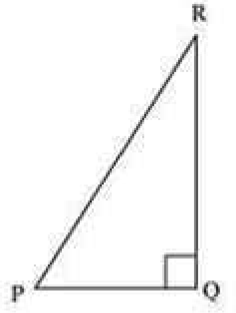
Answer: Given that, PR + QR = 25

PQ = 5

Let PR be x

Therefore,

QR = 25 – x



Pythagoras Theorem: the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$ *[as, $(a + b)^2 = a^2 + b^2 + 2ab$]*

$50x = 650$

$x = 13$

Therefore,

PR = 13 cm

QR = (25 - 13) cm = 12 cm

and we know,

$$\sin\theta = \frac{Perpendicular}{Hypotenuse}$$

$$\cos\theta = \frac{Base}{Hypotenuse}$$

$\tan\theta = \frac{Perpendicular}{Base}$

$\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$

Q. 11 State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1

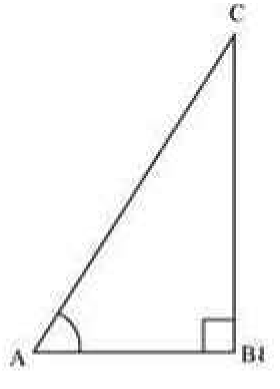
(ii) Sec$A = \frac{12}{5}$for some value of angle A

(iii) Cos A is the abbreviation used for the cosecant of angle A

(iv) Cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$for some angle θ.

Answer: (i) Consider a ΔABC, right-angled at B



$Tan\ A = \frac{12}{5}$

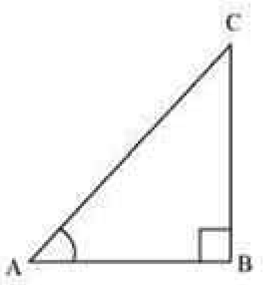
But $\frac{12}{5} > 1$

Tan A > 1

So, Tan A < 1 is not always true

Hence, the given statement is false

(ii) $Sec\ A = \frac{12}{5}$



Let AC be $12k$, AB will be $5k$, where k is a positive integer

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides $AC = 12k$ and $AB = 5k$,

BC should be such that,

$AC - AB < BC < AC + AB$

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\ k$

However, $BC = 10.9k$. Clearly, such a triangle is possible and hence, such value of sec A is possible

Hence, the given statement is true

(iii) Abbreviation used for cosecant of angle A is cosec A. And Cos A is the abbreviation used for cosine of angle A

Hence, the given statement is false

(iv) Cot A is not the product of cot and A. It is the cotangent of ∠A

Hence, the given statement is false

(v) $\sin \theta = \frac{4}{3}$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible

Hence, the given statement is false

Exercise 8.2

Q. 1 Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2tan^2 45° + cos^2 30° - sin^2 60°$

(iii) $\frac{\cos 45°}{\sec 30° + cosec\ 30°}$

(iv) $\frac{\sin 30° + \tan 45° - cosec\ 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\frac{5\ cos^2 60° + 4 sec^2 30° - tan^2 45°}{sin^2 30° + cos^2 30°}$

	0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
cosec	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
cot	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Given is the table of Trigonometrical ratios for different values. Use the values from table for solving the questions.

(i) sin60° cos30° + sin30° cos 60°

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$= 2 + 3/4 - 3/4$

$= 2$

(iii) $\dfrac{\cos(45°)}{\sec(30°)+\text{cosec}(30°)}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{2\left(\frac{1}{\sqrt{3}}+1\right)}$$

$$= \frac{1}{2\sqrt{2}\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}$$

As the denominator has an irrational number, we need to rationalize the fraction

$$= \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} \times \frac{\sqrt{2}(1-\sqrt{3})}{\sqrt{2}(1-\sqrt{3})}$$

$$= \frac{\sqrt{2}(\sqrt{3}-3)}{2(2)(1^2-(\sqrt{3})^2)}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{4(1-3)}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{4(-2)}$$

