Chapter – 7
Coordinate Geometry

Q. 1 Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)

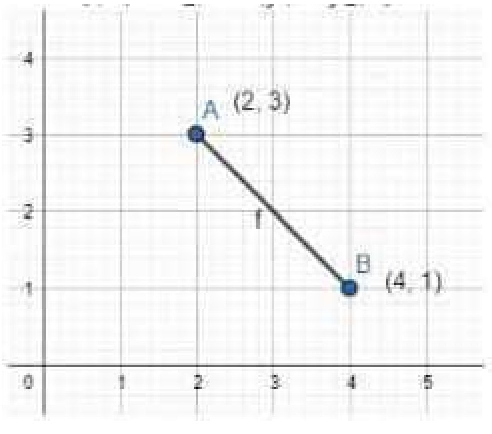
(ii) (– 5, 7), (– 1, 3)

(iii) (a, b), (– a, – b)

Answer:

(i) We know that distance between the two points is given by:

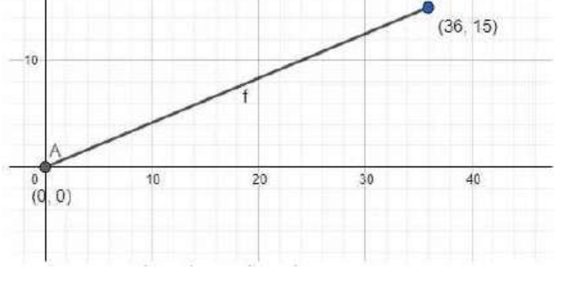
$d = \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$



Q. 2 Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer: To find: Distance between two points

Given: Points A(0, 0), B(36, 15)



For two points $A(x_1, y_1)$ and $B(x_2, y_2)$,
Distance is given by $f = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}$

Distance between (0, 0) and (36, 15) is:

$f = [(36 - 0)^2 + (15 - 0)^2]^{1/2}$

$= [1296 + 225]^{1/2} = (1521)^{1/2}$

$= 39$

Hence Distance between points A and B is 39 units

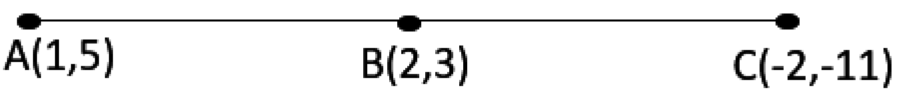
Yes, we can find the distance between the given towns A and B. Let us take town A at origin point (0, 0)

Hence, town B will be at point (36, 15) with respect to town A

And, as calculated above, **the distance between town A and B will be 39 km.**

Q. 3 Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

Answer: Let the points (1, 5), (2, 3), and (- 2,-11) be representing the vertices A, B, and C of the given triangle respectively.
Let A = (1, 5), B = (2, 3) and C = (- 2,-11)Case 1)



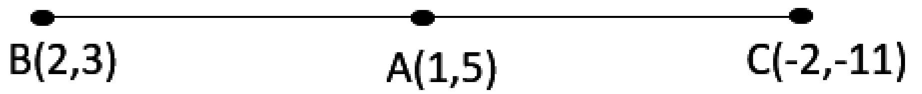
$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$

$$BC = \sqrt{\left(2-(-2)\right)^2 + \left(3-(-11)\right)^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196}$$
$$= \sqrt{212}$$

$$CA = \sqrt{(1-(-2)^2) + (5-(11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9 + 256}$$
$$= \sqrt{265}$$

Since AB + BC $\neq$ CA

Case 2)Now,



$$BA = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{5}$$

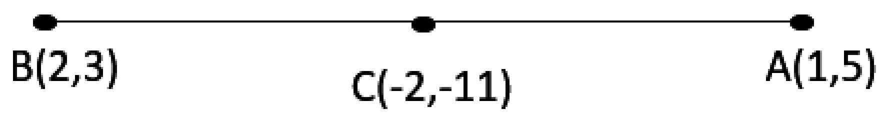
$$AC = \sqrt{(-2-1)^2 + (-11-5)^2}$$

$$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{(4)^2 + (14)^2}$$
$$= \sqrt{16+196} = \sqrt{212}$$

$$BA + AC \neq BC$$

Case 3)Now



$$BC = \sqrt{\left(2-(-2)\right)^2 + \left(3-(-11)\right)^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196}$$
$$= \sqrt{212}$$

$$CA = \sqrt{(1+2)^2 + (5+11)^2} = \sqrt{(3)^2 + (16)^2} = \sqrt{9+256}$$
$$= \sqrt{265}$$

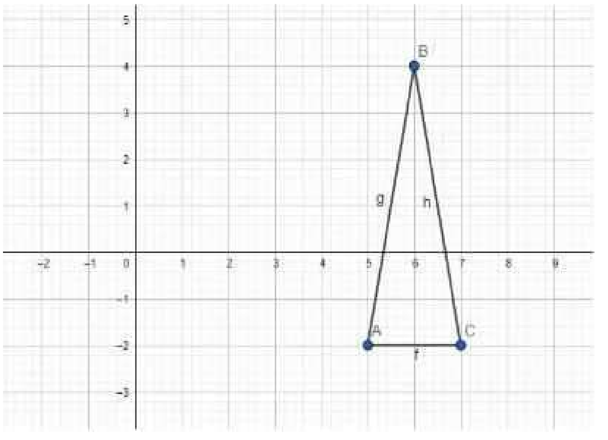
$$BA = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{5}$$

As BC+CA≠BA

As three of the cases are not satisfied. Hence the points are not collinear.

Q. 4 Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Answer:



Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let us assume that points (5, - 2), (6, 4), and (7, - 2) are representing the vertices A, B, and C of the given triangle respectively as shown in the figure.

$AB = [(5- 6)^2 + (-2- 4)^2]^{1/2}$

$= \sqrt{1 + 36}$

$= \sqrt{37}$

$BC = [(6- 7)^2 + (4 + 2)^2]^{1/2}$

$= \sqrt{1 + 36}$

$= \sqrt{37}$

$CA = [(5- 7)^2 + (-2+ 2)^2]^{1/2}$

$= \sqrt{4 + 0}$

$= 2$

Therefore, AB = BC

As two sides are equal in length, therefore, ABC is an isosceles triangle.

Q. 5 In a classroom, 4 friends a reseated at the points A, B, C and D as shown in Fig. 7.8 Champaand Chameli walk into the class and after observing for a few minutes Champa asks Chameli,"Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