$$= \frac{\sqrt{6}-3\sqrt{2}}{-8}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30°+\tan 45°-\text{cosec}\ 60°}{\sec 30°+\cos 60°+\cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$$

$$= \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times = \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-(4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16}$$

$$= \frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

$$= \frac{5\times\left(\frac{1}{2}\right)^2+4\times\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\times\frac{1}{4}+4\times\frac{4}{3}-(1)^2}{\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{5}{4}+\frac{16}{3}-1$$

$$= \frac{5\times3+16\times4-12}{12}$$

$$= \frac{15+64-12}{12}$$

$$= \frac{67}{12}$$

Q. 2 A Choose the correct option and justify your choice:

$\frac{2\tan 30°}{1+\tan^2 30°}$

A. sin 60°
B. cos 60°
C. tan 60°
D. sin 30°

Answer: We know that

$\tan 30° = \frac{1}{\sqrt{3}}$

$= \frac{2\tan 30°}{1+tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$\frac{2\tan 30°}{1+tan^2 30°} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$

Putting values in given equation, $\frac{2\tan 30°}{1+tan^2 30°} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

$\frac{2\tan 30°}{1+tan^2 30°} = \frac{2}{\sqrt{3}} \times \frac{3}{4}$

$= \frac{2\tan 30°}{1+tan^2 30°} = \frac{\sqrt{3}}{2}$

[Since, $\sqrt{3} \times \sqrt{3} = 3$]

in options we have,

sin 60° = $\sqrt{3}/2$

cos 60° = $^1/_2$

tan 60° = $\sqrt{3}$

sin 30° = $^1/_2$

Only (A) is correct.

Q. 2 B Choose the correct option and justify your choice:

$\frac{1-tan^2 45°}{1+tan^2 45°}$

A. tan 90°
B. 1
C. sin 45°
D. 0

Answer: $\frac{1-1*1}{1+1*1}$

$= \frac{1-1}{1+1}$

$= \frac{0}{2}$

= 0

Hence, (D) is correct.

Q. 2 C Choose the correct option and justify your choice:

Sin 2A = 2 sin A is true when A =
A. 0°

B. 30°
C. 45°
D. 60°

Answer: Out of the given alternatives, only A = 0° is correct

As

sin 2A = sin 0° = 0

2 sin A = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

Q. 2 (D) Choose the correct option and justify your choice:

$\frac{2\ tan\ 30°}{1-tan^2 30°}$

A. Cos 60°

B. sin 60°

C. tan 60°

D. sin 30°

Answer: To find: $\frac{2\ tan\ 30°}{1-tan^2 30°}$

$\frac{2\ tan\ 30°}{1-tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ [$\tan 30° = \frac{1}{\sqrt{3}}$]

$\frac{2\ tan\ 30°}{1-tan^2 30°} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}}$

$\frac{2\ tan\ 30°}{1-tan^2 30°} = \sqrt{3}$

$(A) \cos 60° = \frac{1}{2}$

$(B) \sin 60° = \frac{\sqrt{3}}{2}$

Now according to options

$(C) \tan 60° = \sqrt{3}$

$(D) \sin 30° = \frac{1}{2}$

Out of the given alternatives, only tan 60° =$\sqrt{3}$

Hence, (C) is correct.

Q. 3 If $\tan(A+B) = \sqrt{3}\ and \tan(A-B) = \frac{1}{\sqrt{3}}; 0° < A < B \leq 90°; A > B$ Find A and B

Now, we know that

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan 60° = \sqrt{3}$

So, from the question we can tell, $\tan (A + B) = \sqrt{3}$

$\tan (A + B) = \tan 60°$

$A + B = 60°$ ....eq (i)

And $\tan (A - B) = \frac{1}{\sqrt{3}}$

$\tan (A - B) = \tan 30°$

$A - B = 30$ ....eq(ii)