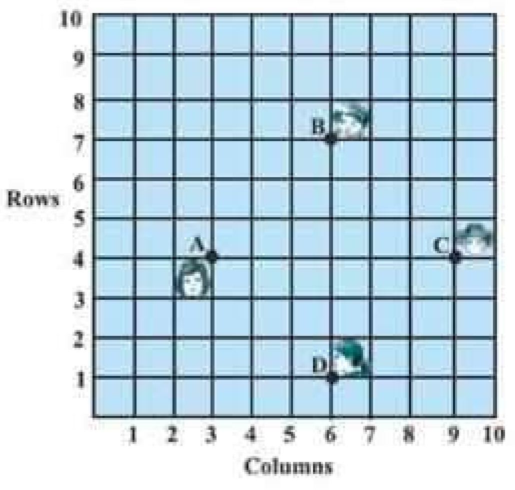
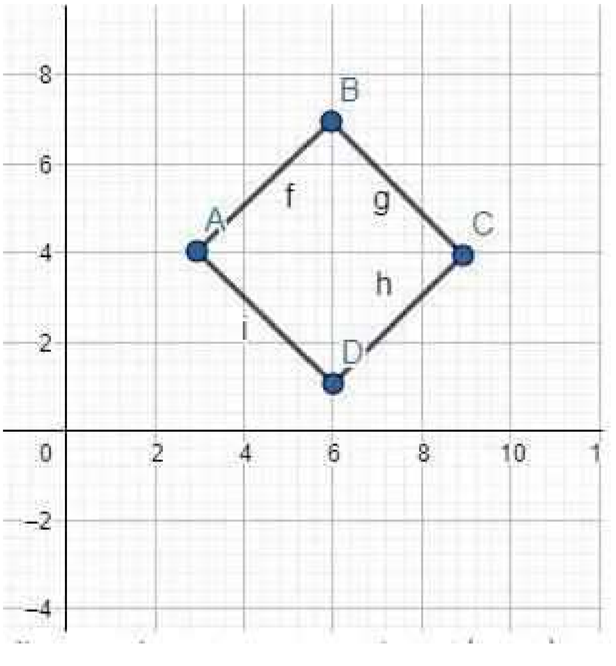


Fig. 7.8

It can be seen that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of 4 friends.



distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Hence,

AB = $[(3- 6)^2 + (4 - 7)^2]^{1/2}$

$\sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$

BC = $[(6 - 9)^2 + (7- 4)^2]^{1/2}$

$\sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$

AD = $[(3 - 6)^2 + (4 - 1)^2]^{1/2}$

$\sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$

Diagonal AC = $[(3 - 9)^2 + (4 - 4)^2]^{1/2}$

$= \sqrt{36+0}$

= 6

Diagonal BD = $[(6- 6)^2 + (7- 1)^2]^{1/2}$

$= \sqrt{36+0}$

= 6

It can be seen that all sides of quadrilateral ABCD are of the same length and diagonals are of the same length

Therefore, ABCD is a square and hence, Champa was correct

Q. 6 Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

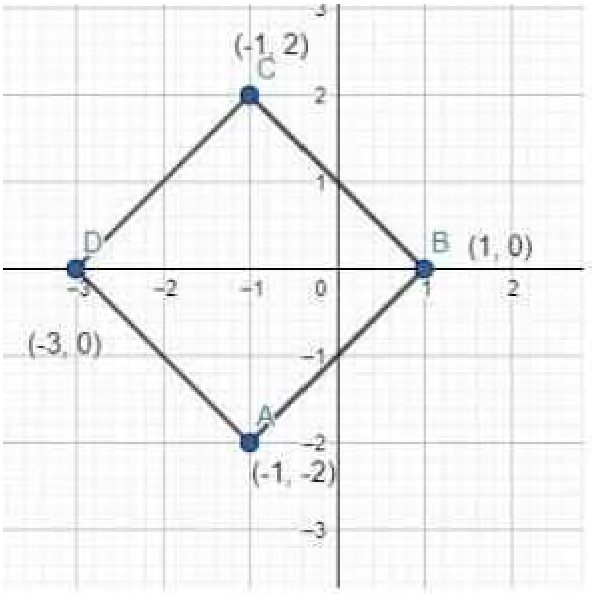
(ii) (–3, 5), (3, 1), (0, 3), (–1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Answer: To Find: Type of quadrilateral formed

(i) Let the points (- 1, - 2), (1, 0), (- 1, 2), and (- 3, 0) be representing the vertices A, B, C,

and D of the given quadrilateral respectively



he distance formula is an algebraic expression used to determine the distance between two points with the coordinates (x_1, y_1) and (x_2, y_2).

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = [(-1- 1)^2 + (-2- 0)^2]^{1/2}$

$= \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

$BC = \sqrt{[(1 + 1)^2 + (0- 2)^2]}$

$= \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

$AD = \sqrt{[(-1+ 3)^2 + (-2- 0)^2]}$

$= \sqrt{4+4} = \sqrt{8}$

$= 2\sqrt{2}$

Diagonal $AC = \sqrt{[(-1 + 1)^2 + (-2 - 2)^2]}$

$\sqrt{0+16}$

$= 4$

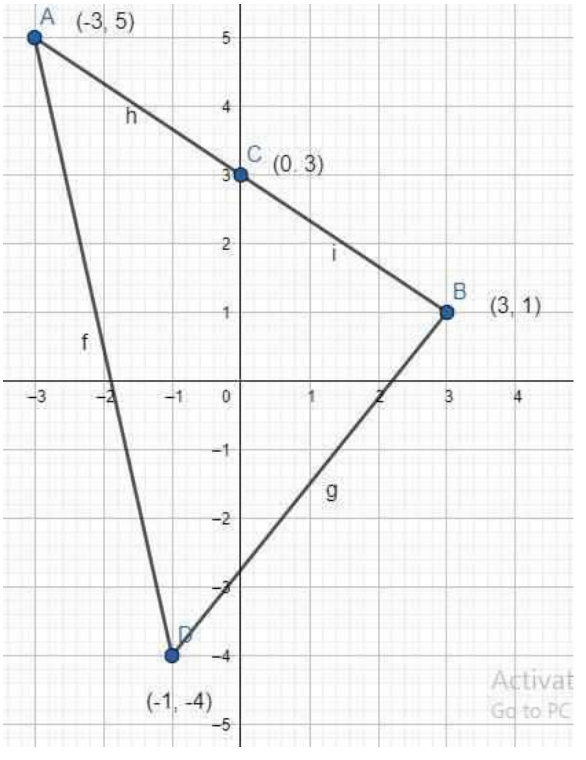
Diagonal BD $= \sqrt{[(1 + 3)^2 + (0- 0)^2]}$

$\sqrt{16 + 0}$

$= 4$

It is clear that all sides of this quadrilateral are of the same length and the diagonals are of the same length. Therefore, the given points are the vertices of a square

(ii)Let the points (- 3, 5), (3, 1), (0, 3), and (- 1, - 4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.



The distance formula is an algebraic expression used to determine the distance between two points with the coordinates (x_1, y_1) and (x_2, y_2).

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB $= \sqrt{[(-3- 3)^2 + (5- 1)^2]}$

$\sqrt{36+16} = \sqrt{52}$
= 2√13

BC = √[(3- 0)^2 + (1- 3)^2]

$= \sqrt{9+4}$
$= \sqrt{13}$
CD = √[(0 + 1)2 + (3+ 4)2]
$= \sqrt{1+49} = \sqrt{50}$
= 5√2