On adding both equations, we obtain

$2A = 90°$

$\Rightarrow A = 45$

From equation (i), we obtain

$45 + B = 60$

$B = 15°$

Therefore, $\angle A = 45°$ and $\angle B = 15°$

Q. 4 State whether the following are true or false. Justify your answer

(i) Sin (A + B) = sin A + sin B
(ii) The value of sin θ increases as θ increases
(iii) The value of cos θ increases as θ increases.
(iv) Sin θ = cos θ for all values of θ.
(v) Cot A is not defined for A = 0°

Answer: (i) $\sin (A + B) = \sin A + \sin B$

Let $A = 30°$ and $B = 60°$

$\sin (A + B) = \sin (30° + 60°) = \sin 90°$

$= 1$

$\sin A + \sin B = \sin 30° + \sin 60°$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2}$

Clearly, sin (A + B) ≠sin A + sin B

Hence, the given statement is false

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as sin 0° = 0

sin 30° = $\frac{1}{2} = 0.5$

sin 45° = $\frac{1}{\sqrt{2}}$ = 0.707

sin 60° = $\frac{\sqrt{3}}{2}$= 0.886

sin 90° = 1

Clearly the value increases.
Hence, the given statement is true

(iii) cos 0° = 1

cos 30° = $\frac{\sqrt{3}}{2}$= 0.886

cos 45° = $\frac{1}{\sqrt{2}}$ = 0.707

cos 60° = $\frac{1}{2} = 0.5$

cos 90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°

Hence, the given statement is false

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As,

$\sin 45^{o} = \frac{1}{\sqrt{2}}$

$\cos 45^{o} = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ

As $\sin 30^{o} = \frac{1}{2}$ and $\cos 30^{o} = \frac{\sqrt{3}}{2}$

Hence, the given statement is false

(v) cot A is not defined for A = 0°

$\cot A = \frac{\cos A}{\sin A}$

$\cot 0^{o} = \frac{\cos 0}{\sin 0}$

$= \frac{1}{0}$ = undefined

Hence, the given statement is true

Exercise 8.3

Q. 1 Evaluate:

(i) $\frac{\sin 18°}{\cos 72°}$

(ii) $\frac{\tan 26°}{\cot 64°}$

(iii) Cos 48° – sin 42°

(iv) Cosec 31° – sec 59°

Answer: Use formulae

$\sin A = \cos(90° - A)$

$\tan A = \cos(90° - A)$

$\operatorname{cosec} A = \sec(90° - A)$

(i) $\frac{\sin 18°}{\cos 72°}$

$(18° = 90° - 72°)$

$= \frac{Sin(90°-72°)}{\text{Cos } 72°}$

$= \frac{\text{Cos } 72°}{\text{Cos } 72°}$

= 1

(ii) $\frac{\tan 26°}{\cot 64°}$

$= \frac{\tan(90°-64°)}{\cot 64°}$

$= \frac{\cot 64°}{\cot 64°}$

=1

(iii) cos 48° - sin 42°

= cos (90° - 42°) – sin 42°

= sin 42° - sin 42°

$= 0$

(iv) $\text{cosec } 31^o - \sec 59^o$

$= \text{cosec}(90^o - 59^o) – \sec 59^o$

$= \sec 59^o - \sec 59^o$

$= 0$

Q. 2 Show that:
(i) $\tan 48° \tan 23° \tan 42° \tan 67° = 1$
(ii) $\cos 38° \text{ Cos } 52° – \sin 38° \sin 52° = 0$

Answer: (i) $\tan 48^o \tan 23^o \tan 42^o \tan 67^o = 1$

LHS: $\tan 48^o \tan 23^o \tan 42^o \tan 67^o$

$= \tan (90^o - 42^o) \tan 23^o \tan 42^o \tan (90^o - 23^o)$
As $\tan(90 - \theta) = \cot \theta$

$= \cot 42^o \tan 23^o \tan 42^o \cot 23^o$

$= \cot 42^o \tan 42^o \tan 23^o \cot 23^o$
As, $\tan\theta \cot\theta = 1$, because $\tan \theta = \frac{1}{\cot \theta}$
$= 1 \times 1$