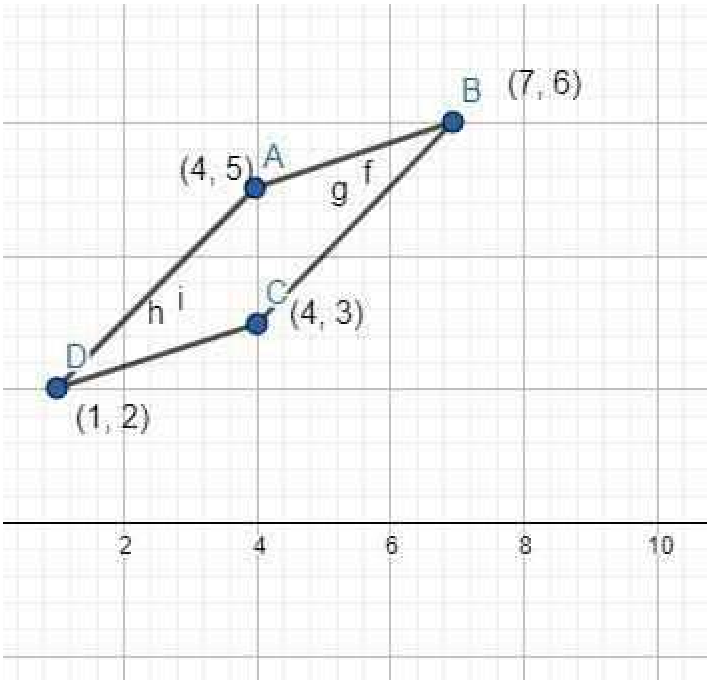
AD = √[(-3+ 1)^2 + (5+ 4)^2]

$= \sqrt{4+81}$

$= \sqrt{85}$

We can observe that all sides of this quadrilateral are of different lengths.
Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii)Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively



The distance formula is an algebraic expression used to determine the distance between two points with the coordinates (x_1, y_1) and (x_2, y_2).

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{[(4-7)^2 + (5-6)^2]}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$BC = \sqrt{[(7-4)^2 + (6-3)^2]}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$CD = \sqrt{[(4-1)^2 + (3-2)^2]}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$AD = \sqrt{[(4-1)^2 + (5-2)^2]}$

$= \sqrt{9+9}$

$= \sqrt{18}$

Diagonal AC $=\sqrt{[(4 - 4)^2 + (5- 3)^2]}$

$= \sqrt{0+4}$

$= 2$

Diagonal BD $= \sqrt{[(7 - 1)^2 + (6 - 2)^2]}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

We can observe that opposite sides of this quadrilateral are of the same length.

However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Q. 7 Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).

Answer: We have to find a point on *x*-axis.

Hence, its *y*-coordinate will be 0

Let the point on *x*-axis be (x, 0)

By distance formula, Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Distance between (x, 0) and (2, -5) $= \sqrt{[(x- 2)^2 + (0+ 5)^2]}$

$= \sqrt{[(x- 2)^2 + (5)^2]}$

Distance between (x, 0) and (-2, 9) $= \sqrt{[(x + 2)^2 + (0 + 9)^2]}$

$= \sqrt{[(x + 2)^2 + (9)^2]}$

By the given condition, these distances are equal in measure

$\sqrt{[(x - 2)^2 + (5)^2]} = \sqrt{[(x + 2)^2 + (9)^2]}$

Squaring both sides we get,

$(x - 2)^2 + 25 = (x + 2)^2 + 81$

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

$\Rightarrow (x^2 - x^2) - 4x - 4x = 81 - 25$

$\Rightarrow - 8 x = 56$

$\Rightarrow 8x = -56$

$\Rightarrow x = - 7$

Therefore, the point is (- 7, 0)

Q. 8 Find the values of y for which the distance between the points P(2, – 3) and Q (10, y) is 10 units.

Answer:

Given: Distance between (2, - 3) and (10, *y*) is 10

To find: y

By distance formula, Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Therefore,

$\sqrt{[(2 - 10)^2 + (-3- y)^2]} = 10$

$\sqrt{[(-8)^2 + (3+ y)^2]} = 10$

Squaring both sides to remove the square root,

$\Rightarrow 64 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 100 - 64$

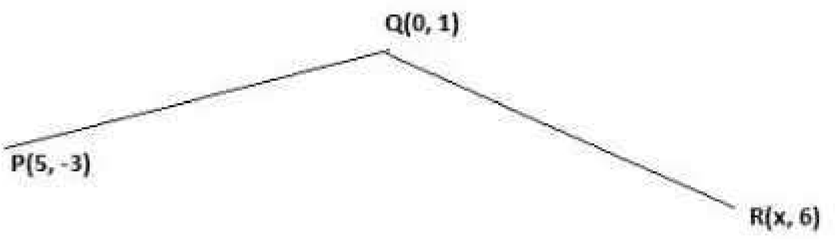
$\Rightarrow (y + 3)^2 = 36$

$\Rightarrow y + 3 = \pm 6$
$\Rightarrow (y + 3)$ will give two values 6 and -6 as answer because both the values when squared will give 36 as answer.)

$y + 3 = 6$ or $y + 3 = -6$

Therefore, $y = 3$ or -9

Q. 9 If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.



We know, By distance formula distance between two coordinates $A(x_1, y_1)$ and $B(x_2, y_2)$ is
$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
Now, as PQ = QR

$\sqrt{[(5 - 0)^2 + (-3 - 1)^2]} = \sqrt{[(0 - x)^2 + (1 - 6)^2]}$

$\sqrt{25 + 16} = \sqrt{x^2 + 25}$

Squaring both sides,
$41 = x^2 + 25$

$x^2 = 16$

$x = \pm 4$

Hence, point R is (4, 6) or (- 4, 6).

When point R is (4, 6)

PR = $[(5 - 4)^2 + (-3 - 6)^2]^{1/2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

QR = $[(0 - 4)^2 + (1 - 6)^2]^{1/2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

When point R is (- 4, 6),

PR = $[(5 + 4)^2 + (-3 - 6)^2]^{1/2}$

$\sqrt{81 + 81}$

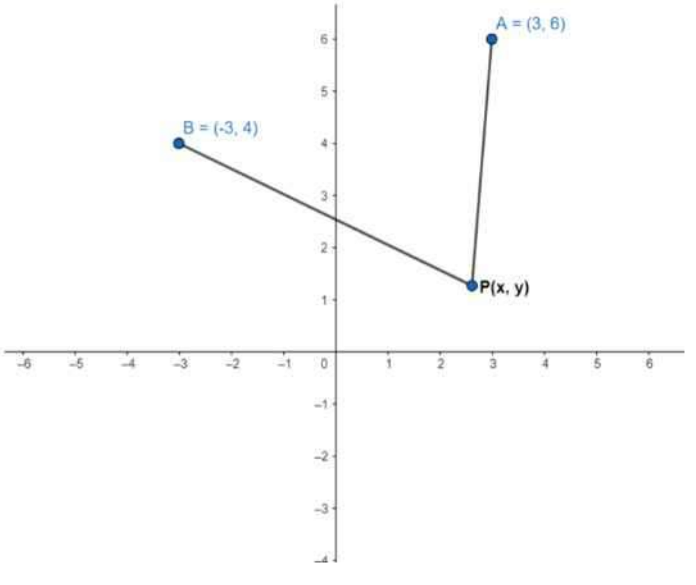
$9\sqrt{2}$

QR = $[(0+ 4)^2 + (1 - 6)^2]^{1/2}$

$\sqrt{16 + 25}$

$\sqrt{41}$

Q. 10 Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).



To Find: Relation between x and y
Given: (x, y) is equidistant from (3, 6) and (-3, 4)
From the figure it can be seen that Point (x, y) is equidistant from (3, 6) and (- 3, 4)
This means that the distance of (x, y) from (3, 6) will be equal to distance of (x, y) from (- 3, 4)We know by distance formula that, distance between two points $A(x_1, y_1)$, $B(x_2, y_2)$ is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore,