$= 1 = \text{RHS}$

Hence $\tan 48^o \tan 23^o \tan 42^o \tan 67^o = 1$, Proved

(ii) $\cos 38^o \cos 52^o - \sin 38^o \sin 52^o = 0$

LHS= $\cos(90^o - 52^o) \cos(90^o - 38^o) - \sin 38^o \sin 52^o$

As we know,
$\cos(90 - \theta) = \sin \theta = \sin 38^o \sin 52^o - \sin 38^o \sin 52^o$

$= 0 = \text{RHS}$

Hence $\cos 38^o \cos 52^o - \sin 38^o \sin 52^o = 0$, proved

Q. 3 If $\tan 2A = \cot (A – 18°)$, where $2A$ is an acute angle, find the value of A.

Answer: Given: tan 2A = cot(A - 18)
By complementary angle formula: cot(90 - A) = tan A
From the question, tan 2 A = cot (90 - 2A)Therefore,

cot(90 - 2A) = cot(A - 18)Now both the angles will be equal, so

$90° – 2A = A – 18°$

$108° – 2A = A$

$3A = 108°$

$A = 108/3$
$= 36°$**A = 36°**

Q. 4 If tan A = cot B, prove that A + B = 90°

Answer: To Prove: A + B = 90°
Formula to use = [tan (90° - A) = cot A] and [cot(90° - A) = tan A]
Proof:
tan A = cot (90°- A) (by complementary angles formula)tan A = cot B

cot (90°- A) = cot B [As, cot(90° - A) = tan A]
Therefore,

$90° - A = B$

$A + B = 90°$

Hence proved

Q. 5 If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A

Answer: sec 4A = cosec (90° - 4A) = cosec (A - 20°)

Therefore,

$90° – 4A = A - 20°$

$110°– 4A = A$

$5A = 110°$

A = 22°

Q. 6 If A, B and C are interior angles of a triangle ABC, then show that

$$sin\left(\frac{B+C}{2}\right) = cos\,\frac{A}{2}$$

Answer: To Prove

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Proof:

We know, By angle sum property of a triangle

∠A + ∠B + ∠C = 180°

⇒ B + C = 180°- A

Taking sin on both sides of equation

Therefore,

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180-A}{2}\right) = \sin\left(90-\frac{A}{2}\right)$$

$$= \cos\frac{A}{2}$$

As, $\sin(90-\theta)\cos\theta$ therefore, $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Hence proved.

Q. 7 Express sin 67° + Cos 75° in terms of trigonometric ratios of angles between 0° and 45°

Answer: sin 67° + cos 75°

For expressing this into between 0° and 45° we need to know complementary angle formulas:

sin(90° - A) = cos A cos(90° - A) = sin A

Hence, Now we can break 67° as 90° – 23 °.And 75° as 90° – 15°.So,sin 67° + cos 75° = sin (90° – 23°) + cos (90° – 15°)

sin 67° + cos 75° = cos 23° + sin 15°

(Since, sin(90°- A) = cos A)

Exercise 8.4

Q. 1 Express the trigonometric ratios sin A, sec A and tan A in terms of cot A

Answer: sin A can be expressed in terms of cot A as:

And we know that: $cosec^2$ A - cot^2 A = 1, $cosec^2A=1+cot^2A$,

so cosec A=√(1+cot^2A)

$$\sin A = \frac{1}{cosec\ A}$$

Therefore, $\sin A = \frac{1}{\sqrt{cos^2 A+1}}$

Now, And we know that: $cosec^2$ A - cot^2 A = 1, $cosec^2A=1+cot^2A$,

so cosec A=√(1+cot^2A)

$$\sin A = \frac{1}{cosec\ A}$$

Therefore,

$$\sin A = \frac{1}{\sqrt{cos^2 A+1}}$$

Now, Sec A can be expressed in terms of cot A as:
We know that: sec^2 A - tan^2 A = 1, sec A = √(1 + tan^2 A)

$$\sec A = \sqrt{1 + tan^2 A}$$

$$\sec A = \sqrt{1 + \frac{1}{cot^2 A}}$$

And also, tan A = 1/ cot A

Therefore,

$$\sec A = \sqrt{\frac{1+cot^2 A}{cot^2 A}}$$

$$\sec A = \frac{1}{\cot A}\sqrt{1 + cot^2 A}$$

tanA can be expressed in terms of cotA as:

$\tan A = \frac{1}{\cot A}$

Q. 2 Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

Sin A can be expressed in terms of sec A as:

Sin A = $\sqrt{\sin^2 A}$

sin A = $\sqrt{(1 - \cos^2 A)}$

$$Si\,nA = \sqrt{1 - \frac{1}{sec^2 A}}$$

$$\sin A = \sqrt{\frac{sec^2 A - 1}{sec^2 A}}$$

$$\sin A = \frac{1}{\sec A}\sqrt{sec^2 A - 1}$$

Now,

cos A can be expressed in terms of secA as:

$$\cos A = \frac{1}{secA}\ \frac{1}{\sec A}$$

tanA can be expressed in the form of sec A as:

As, $1 + \tan^2 A = \sec^2 A$

⇒ tan A $= \pm \sqrt{(\sec^2 A - 1)}$

As A is acute angle, And tan A is positive when A is acute,So,tan A = $\sqrt{(\sec^2 A - 1)}$

cosec A can be expressed in the form of sec A as:

$$cosec\, A = \frac{1}{\sin A}$$

$$\frac{1}{\frac{\sec A}{\sqrt{1 - sec^2 A}}}$$

$$= \frac{\sqrt{1 - sec^2 A}}{\sec A}$$

cot A can be expressed in terms of sec A as:

cot A= $\frac{1}{\tan A}$

as, 1 + $\tan^2$A = $\sec^2$A

$\frac{1}{\sqrt{sec^2 A-1}}$

Q. 3 Evaluate:

(i) $\frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°}$

(ii) Sin 25° Cos 65° + Cos 25° Sin 65°

Answer:

(i) To Prove: $\frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°} = 1$

Proof:

L.H.S. $= \frac{sin^2 63°+sin^2 27°}{cos^2 17°+cos^2 73°}$

$= \frac{sin^2 63°+sin^2 (90°-63°)}{cos^2 17°+cos^2 (90°-17°)}$

$(By\ complementary\ angle\ formula\ that, \sin(90 - A = \cos A\ and \cos(90 - A)))$

$= \frac{sin^2 63°+cos^2 63°}{cos^2 17°+sin^2 17°}$

$(By\ sin^2 A + cos^2 A = 1)$

$= \frac{1}{1}$

= 1 = R.H.S

L.H.S = R.H.S.

Hence, Proved

(ii) sin 25°cos 65°+ cos 25°sin 65°

= sin 25° cos (90°-25°) + cos 25°sin 65°

= sin 25°sin 25° + cos 25°sin 65°= $\sin^2$ 25° + cos 25° sin (90°-25°)

= $\sin^2$ 25° + $\cos^2$ 25°

=1

Q. 4 A Choose the correct option. Justify your choice.

$9sec^2A - 9tan^2A$

A. 1

B. 9

C. 8

D. 0

Answer: Following is the explanation:

9 $\sec^2$ A – 9 $\tan^2$ A

= 9 ($\sec^2$ A – $\tan^2$ A)

= 9 × 1 = 9

Q. 4 (B) Choose the correct option. Justify your choice.

$(1 + tan\ \theta + sec\ \theta)(1 + cot\,\theta - cosec\,\theta) =$

A. 0

B. 1

C. 2

D. –1

Answer: Consider $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - cosec\,\theta)$

By applying formulae

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, cosec\,\theta = \frac{1}{\sin\theta}$$

$$\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{1+\cos\theta+\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