$$\sqrt{[(x- 3)^2 + (y - 6)^2]} = \sqrt{[(x + 3)^2 + (y- 4)^2]}$$

Squaring both sides, we get

$$(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

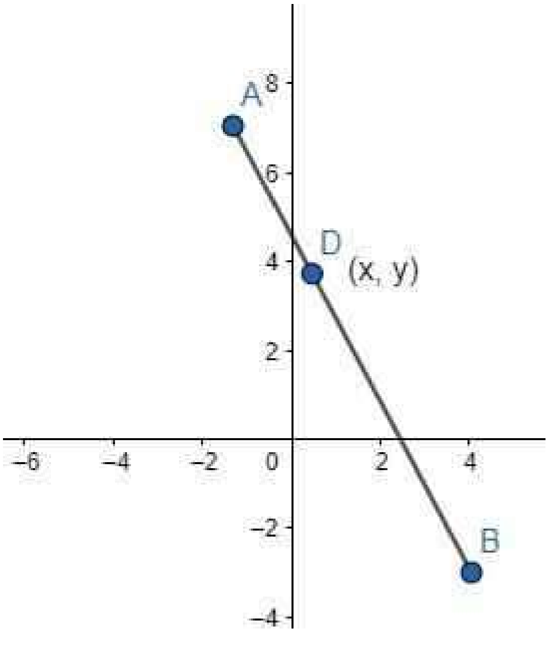
$$3x + y = 5$$

$3x + y - 5 = 0$ is the relation between x and y.

Exercise 7.2

Q. 1 Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2: 3

Answer:



If (x_1, y_1) and (x_2, y_2) are points that are divided in ratio m:n then,

$$(x, y) = \frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}$$

Let D(x, y) be the required point.

Now,

Using the section formula, we get

$$x = \frac{2\times4+3\times-1}{2+3}$$

$$x = \frac{8-3}{5}$$

x = 1

$$y = \frac{2\times-3+3\times7}{2+3}$$

$$y = \frac{-6+21}{5}$$

y = 3

Therefore, the point is (1, 3).

Q. 2 Find the coordinates of the points of trisection of the line segment joining (4,–1) and (–2, –3).

Answer:

The points of trisection means that the points which divide the line in three equal parts. From the figure C, and D are these two points.Let C (x_1, y_1) and D (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., BC = CD = DA

Let BC = CD = DA = kPoint C divides the BC and CA as:BC = kCA = CD + DA = k + k = 2kHence ratio between BC and CA is:

$$\frac{BC}{CA} = \frac{k}{2k} = \frac{1}{2}$$

Therefore, point C divides BA internally in the ratio 1:2

then by section formula we have that if a point P(x, y) divide two points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n then,

the point (x, y) is given by $(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Therefore C(x, y) divides B(–2, –3) and A(4,–1) in the ratio 1:2, then

$$C(x,y) = \frac{(1\times4)+(2\times-2)}{1+2}, \frac{(1\times-1)+(2\times-3)}{1+2}$$

$$C(x,y) = \frac{4-4}{1+2}, \frac{-1-6}{1+2}$$

$$C(x,y) = 0, \frac{-7}{3}$$

Point D divides the BD and DA as:DA = kBD = BC + CD = k + k = 2k Hence ratio between BD and DA is:

$$\frac{BD}{DA} = \frac{2k}{k} = \frac{2}{1}$$

The point D divides the line BA in the ratio 2:1

So now applying section formula again we get

$$D(x,y) = \frac{(2\times4)+(1\times-2)}{2+1}, \frac{(2\times-1)+(1\times-3)}{2+1}$$

$$D(x,y) = \frac{8-2}{3}, \frac{-2-3}{3}$$

$$D(x,y) = \frac{6}{3}, \frac{-5}{3}$$

$$D(x,y) = 2, \frac{-5}{3}$$

Q. 3 To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. 7.12. Niharika runs $\frac{1}{4}$th thedistance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

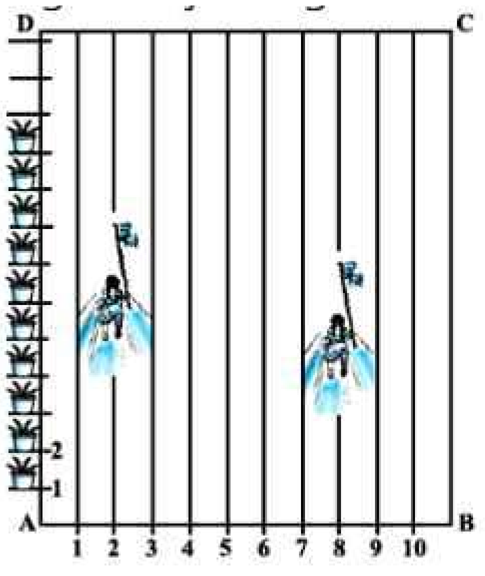


Fig. 7.12

Answer:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $1/4 \times 100 = 25$ m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25)

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., 1/5 x 100 = 20 m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20)

Now we have the positions of posts by Preet and Niharika
According to distance formula, distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance between these flags by using distance formula, D

$= [(8 - 2)^2 + (25 - 20)^2]^{1/2}$

$= (36 + 25)^{1/2}$

$= \sqrt{61}m$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x,y)

Now by midpoint formula,

$$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$x = \frac{2+8}{2} = 5$$

$$y = \frac{25+20}{2} = 22.5$$

Hence, A (x, y) = (5, 22.5)

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Q. 4 Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Answer: Let the ratio in which the line segment joining (- 3, 10) and (6, - 8) is divided by point (- 1, 6) be k : 1

Using section formula

i.e. the coordinates of the points P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2,y_2)$, internally in the ratio m : n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$(-1, 6) = \left(\frac{k(6)+1(-3)}{k+1}, \frac{k(-8)+1(10)}{k+1}\right)$$

Therefore,

$$-1 = \frac{6k-3}{k+1}$$

⇒ -k – 1 = 6k – 3

⇒ - k -6k = -3 + 1

⇒ - 7k = -2

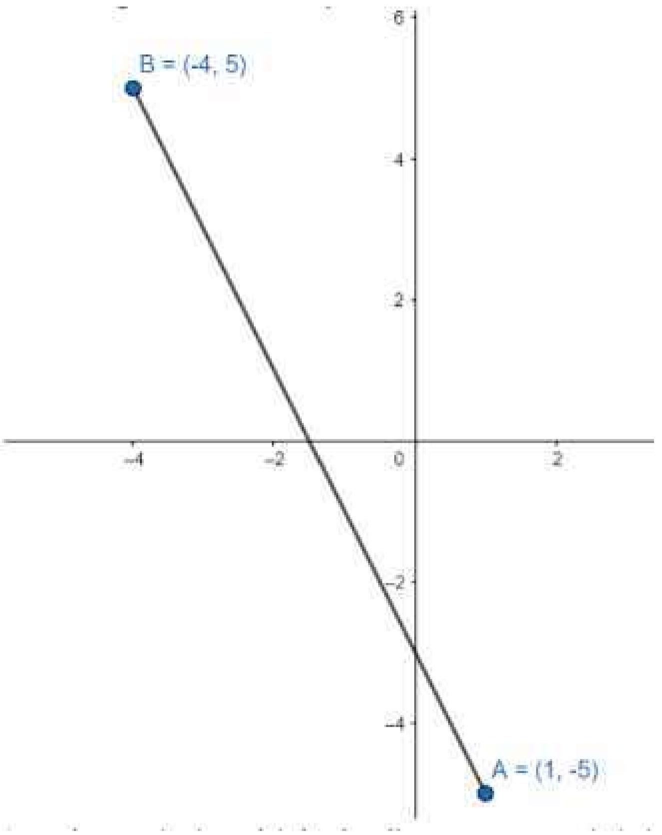
⇒ 7k = 2

$$\Rightarrow k = \frac{2}{7}$$

Therefore, the required ratio is 2: 7.