Multiplying both terms, we get

$$= \frac{\sin\theta+\sin\theta\cos\theta+\sin^2\theta+\cos\theta+\cos^2\theta+\sin\theta\cos\theta-1-\cos\theta-s}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$$

$$= \frac{1+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

= 2

Therefore, $(1 + tan\theta + sec\theta)(1 + cot\theta - cosec\theta) = 2$

Q. 4 C Choose the correct option. Justify your choice.

(Sec A + tan A) (1 – sin A) =

A. sec A

B. sin A

C. cosec A

D. Cos A

Answer: (Sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{cos\,A}+\frac{sin\,A}{cos\,A}\right)(1-sin\,A)$$

$$= \frac{(1+\sin A)(1-\sin A)}{\cos\theta}$$

$= (1 - \sin^2 A)/\cos A$

$= \cos^2 A/\cos A$

$= \cos A$

Q. 4 D Choose the correct option. Justify your choice.

$\frac{1+tan^2 A}{1+cot^2 A} =$

A. $\sec^2 A$
B. -1
C. $\cot^2 A$
D. $\tan^2 A$

Answer: We know,

$1 + \tan^2\theta = \sec^2\theta$

and
$1 + \cot^2\theta = \text{cosec}^2\theta$

Therefore,

$$\frac{1+tan^2 A}{1+cot^2 A} = \frac{sec^2 A}{cosec^2 A}$$

$$\frac{sec^2 A}{cosec^2 A} = \frac{\frac{1}{cos^2 A}}{\frac{1}{sin^2 A}} = \frac{sin^2 A}{cos^2 A}$$

$$\frac{sin^2 A}{cos^2 A} = tan^2 A$$

$$\Rightarrow \frac{1+tan^2 A}{1+cot^2 A} = tan^2 A$$

Therefore, option (D) is correct

Q. 5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(cosec\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta cosec\,\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=cosec\,A+\cot A$ using the identity $cosec^2 A=1+cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2sin^3\theta}{2cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+cosec\,A)^2(\cos A+\sec A)^2=7+tan^2 A+cot^2 A$

(ix) $(cosec\,A-\ sinA)(sec\,A-\ cos\,A)\ =\frac{1}{tan\,A+cot\,A}$

[Hint: simplify LHS and RHS separately]

(x) $\left(\frac{1+tan^2 A}{1+cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=tan^2 A$

Answer: To Prove

$(cosec\,\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

Proof: LHS $=(cosec\,\theta-\cot\theta)^2$

Apply formulas: $cosec\,\theta=\frac{1}{\sin\theta},\cot\theta=\frac{\cos\theta}{\sin\theta}$

$\left(\frac{1}{sin\,\theta}-\frac{cos\,\theta}{sin\,\theta}\right)^2$

$\left[\frac{(1-\cos\theta)}{\sin\theta}\right]^2$

Since, $sin^2\theta=1-\ cos^2\theta$

$=\frac{(1-\cos\theta)^2}{1-cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

Hence, proved

(ii) To Prove: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

Proof:

LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

Proof

LHS: $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

Use the identity $sin^2\theta + cos^2\theta = 1$

$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)}$

$= \frac{2+2\sin A}{(1+\sin A)(\cos A)}$

$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)}$

$= \frac{2}{\cos A}$

$= 2\sec A$

= RHS

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta cosec\,\theta$

[Hint: Write the expression in terms of sin θ and Cos θ]

$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{sin^3\theta-cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

Use the formula $a^3 - b^3 = (a^2 + b^2 + ab)(a - b)$

$$\frac{\sin\theta-\cos\theta(sin^2\theta+cos^2\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

Cancelling (sin θ - cos θ) from numerator and denominator

$$= \frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

As cosθ = 1/secθ and sinθ = 1/cosecθ

= 1 + sec θ cosec θ

= RHS

(iv) $\frac{1+\sec A}{\sec A} = \frac{sin^2 A}{1-\cos A}$

[Hint: Simplify LHS and EHS separately]

LHS

$$\frac{1+\sec A}{\sec A}$$

Use the formula $\sec A = 1/\cos A = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

= cos A + 1

RHS

$\frac{sin^2 A}{1-\cos A}$

Use the identity $sin^2\theta + cos^2\theta = 1$

$= \frac{1-cos^2 A}{1-\cos A}$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$= \frac{(1-\cos A)(1+\cos A)}{1-\cos A}$