Q. 5 Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also, find the coordinates of the point of division.

Answer: The diagram for the question is



Let the ratio in which the line segment joining A (1, - 5) and B (- 4, 5) is divided by x-axis be k: 1

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$

We know that y-coordinate of any point on x-axis is 0

Therefore,

$\frac{5k-5}{k+1} = 0$

k = 1

Therefore, x-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right)$

$= \left(\frac{-4+1}{2}, \frac{5-5}{2}\right)$

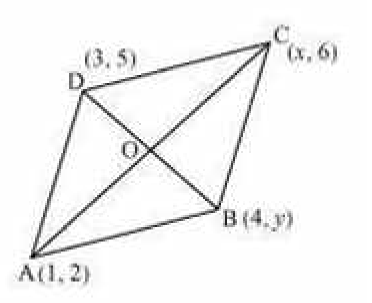
$= \left(\frac{-3}{0}, 0\right)$

Q. 6 If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer: Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD.

Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.



By mid point formula, If (x, y) is the midpoint of the line joining points A(x_1, y_1) and B(x_2, y_2) Then,

$$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

If O is the mid-point of AC, then the coordinates of O are:

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are:

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O

Comparing the x coordinates we get,

$$\frac{x+1}{2} = \frac{7}{2}$$

x + 1 = 7

x = 6

And,

Comparing the y coordinates we get,

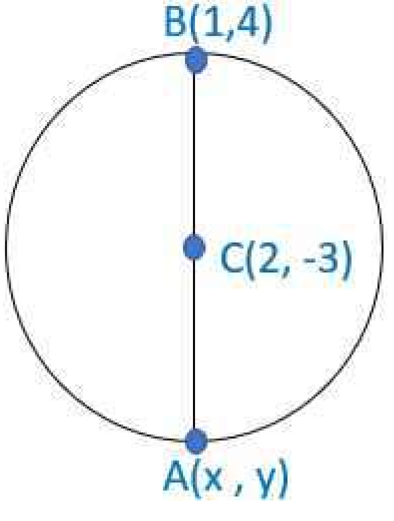
$$\frac{5+y}{2} = 4$$

5 + y = 8

y = 3

Hence, x = 6 and y = 3.

Q. 7 Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4)



For points A(x_1, y_1) and B(x_2, y_2), the midpoints (x, y) is given by

$$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

Let the coordinates of point A be (x, y)

Mid-point of AB is (2, - 3), which is the center of circle and point B is (1, 4)

Therefore,

$$(2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

Equating the coordinates we get,

$\frac{x+1}{2} = 2, \frac{y+4}{2} = -3$

x = 4 – 1, y = -6 – 4

x= 3, y = -10

Therefore point is A (3, -10)

Therefore, the coordinates.

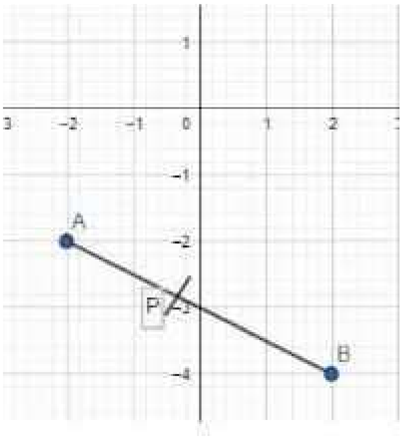
Q. 8 If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB

Answer: To find: The coordinates of P

Given: Points A(-2, -2) and B(2, -4) and ratio AP:AB = 3:7

The coordinates of point A and B are (- 2, - 2) and (2, - 4) respectively

AP $= \frac{3}{7}$ AB



Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4

By section formula,
If a point divides the point (x_1, y_1) and (x_2, y_2) in the ratio m:n

Then, $(X, Y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

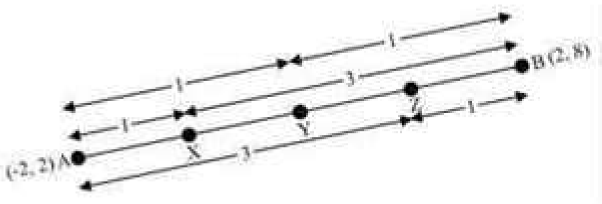
$(X, Y) = \left(\frac{3\times2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$

$(X, Y) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

(-2/7, -20/7) is the point which divides line in the ratio of 3:4.

Q. 9 Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Answer: It can be observed from the figure that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1respectively



Coordinate of X $= \left(\frac{1*2+3*-2}{1+3}, \frac{1*8+3*2}{1+3}\right)$

$= \left(-1, \ \frac{7}{2}\right)$

Coordinate of Y $= \left(\frac{2\pm2}{2}, \frac{2+8}{2}\right)$

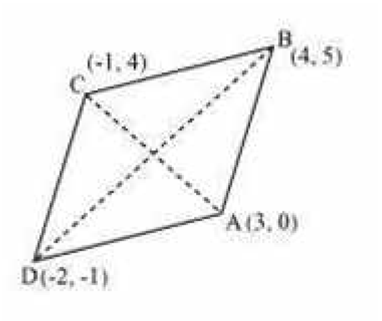
= (0, 5)

Coordinates of Z $= \left(\frac{3*2+1*-2}{3+1}, \frac{3*8+1*2}{3+1}\right)$

$= \left(1, \frac{13}{2}\right)$

Q. 10 Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order. [Hint: Area of a rhombus $= \frac{1}{2}$ (product of its diagonals)]

Answer: Let (3, 0), (4, 5), (- 1, 4) and (- 2, - 1) are the vertices A, B, C, D of a rhombus ABCD



Length of diagonal AC = $[(3 + 1)^2 + (0 - 4)^2]^{1/2}$

$= \sqrt{16 + 16}$

$= 4\sqrt{2}$

Length of diagonal BD = $[(4 + 2)^2 + (5 + 1)^2]^{1/2}$

$= \sqrt{36 + 36}$

$= 6\sqrt{2}$

Therefore,

Area of rhombus = 1/2 (Product of the length of the diagonals)

Therefore,

Area of rhombus ABCD = $1/2 \times 4\sqrt{2} \times 6\sqrt{2} = 1/2 \times 48$

= 24 square units

Exercise 7.3

Q. 1 Find the area of the triangle whose vertices are:

(i) (2, 3), (–1, 0), (2, – 4)
(ii) (–5, –1), (3, –5), (5, 2)

Answer:

For a triangle with points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3)

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

(i) For triangle given: A(2, 3), B(–1, 0), C(2, – 4)
Area of the triangle $= \frac{1}{2}$ [2 (0+ 4) - 1 (-4 – 3) + 2 (3 – 0)

$= \frac{1}{2}$ (8 + 7 + 6)

$= \frac{21}{2}$ Square units

(ii) For triangle given: A(–5, –1), B(3, –5), C(5, 2)
Area of the triangle $= \frac{1}{2}$ [-5 (-5 – 2) + 3 (2+ 1) + 5 (-1+ 5)