= cos A +1

LHS= RHS

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing numerator and Denominator by sin A

$= \frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}}$

$= \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$

Use the formula cotθ = cosθ / sinθ $= \frac{\cot A-1+cosec\ A}{\cot A+1-cosec\ A}$

Using the identity $cosec^2 A = 1 + cot^2 A$

$= \frac{\cot A-(cosec^2 A-cot^2 A)+cosec\ A}{\cot A+1-cosec\ A}$

$= \frac{(\cot A+cosec\ A)-(cosec^2 A-cot^2 A)}{\cot A+1-cosec\ A}$

Use the formula $a^2 - b^2 = (a - b)(a + b)$

$$= \frac{(\cot A + cosec\ A) - [(cosec\ A - \cot A)(cosec\ A + \cot A)]}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A)[1 - (cosec\ A - \cot A)]}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A)(1 - cosec\ A + \cot A)}{\cot A + 1 - cosec\ A}$$

= cot A + cosec A

= RHS

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Dividing numerator and denominator of LHS by cos A

$$\sqrt{\frac{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}}$$

$As\ cos\theta = 1/sec\theta\ and\ tan\theta = sin\theta/cos\theta = \sqrt{\frac{\sec A+\tan A}{\sec A-\tan A}}$

Rationalize the square root to get, $= \sqrt{\frac{(\sec + \tan A)\times(\sec A+\tan A)}{(\sec A-\tan A)\times(\sec A+\tan A)}}$

Use the formula $a^2 - b^2 = (a - b)(a + b)$ to get,

$$= \sqrt{\frac{(sec\ A + tan\ A)^2}{(sec^2 A - tan^2 A)}}$$

$$= \frac{\sqrt{(sec\ A + tan\ A)^2}}{\sqrt{sec^2 A - tan^2 A}}$$

Use the identity $\sec^2\theta = 1 + \tan^2\theta$ to get,

$$= \frac{sec\ A + tan\ A}{1}$$

= sec A + tan A

= RHS

(vii) To prove $\frac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta} = \tan\theta$

Proof: LHS

$\frac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2sin^2\theta)}{\cos\theta(2cos^2\theta - \cos\theta)}$

Since $sin^2\theta = 1 - cos^2\theta$

$= \frac{\sin\theta(1-2(1-cos^2\theta))}{\cos\theta(2cos^2\theta - 1)}$

$= \frac{\sin\theta(1-2+2cos^2\theta)}{\cos\theta(2cos^2\theta - 1)}$

$= \frac{\sin\theta(2cos^2\theta - 1)}{\cos\theta(2\ cos^2\theta - 1)}$

As tanθ = sinθ/cosθ= tanθ = R.H.SHence, Proved.**(viii)**
To Prove: $(\sin A + cosec\,A)^2(\cos A + \sec A)^2 = 7 + tan^2 A + cot^2 A$

Proof:

LHS: = (sin A + cosec A)^2 + (cos A + sec A)^2

Use the formula (a+b)^2 = a^2 + b^2 + 2ab to get,

= (sin^{2}A + cosec^{2}A + 2sin A cosec A) + (cos^{2}A + sec^{2}A + 2 cos A sec A)
Since sinθ=1/cosecθ and cosθ = 1/secθ

$= \left(sec^2 A + cosec^2 A + 2\sin A\ \frac{1}{\sin A}\right) + \left(cos^2 A + sec^2 A + 2\cos A \frac{1}{\cos A}\right)$

$= \sin^2 A + cosec^2 A + 2 + \cos^2 A + \sec^2 A + 2$

$=(\sin^2 A + \cos^2 A) + cosec^2 A + \sec^2 A + 2 + 2$

Use the identities $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$ and $cosec^2 A = 1 + \cot^2 A$ to get

$= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2 + 2$

$= 1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A$

$= 7 + \tan^2 A + \cot^2 A$

= RHS

(viii) Not available in ncert solution

(ix) To prove: $(cosec\ A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[Hint: Simplify LHS and EHS separately]

Proof: **LHS** = (cosec A – sin A) (sec A – cos A)