$= \frac{1}{2}$ (35 + 9 + 20)

= 32 square units

Q. 2 In each of the following find the value of 'k', for which the points are collinear

(i) (7, –2), (5, 1), (3, k)
(ii) (8, 1), (k, – 4), (2, –5)

Answer: For three points A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)
Area of triangle ABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

(i) Collinear points mean that the points lie on a straight line. So, For collinear points, area of triangle formed by them is zero

Therefore, for points (7, - 2) (5, 1), and (3, k), area = 0

$\frac{1}{2}$ [7 (1 – k) + 5 (k+ 2) + 3 (-2 – 1) = 0

7 – 7 k + 5 k + 10 – 9 = 0

-2 k + 8 = 0

k = 4

(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, - 4), and (2, - 5), area = 0

$\frac{1}{2}$ [8 (-4+ 5) + k (-5- 1) + 2 (1+ 4) = 0

8 – 6k + 10 = 0

6k = 18

k = 3

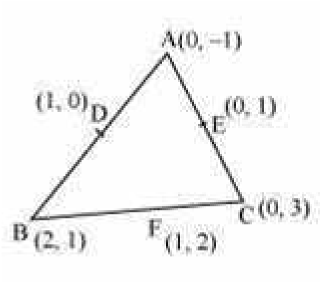
Q. 3 Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer: Let the vertices of the triangle be A (0, - 1), B (2, 1), C (0, 3)

We know, mid-points of a line joining points (x_1, y_1) and (x_2, y_2) is given by

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by



$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

Also, we know area of a triangle having (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2}(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$$

Area of the triangle (DEF) = $\frac{1}{2}$ [1 (2 – 1) + 1(1- 0) + 0 (0- 2)

= $\frac{1}{2}$ (1 + 1)

= 1 square unit

Area of the triangle (ABC) = = $\frac{1}{2}$ [0 (1 – 3) + 2(3 + 1) + 0 (-1 - 1)

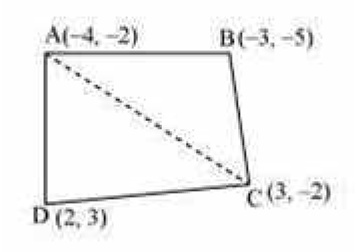
= $\frac{1}{2}$ × 8

= 4 square units

Therefore, required ratio = 1: 4.

Q. 4 Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2),(–3, – 5), (3, – 2) and (2, 3).

Answer: Let the vertices of the quadrilateral be A (- 4, - 2), B (- 3, - 5), C (3, - 2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD



Area of the triangle (ABC) = = $\frac{1}{2}$ [-4 (-5 + 2) -3 (-2+ 2) + 3 (-2+ 5)]

= $\frac{1}{2}$ (12 + 0 + 9)

= $\frac{21}{2}$ Square units

Area of the triangle (ACD) = $\frac{1}{2}$ [-4 (-2 – 3) + 3(3 + 2) + 2 (-2+ 2)]

= $\frac{1}{2}$ (20 + 15 + 0)

= $\frac{35}{2}$ Square units

Area of Quadrilateral ABCD = Area of the triangle (ABC) + Area of the triangle (ACD)

$= \frac{21}{2} + \frac{35}{2}$

$= \frac{56}{2}$

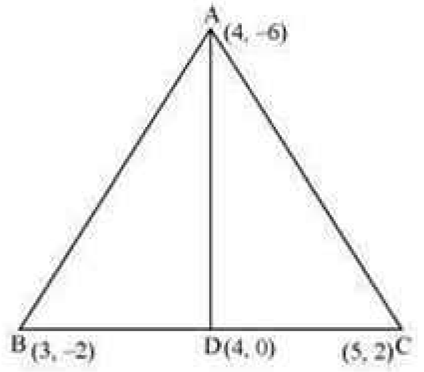
= 28 Square units

Q. 5 You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A(4, – 6), B(3, –2) and C (5, 2)

Answer: Let the vertices of the triangle be A (4, - 6), B (3, - 2), and C (5, 2)

Let D be the mid-point of side BC of ΔABC Therefore, AD is the median in ΔABC

To prove: Area ΔABC = Area ΔABD + Area ΔADC



Midpoint of the line joining the points A(x_1, y_1) and B(x_2, y_2) is given by

$X = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

Therefore, by applying the formula we can get the coordinate of D

Coordinate of point $D = \left(\frac{3+5}{2}, \frac{-2+2}{0}\right) = (4, 0)$

We also know that if A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are vertices of a triangle then

Area of triangle is given by,

$$Area = \frac{1}{2}[x_1(y_2 - y_3) - x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

For A (4, - 6), B (3, - 2), and C (5, 2)

Area of the Δ (ABC) = $\frac{1}{2}$ [4 (-2 - 2) + 3 (0+ 8) + 5 (-6+ 2)]

$= \frac{1}{2}(-8 + 32 - 30)$

= - 3 Square units

However, area cannot be negative. Therefore, area of ΔADC is 3 square units

Now for for A(4, -6), B(3, -2) and D(4, 0)Area of Δ ABD

$= \frac{1}{2}[4(-2 - 0) + 3(0 + 6) + 4(-6 + 2)]$

Area $= \frac{1}{2}$ [-8 + 18 - 16]

Area = -3 square units Now the Area cannot be negative so **Area of ΔABD = 3 units**

Area of Δ ABD + Area of Δ ADC = 3 + 3 square units

Area of Δ ABD + Area of Δ ADC = 6 = Area of Δ ABC

Clearly, median AD has divided ΔABC in two triangles of equal areas

Exercise 7.4

Q. 1 Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, – 2) and B (3, 7).

Answer: Let the given line divide the line segment joining the points A(2, - 2) and B(3, 7) in a ratio $k : 1$

Coordinates of the point of division = $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

Therefore,

$$2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$$\frac{6k+4+7k-2-4k-4}{k+1} = 0$$

$9k - 2 = 0$

$$k = \frac{2}{9}$$

Hence, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7) is **2:9**.

Q. 2 Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear

Answer: If the given points are collinear, then the area of triangle formed by these points will be 0

Area of the triangle $= \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}[x (2 - 0) + 1 (0 - y) + 7 (y - 2)]$

$0 = \frac{1}{2}(2x - y + 7y - 14)$

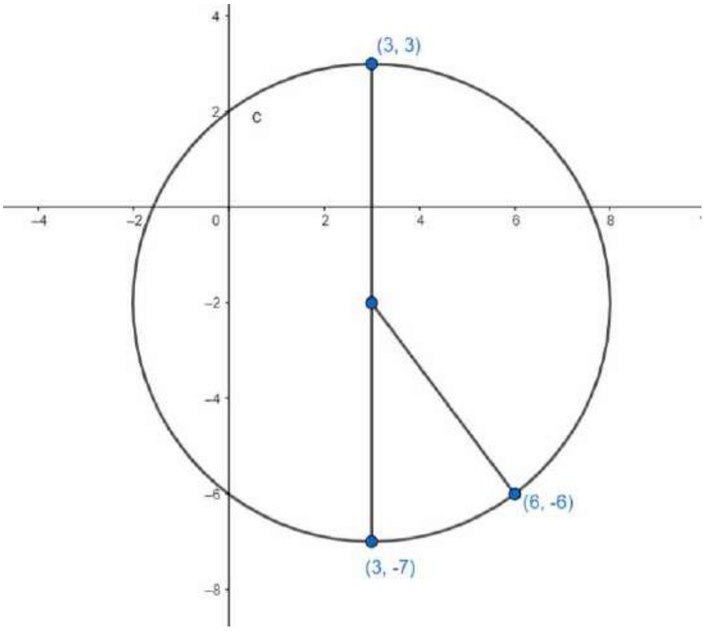
$2x + 6y - 14 = 0$

$x + 3y - 7 = 0$

This is the required relation between x and y.

Q. 3 Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3)

The figure for the question is:



Let O (x, y) be the centre of the circle. And let the points (6, - 6), (3, - 7), and (3, 3) be representing the points A, B,and C on the circumference of the circle

By distance formula, Distance between two points A(x_1, y_1) and B(x_2, y_2) is

$AB = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Then,

$OA = \sqrt{[(x- 6)^2 + (y+ 6)^2]}$

$OB = \sqrt{[(x- 3)^2 + (y+ 7)^2]}$

$OC = \sqrt{[(x- 3)^2 + (y- 3)^2]}$

As OA, OB and OC are radii of same circle, OA = OB

$[(x - 6)^2 + (y + 6)^2]^{1/2} = [(x - 3)^2 + (y + 7)^2]^{1/2}$

$-6x - 2y + 14 = 0$

$3x + y = 7$ (i)

Similarly, OA = OC

$[(x - 6)^2 + (y + 6)^2]^{1/2} = [(x - 3)^2 + (y - 3)^2]^{1/2}$

$-6x + 18y + 54 = 0$

$-3x + 9y = -27$ (ii)

On adding equation (i) and (ii), we obtain

$10y = - 20$

$y = - 2$

From equation (i), we obtain

$3x - 2 = 7$

$3x = 9$

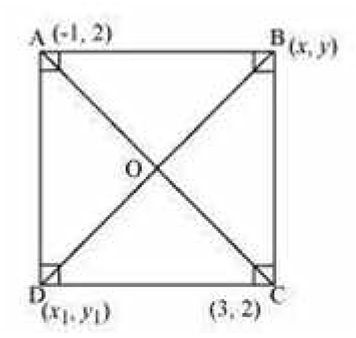
$x = 3$

Therefore, the centre of the circle is (3, - 2).

Q. 4 The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices

Let ABCD be a square having (- 1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively

We know that the sides of a square are equal to each other



From the figure we can see that the sides of square AB and BC are equal

And also we know that for two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, The distance between points P and Q is given by the formula

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

From the figure,

AB = BCTherefore,$\sqrt{[(x + 1)^2 + (y - 2)^2]} = \sqrt{[(x - 3)^2 + (y - 2)^2]}$

Squaring both side, and evaluating

$x^2 + 2x + 1 + y^2 – 4y + 4 = x^2 + 9 – 6x + y^2 + 4 – 4y$

$8x = 8$

$x = 1$

We know that in a square, all interior angles are of 90°.

Pythagoras theorem: the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

In ΔABC,

Applying pythagoras theorem we get,

$AB^2 + BC^2 = AC^2$

$\sqrt{[(x + 1)^2 + (y - 2)^2]}^2 + \sqrt{[(x - 3)^2 + (y - 2)^2]}^2 = \sqrt{[(3 + 1)^2 + (2 - 2)^2]}^2$

$4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$2y^2 + 16 - 8y = 16$

$2y^2 - 8y = 0$

$y(y - 4) = 0$

$y = 0$ or 4

Now we have obtained the B(x, y) point and we are left to find point C(x_1, y_1)

We know that in a square, the diagonals are equal in length and bisect each other at 90°.

Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD

If O is mid-point of AC, then
we know, mid-point of (x_1, y_1) and (x_2,y_2) is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

i.e. $O = \left(\frac{-1+3}{2},, \frac{2+2}{2}\right)$

therefore, O = (1, 2)Also, if we take O as midpoint of BD then

$O = \left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

If y = 0

$\Rightarrow (1, 2) = \left(\frac{1+x_1}{2}, \frac{0+y_1}{2}\right)$

$\Rightarrow \frac{1+x_1}{2} = 1 \; and \; \frac{0+y_1}{2} = 2$

$x_1 = 1 \; and \; y_1 = 4$

If y = 4

$\Rightarrow (1, 2) = \left(\frac{1+x_1}{2}, \frac{4+y_1}{2}\right)$

$\Rightarrow \frac{1+x_1}{2} = 1 \; and \; \frac{4+y_1}{2} = 2$

$x_1 = 1 \; and \; y_1 = 0$

Therefore, The sides of square are A(-1, 2), B(1, 4), C(1, 0) and D(3, 2)or A(-1, 2), B(1, 0), C(1, 4) and D(3, 2).

Q. 5 The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the Fig. 7.14. The students are to sow seeds of flowering plants on the remaining area of the plot

(i) Taking A as origin, find the coordinates of the vertices of the triangle.
(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?
Also calculate the areas of the triangles in these cases. What do you observe?

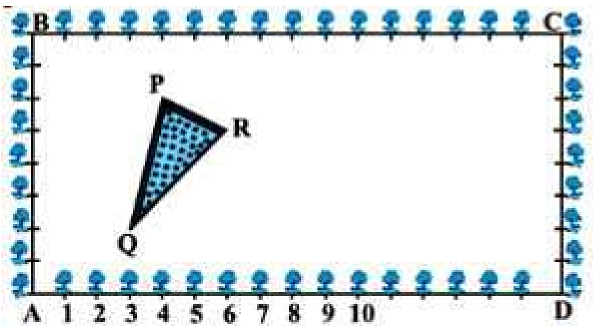


Fig. 7.14

Answer: (i) Taking A as origin,

We will take AD as *x*-axis and AB as *y*-axis.

It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively

Area of the triangle PQR $= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$

$= \frac{1}{2}$ [4 (2 – 5) + 3 (5 – 6) + 6 (6 – 2)

$= \frac{1}{2}$ (-12 – 3 + 24)

$= \frac{9}{2}$ Square units

(ii) Taking C as origin, CB as x-axis, and CD as y-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6),and (10, 3) respectively

Area of the triangle PQR $= \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$

$= \frac{1}{2}$ [12 (6 – 3) + 13 (3 – 2) + 10 (2 – 6)

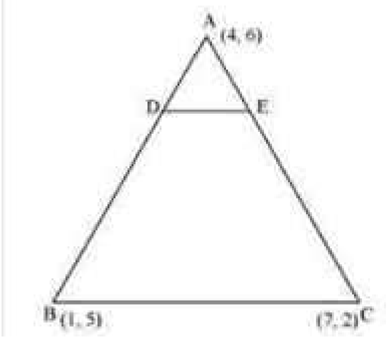
$= \frac{1}{2}$ (36 + 13 – 40)

$= \frac{9}{2}$ Square units

It can be observed that the area of the triangle is same in both the cases.

Q. 6 The vertices of a Δ ABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$ Calculate the area of the Δ ADE and compare it with the area of Δ ABC (Recall Theorem 6.2 and Theorem 6.6).