Use the formula $\sin\theta = 1/cosec\theta$ and $\cos\theta = 1/\sec\theta$

$= \left(\frac{1}{si\,nA} - si\,nA\right)\left(\frac{1}{cos\ A} - cos\ A\right)$

$= \frac{(1-si\,n^2 A)}{si\,nA} \times \frac{(1-cos^2 A)}{si\,nA}$

$= \frac{cos^2 A}{si\,nA} \times \frac{si\,n^2 A}{cos\ A}$ $\quad [(1 - si\,n^2 A) = cos^2 A]$

$[(1 - cos^2 A) = cos^2 A]$

$= \cos A \sin A$

RHS

$\frac{1}{tan\ \theta + cot\ \theta}$

use the formula $\tan\theta = \sin\theta/\cos\theta$ and $\cot\theta = \cos\theta/\sin\theta$ $= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\frac{sin^2 A + cos^2 A}{\cos A \sin A}}$

$= \frac{\cos A \sin A}{sin^2 A + cos^2 A}$

Use the identity $\sin^2 A + \cos^2 A = 1$

$= \frac{\cos A \sin A}{1}$

= cos A sin A

LHS = RHS

(x) To Prove: $\left(\frac{1+tan^2 A}{1+cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = tan^2 A$

Proof

Taking left most term Since, cot A is the reciprocal of tan A, we have

$\frac{1+tan^2 A}{1+cot^2 A} = \frac{1+tan^2 A}{1+\frac{1}{tan^2 A}}$

$= \frac{1}{\frac{1}{tan^2 A}}$

$= tan^2 A$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$

= right most part Taking middle part: $= \left(\frac{1}{\frac{-1}{\tan A}}\right)^2$

$= (-\tan A)^2$

$= tan^2 A$

= right most part Hence, Proved.

Testing

Q. 4 Draw the graph of 2x – 3y = 4. From the graph, find whether x = -1, y = -2 is a solution or not. (Final)

Answer: Given equation, 2x – 3y = 4

⇒ 3y = 2x- 4

$y = \frac{2x-4}{3}$

When x=-4, then,

$y = \frac{2x-4}{3}$

$\Rightarrow \frac{2\times(-4)-4}{3}$

$\Rightarrow y = \frac{-8-4}{3}$

$\Rightarrow y = \frac{-12}{3}$

⇒ y = -4

When x=-1, then,

$y = \frac{2x-4}{3}$

$y = \frac{2\times(-1)(-4)}{3}$

$\Rightarrow y = \frac{-2-4}{3}$

$\Rightarrow y = \frac{-6}{3}$

⇒ y = -2

When x=2, then,

$y = \frac{2x-4}{3}$

$\Rightarrow y = \frac{2\times2-4}{3}$

$\Rightarrow y = \frac{4-4}{3}$

$\Rightarrow$ y = 0

When x=5, then,

$y = \frac{2x-4}{3}$

$\Rightarrow y = \frac{2\times5-4}{3}$

$\Rightarrow y = \frac{10-4}{3}$

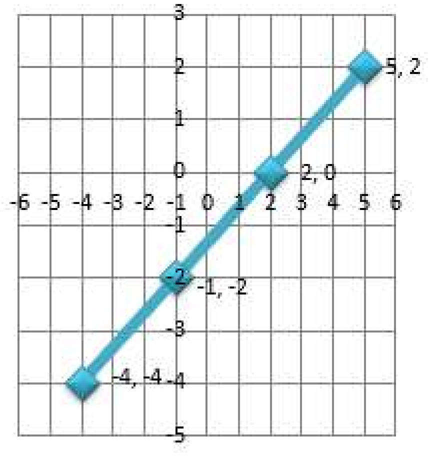
$\Rightarrow y = \frac{6}{3}$

$\Rightarrow$ y = 2

Thus we have the following table,

X	-4	-1	2	5
Y	-4	-2	0	2

On plotting these points we have the following graph,



Clearly, from the graph (-1, -2) is the solution of the line 2x – 3y = 4