Answer: Given that



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

$$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3

Coordinates of point D = $\left(\frac{1*1+3*4}{1+3}, \frac{1*5+3*6}{1+3}\right)$

$= \left(\frac{13}{4}, \frac{23}{4}\right)$

Coordinates of point D = $\left(\frac{1*7+3*4}{1+3}, \frac{1*2+3*6}{1+3}\right)$

$= \left(\frac{19}{4}, \frac{20}{4}\right)$

Area of the triangle = $\frac{1}{2}$ [x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 - y_2)

$= \frac{1}{2}\left[4\left(\frac{23}{4} - \frac{20}{4}\right) + \frac{13}{4}\left(\frac{20}{4} - 6\right) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right]$

$= \frac{1}{2}\left[3 - \frac{13}{4} + \frac{19}{16}\right]$

$= \frac{1}{2}\left[\frac{48-52+19}{16}\right]$

= 15/32 square units

Area of triangle ABC = [4 (5 – 2) + 1 (2 - 6) + 7 (6 – 5)] /2

= (12 – 4 + 7)/ 2

= 15/2 square units

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle.

These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles

And, ratio between the areas of ΔADE and ΔABC = $(1/4)^2$

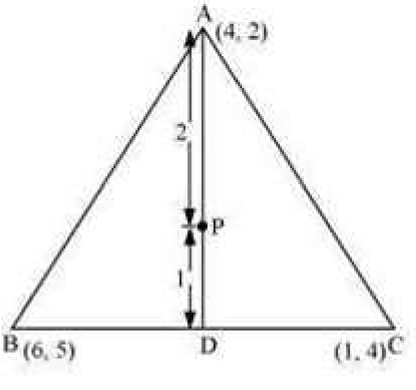
= 1/16

Q. 7 Let A (4, 2), B(6, 5) and C (1, 4) be the vertices of Δ ABC

(i) The median from A meets BC at D. Find the coordinates of the point D
(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR: RF = 2: 1
(iv) What do you observe?
[Note: The point which is common to all the three medians is called the centroid and this point divides each median in the ratio 2: 1]
(v) If A(x_1, y_1), B(x_2, y_2) and B(x_3, y_3) are the vertices of Δ ABC, find the coordinates of the centroid of the triangle.

Answer: (i) Median AD of the triangle will divide the side BC in two equal parts

Therefore, D is the mid-point of side BC



And according to midpoint formula, midpoints of (x_1, y_1), (x_2, y_2) is given by

$$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

Coordinate of D = $\left(\frac{6+1}{2}, \frac{5+4}{2}\right)$

$= \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Point P divides the side AD in a ratio 2:1

By section formula if (x, y) divides line joining points A(x_1, y_1), B(x_2, y_2) in ratio m:n

Then, $x, y = \frac{mx_2+nx_1}{m+n} + \frac{my_2+ny_1}{m+n}$

Coordinates of p = $\left(\frac{2*\frac{7}{2}+1*4}{2+1}, \frac{2*\frac{9}{2}+1*2}{2+1}\right)$

$= \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC

Coordinates of E = $\left(\frac{4+1}{2}, \frac{2+4}{2}\right)$

$= \left(\frac{5}{2,3}\right)$

Point Q divides the side BE in a ratio 2:1

By section formula if (x, y) divides line joining points A(x_1, y_1), B(x_2, y_2) in ratio m:n

Then, $x, y = \frac{mx_2+nx_1}{m+n} + \frac{my_2+ny_1}{m+n}$

Coordinates of p = $\left(\frac{2*\frac{5}{2}+1*6}{2+1}, \frac{2*3+1*5}{2+1}\right)$

$= \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB

Coordinates of F = $\left(\frac{4+6}{2}, \frac{2+5}{2}\right)$

$= \left(5, \frac{7}{2}\right)$

Point R divides the side CF in a ratio 2:1

By section formula if (x, y) divides line joining points A(x_1, y_1), B(x_2, y_2) in ratio m:n

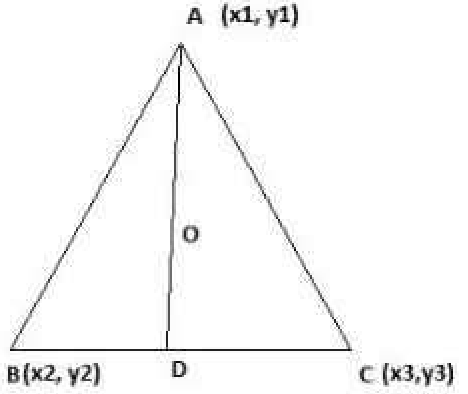
Then, $x, y = \frac{mx_2+nx_1}{m+n} + \frac{my_2+ny_1}{m+n}$

Coordinates of p = $\left(\frac{2*5+1*6}{2+1}, \frac{2*\frac{7}{2}+1*4}{2+1}\right)$

$= \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) It can be observed that the coordinates of point P, Q, R are the same. Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v)



Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3,y_3)

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC

Coordinates of D = $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Let the centroid of this triangle be O. Point O divides the side AD in a ratio 2:1

By section formula if (x, y) divides line joining points A(x_1, y_1), B(x_2, y_2) in ratio m:n

then $x, y = \frac{mx_2+nx_1}{m+n} + \frac{my_2+ny_1}{m+n}$

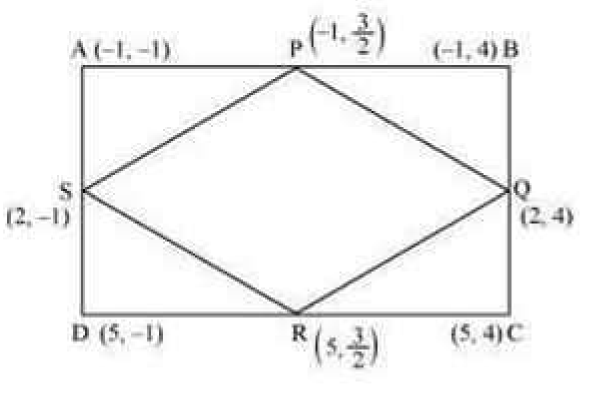
Coordinates of O = $\left(\frac{2*\frac{x_2}{3}+\frac{x_3}{2}+1*x_1}{2+1}, \frac{2*\frac{y_2}{3}+\frac{y_3}{2}+1*y_1}{2+1}\right)$

Centroid of ABC = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Q. 8 ABCD is a rectangle formed by the points A(–1, –1), B(– 1, 4), C(5, 4) and D(5, – 1). P, Q,R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Answer:

P is the mid-point of AB



Coordinates of $P = \left(-1, \frac{3}{2}\right)$

Similarly coordinates of Q, R and S are (2, 4), (5, $\frac{3}{2}$) and (2, -1) respectively.

Length of PQ = $[(-1 - 2)^2 + (\frac{3}{2} - 4)^2]^{1/2}$

$= \sqrt{\frac{61}{4}}$

Length of QR = $[(2-5)^2 + (4 - \frac{3}{2})^2]^{1/2}$

$= \sqrt{\frac{61}{4}}$

Length of RS = $[(5\text{-}2)^2 + (\frac{3}{2} + 1)^2]^{1/2}$

$= \sqrt{\frac{61}{4}}$

Length of SP = $[(2+1)^2 + (-1 - \frac{3}{2})^2]^{1/2}$

$= \sqrt{\frac{61}{4}}$

Length of SP = $[(-1-5)^2 + (\frac{3}{2} - \frac{3}{2})^2]^{1/2}$

= 6

Length of QS = $[(2-2)^2 + (4+1)^2]^{1/2} = 5$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus