Chapter – 6
Triangle

Exercise - 6.1

Q. 1 Fill in the blanks using the correct word given in brackets:

(i) All circles are ___________. (congruent, similar)

(ii) All squares are ____________. (similar, congruent)

(iii) All _____________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are ____________. (equal, proportional)

Answer: The solutions of the fill ups are:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

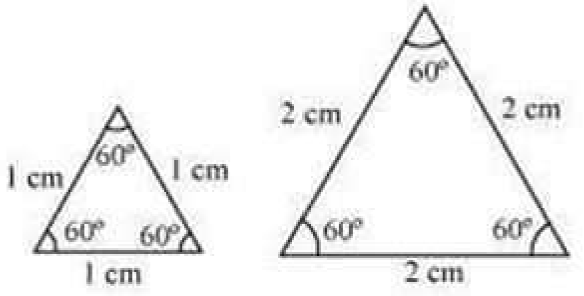
Q. 2 Give two different examples of pair of

(i) Similar figures

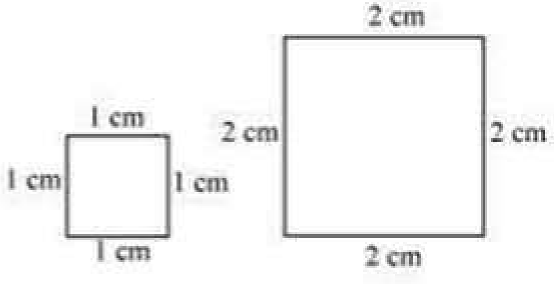
(ii) Non-similar figures

Answer: Two examples of similar figures are:

(i) Two equilateral triangles with sides 1 cm and 2 cm

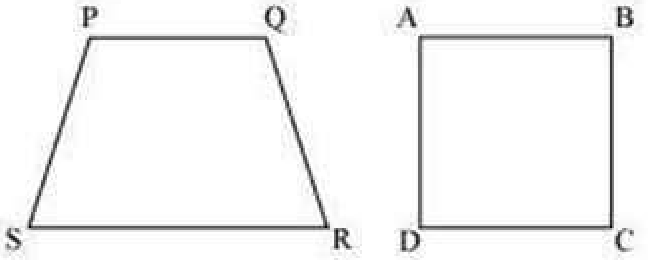


(ii) Two squares with sides 1 cm and 2 cm

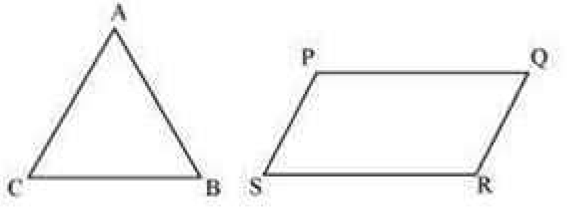


Now two examples of non-similar figures are:

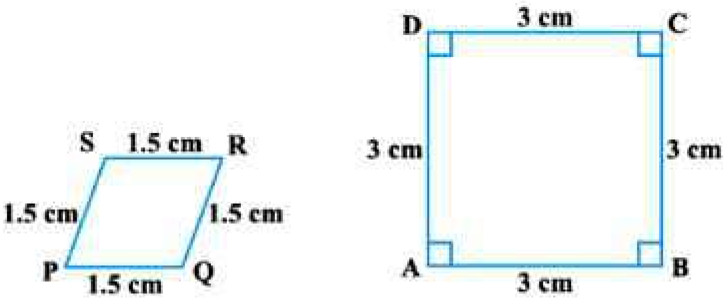
(i) Trapezium and square



(ii) Triangle and parallelogram



Q. 3 State whether the following quadrilaterals are similar or not:



Answer: The given quadrilateral PQRS and ABCD are not similar because though their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2

Q. 1 In Fig. 6.17, (i) and (ii), DE ∥ BC. Find EC in (i) and AD in (ii)

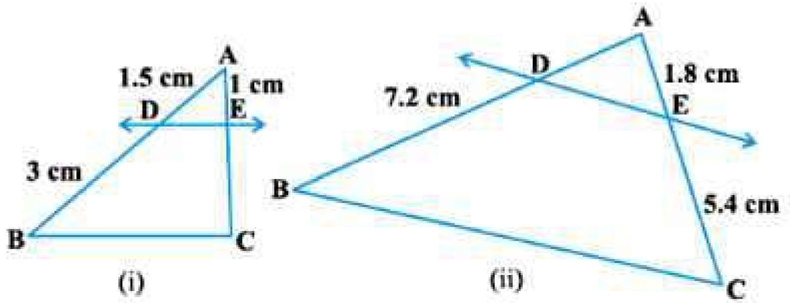


Fig. 6.17

B

Answer: (i) Let us take EC = x cm

Given: DE ∥ BC

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

Now, using basic proportionality theorem, we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{x}$$

$$X = \frac{3\times1}{1.5}$$

x = 2 cm

Hence, EC = 2 cm

(ii) Let us take AD = x cm

Given: DE ∥ BC

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

Now, using basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

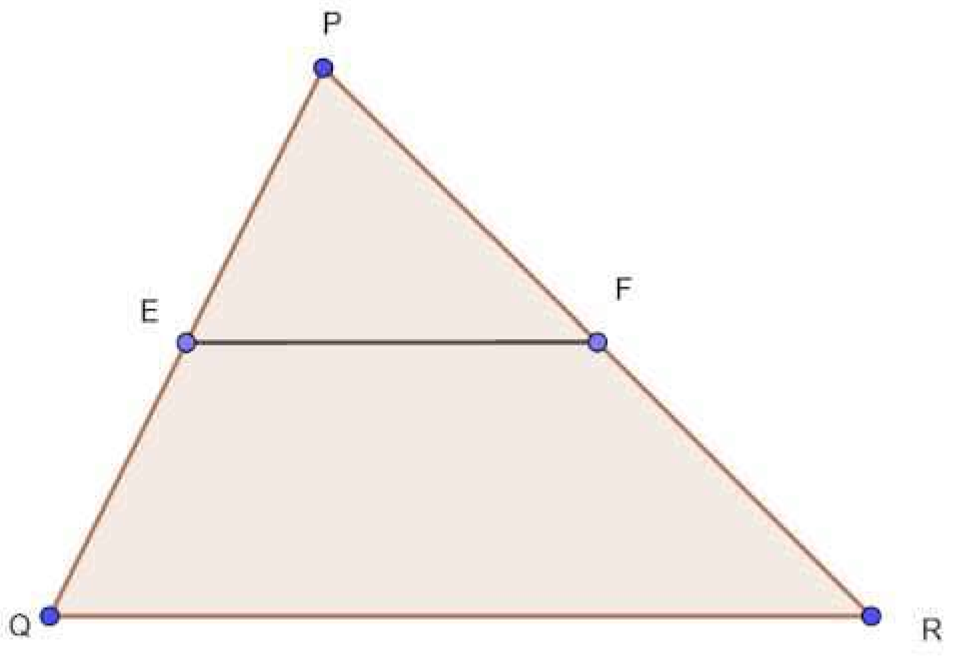
Hence, AD = 2.4 cm

Q. 2 E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF ∥ QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm



Given :

PE = 3.9 cm,

EQ = 3 cm,

PF = 3.6 cm,

FR = 2.4 cm

Now we know,

Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally.

So, if the lines EF and QR are to be parallel, then ratio PE:EQ should be proportional to PF:PR

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

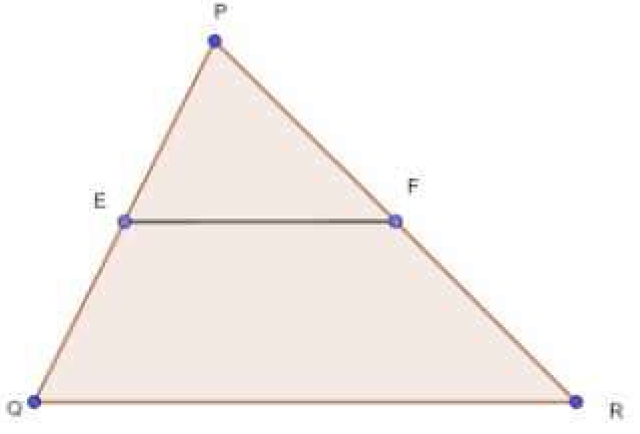
$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence,

$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR

(ii)



We know that, Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally.

So, if the lines EF and QR are to be parallel, then ratio PE:EQ should be proportional to PF:PR

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

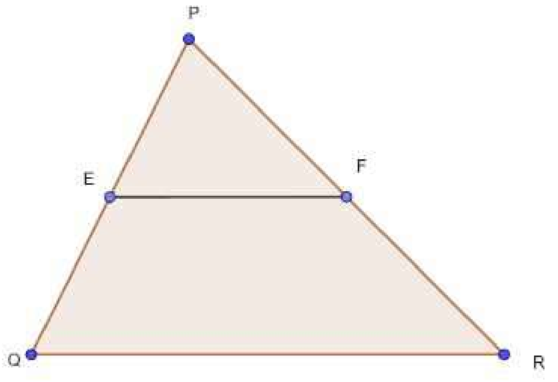
$$\frac{PF}{FR} = \frac{8}{9}$$

Hence,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF is parallel to QR

(iii)



In this we know that,

Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides of the triangle, then the line divides these two sides proportionally.

So, if the lines EF and QR are to be parallel, then ratio PE:EQ should be proportional to PF:PR

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{8}{128} = \frac{9}{64}$$

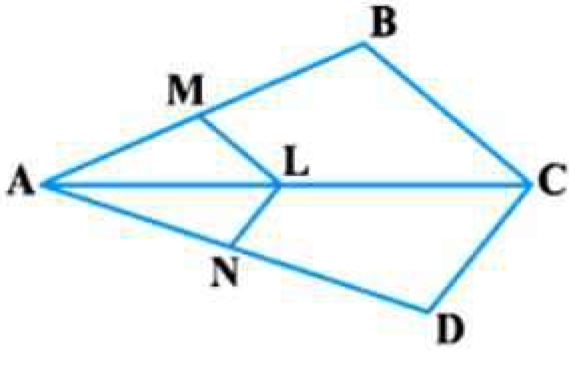
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence,

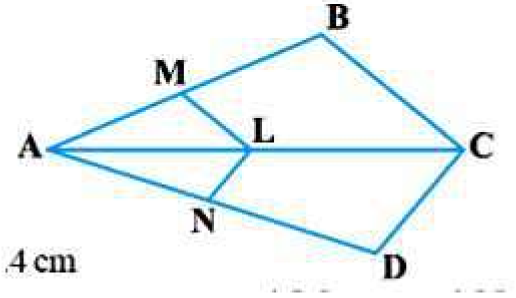
$$\frac{PE}{PQ} = \frac{PF}{PR}$$

EF is parallel to QR

Q. 3 In Fig. 6.18, if LM ∥ CB and LN ∥ CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$



Answer:



To Prove: $\frac{AM}{AB} = \frac{AN}{AD}$

Given: LM ll CB and LN ll CD From the given figure: In ΔALM and ΔABC

LM ∥ CB

Proportionality theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion

Now, using basic proportionality theorem that the corresponding sides will have same proportional lengths, we get:

$\frac{AM}{AB} = \frac{AL}{AC}$ (i)

Similarly, LN parallel to CD

Therefore,

$\frac{AN}{AD} = \frac{AL}{AC}$ (ii)

From (i) and (ii), we obtain

$\frac{AM}{AB} = \frac{AN}{AD}$

Hence, proved.

Q. 4 In Fig. 6.19, DE ∥ AC and DF ∥ AE. Prove that

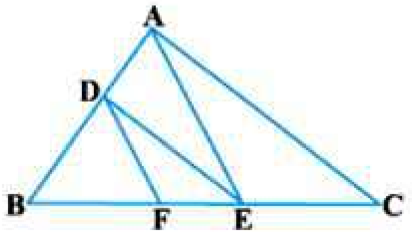
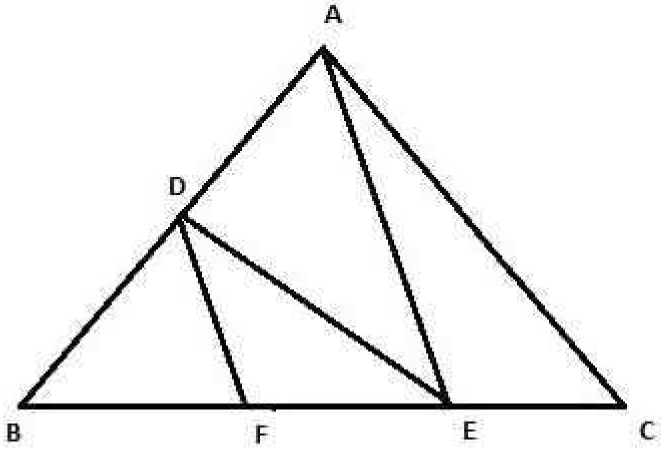


Fig. 6.19

$\frac{BF}{FE} = \frac{BE}{EC}$

Answer:



To Prove $\frac{BF}{FE} = \frac{BE}{EC}$

Given:

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion. In triangle ABC, DE is parallel to AC

Therefore,

By Basic proportionality theorem

$\frac{BF}{FE} = \frac{BE}{EC}$ ….. (1)

In triangle BAE, DF is parallel to AE

In triangle BAE, DF is parallel to AE

Therefore, By Basic proportionality theorem

$\frac{BD}{DA} = \frac{BF}{FE}$ …… (2)

From (1) and (2), we get

$\frac{BE}{EC} = \frac{BF}{FE}$

Hence, Proved.

Q. 5 In Fig. 6.20, DE ∥ OQ and DF ∥ OR. Show that EF ∥ QR.

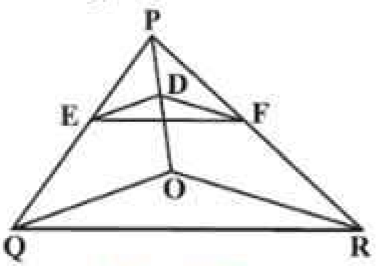


Fig. 6.20

Answer: To Prove: EF ∥ QR

Given: In triangle POQ, DE parallel to OQ Proof:

In triangle POQ, DE parallel to OQ

Hence,

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

$\frac{PE}{Eq} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)

Now,

In triangle POR, DF parallel OR

Hence,

$\frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we get

$\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore,

EF is parallel to QR (Converse of basic proportionality theorem)

Hence, Proved.

Q. 6 In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB ∥ PQ and AC ∥ PR. Show that BC ∥ QR.

Answer: To Prove: BC ll QR

Given that in triangle POQ, AB parallel to PQ

Hence,

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

$\frac{OA}{AP} = \frac{OB}{BQ}$ (Basic proportionality theorem)

Now,

Therefore,

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

Using Basic proportionality theorem, we get:

$\frac{OA}{AP} = \frac{OC}{CR}$

From above equations, we get

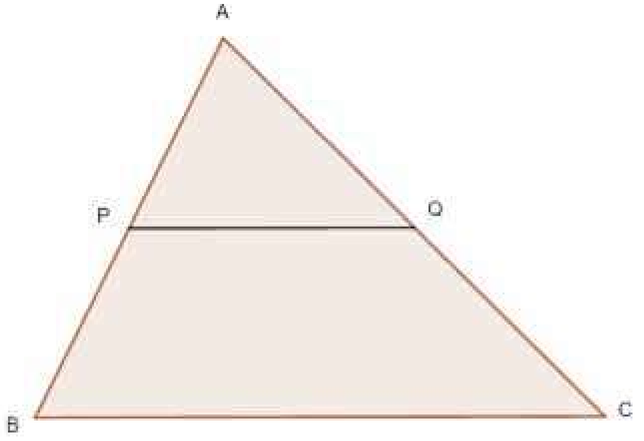
$\frac{OB}{BQ} = \frac{OC}{CR}$

BC is parallel to QR (By the converse of Basic proportionality theorem)

Hence, Proved.

Q. 7 Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX)

Answer: Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that PQ is parallel to BC.



To Prove: PQ bisects AC

Given: PQ ll BC and PQ bisects AB

Proof:

According to Theorem 6.1: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion. Now, using basic proportionality theorem, we get

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1}$$

[As AP = PB coz P is the mid-point of AB]

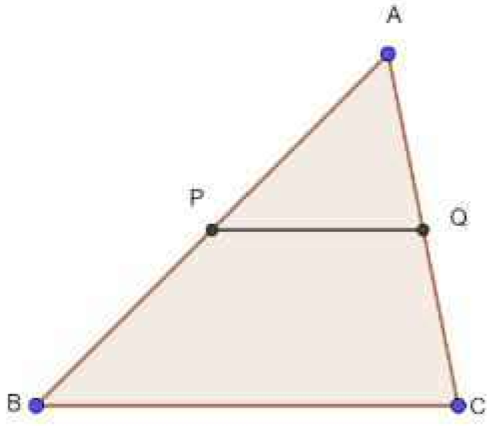
Hence,

AQ = QC

Or, Q is the mid-point of AC

Hence proved.

Q. 8 Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX)



To Prove: PQ || BC

Given: P and Q are midpoints of AB and AC

Proof:

Let us take the given figure in which PQ is a line segment which joins the mid-points P and Q of line AB and AC respectively

i.e., AP = PB and AQ = QC

We observe that,

$$\frac{AP}{PB} = \frac{1}{1}$$

And,

$$\frac{AQ}{QC} = \frac{1}{1}$$

Therefore,

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.

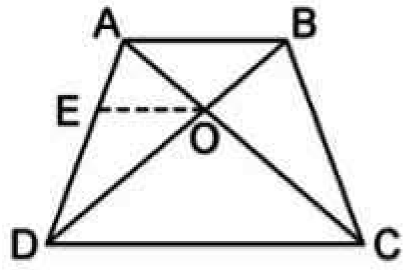
Hence, using basic proportionality theorem we get:

PQ parallel to BC

Hence, Proved.

Q. 9 ABCD is a trapezium in which AB ∥ DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer: The figure is given below:



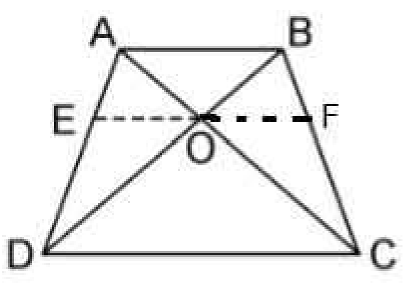
Given: ABCD is a trapezium

AB ||| CD

Diagonals intersect at O

To Prove = $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Construct a line EF through point O, such that EF is parallel to CD.



Proof:

In ΔADC, EO is parallel to CD

According to basic proportionality theorem, if a side is drawn parallel to any side of the triangle then the corresponding sides formed are proportional.

Now, using basic proportionality theorem in ΔABD and ΔADC, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

In ΔABD, OE is parallel to AB

So, using basic proportionality theorem in ΔEOD and ΔABD, we get

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad \text{(ii)}$$

From (i) and (ii), we get

$$\frac{AO}{OC} = \frac{BO}{OD}$$

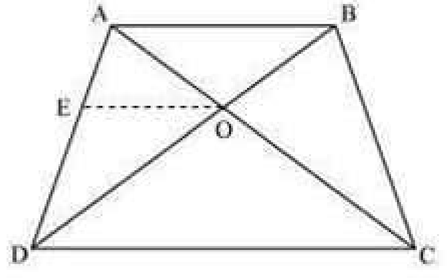
Therefore by cross multiplying we get,

$$\frac{AO}{BO} = \frac{OC}{OD}$$

Hence, Proved.

Q. 10 The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium

Answer: The quadrilateral ABCD is shown below, BD and AC are the diagonals.



Construction: Draw a line OE parallel to AB

Given: In ΔABD, OE is parallel to AB

To prove: ABCD is a trapezium

According to basic proportionality theorem, if in a triangle another line is drawn parallel to any side of triangle, then the sides so obtain are proportional to each other.

Now, using basic proportionality theorem in ΔDOE and ΔABD, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad \text{.... (i)}$$

It is given that,

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad \text{... (ii)}$$

From (i) and (ii), we get

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad \text{.... (iii)}$$

Now for ABCD to be a trapezium AB has to be parallel of CD

Now From the figure we can see that If eq(iii) exists then,

EO ∥ DC (By the converse of basic proportionality theorem)

Now if,

⇒ AB ∥ OE ∥ DC

Then it is clear that

⇒ AB ∥ CD

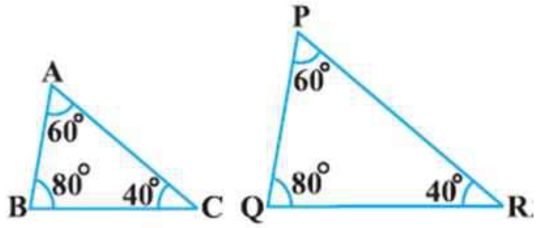
Thus the opposite sides are parallel and therefore it is a trapezium.

Hence,

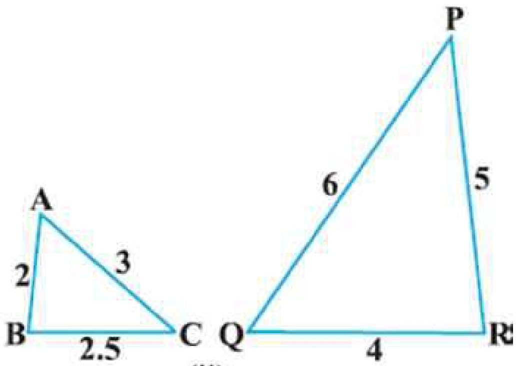
ABCD is a trapezium.

Exercise 6.3

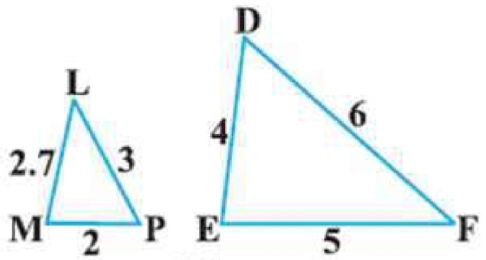
Q. 1 State which pairs of triangles in Fig. are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



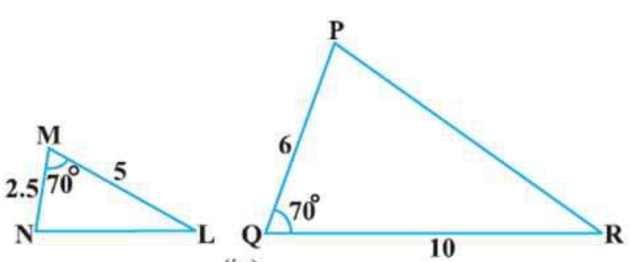
(i)



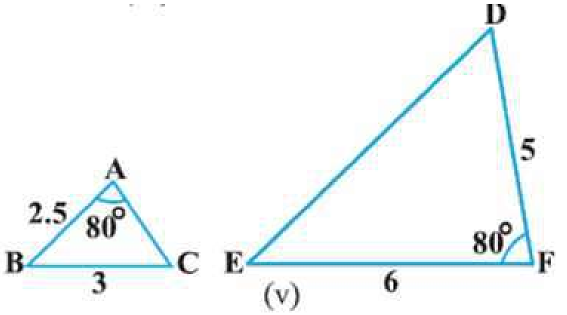
(ii)



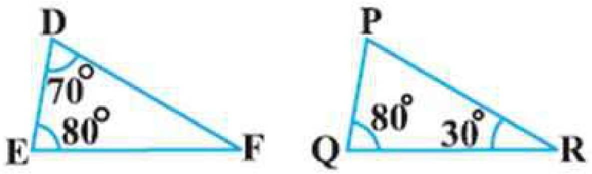
(iii)



(iv)



(v)



(vi)

Answer: (i) From the figure:

∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC ~ ΔPQR [By AAA similarity]

Now corresponding sides of triangles will be proportional,

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

(ii) From the triangle, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = 0.5$

Hence the corresponding sides are propotional. Thus the corresponding angles will be equal. The triangles ABC and QRP are similar to each other by SSS similarity

(iii) The given triangles are not similar because the corresponding sides are not proportional

(iv) In triangle MNL and QPR, we have

∠M = ∠Q = 70°

But

$\frac{MN}{PQ} = \frac{2.5}{6} = \frac{5}{12}$

$\frac{ML}{PR} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \frac{MN}{PQ} \neq \frac{ML}{PR}$

Therefore, MNL and QPR are not similar.

(v) In triangle ABC and DEF, we have

AB = 2.5, BC = 3

∠A = 80°

EF = 6

DF = 5

∠F = 80°

$\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

And, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$
∠B ≠ ∠F

Hence, triangle ABC and DEF are not similar

(vi) In triangle DEF, we have
$\angle D + \angle E + \angle F = 180^o$ (Sum of angles of triangle)

$70^o + 80^o + \angle F = 180^o$
$\angle F = 30^o$
In PQR, we have
$\angle P + \angle Q + \angle R = 180^o$
$\angle P + 80^o + 30^o = 180^o$
$\angle P = 70^o$
In triangle DEF and PQR, we have

$\angle D = \angle P = 70^o$
$\angle F = \angle Q = 80^o$

∠F = ∠R = 30°

Hence, ΔDEF ~ ΔPQR (AAA similarity)

Q. 2 In Fig. 6.35, ΔODC ~ ΔOBA, ∠BOC = 125° and ∠CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB

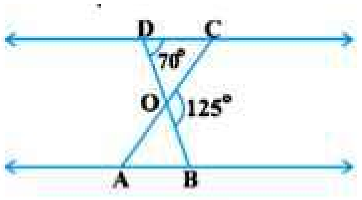


Fig. 6.35

Answer: From the figure,

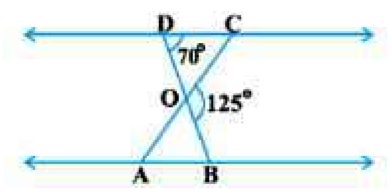


Fig. 6.35

We see, DOB is a straight line

∠ DOC + ∠ COB = 180° (angles on a straight line form a supplementary pair)

∠ DOC = 180° - 125°

∠ DOC = 55°

Now, In ΔDOC,

∠ DCO + ∠ CDO + ∠ DOC = 180°

(Sum of the measures of the angles of a triangle is 180°)

∠ DCO + 70° + 55° = 180°

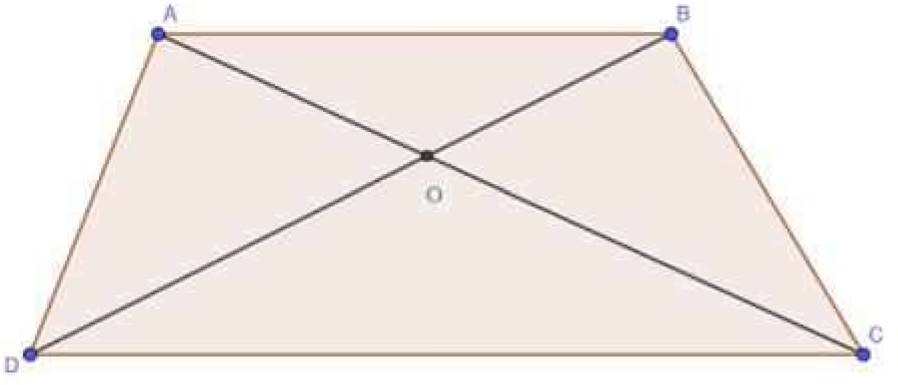
∠ DCO = 55°

It is given that ΔODC ~ΔOBA

∠ OAB = ∠ OCD (Corresponding angles are equal in similar triangles)

Thus, ∠ OAB = 55°.

Q. 3 Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.



In ΔDOC and ΔBOA,

∠CDO = ∠ABO (Alternate interior angles as AB ∥ CD)

∠DCO = ∠BAO (Alternate interior angles as AB ∥ CD)

∠DOC = ∠BOA (Vertically opposite angles)

Therefore,

ΔDOC ~ ΔBOA [BY AAA similarity]Now in similar triangles, the ratio of corresponding sides are proportional to each other. Therefore,

$\frac{OA}{OC} = \frac{OB}{OD}$ ….. (Corresponding sides are proportional)

or $\frac{AO}{CO} = \frac{BO}{DO}$

Hence proved.

Q. 4 In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and < 1 = <2. Show that $\Delta PQS \sim \Delta TQR$.

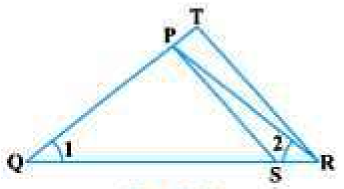


Fig. 6.36

Answer:

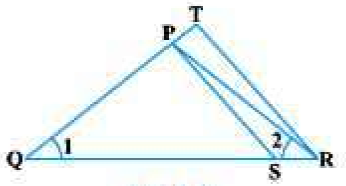


Fig. 6.36

To Prove: Δ PQS ~ Δ TQR
Given: In ΔPQR,

∠PQR = ∠PRQ

Proof: As ∠PQR = ∠PRQ

PQ = PR [sides opposite to equal angles are equal] (i)
Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\frac{QR}{QS} = \frac{QT}{QP} \quad \text{by (1)}$$

In Δ PQS and Δ TQR, we get

$$\frac{QR}{QS} = \frac{QT}{QP}$$

∠Q = ∠Q

Therefore,

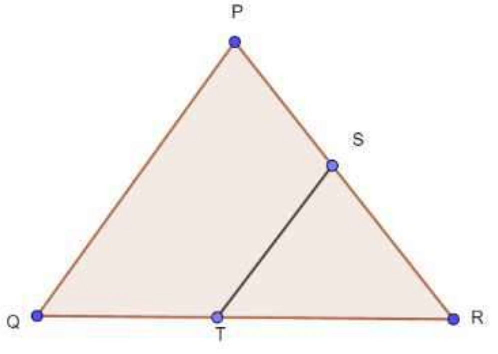
By SAS similarity Rule which states that Triangles are similar if two sides in one triangle are in the same proportion to the corresponding sides in the other, and the included angle are equal.

Δ PQS ~ Δ TQR

Hence, Proved.

Q. 5 S and T are points on sides PR and QR of Δ PQR such that ∠P = ∠RTS. Show that Δ RPQ ~ Δ RTS

Answer:



In ΔRPQ and ΔRST,

∠ RTS = ∠ QPS (Given)

∠ R = ∠ R (Common to both the triangles)

If two angles of two triangles are equal, third angle will also be equal. As the sum of interior angles of triangle is constant and is 180°

∴ ΔRPQ ~ ΔRTS (By AAA similarity).

Q. 6 In Fig. 6.37, if Δ ABE ≅Δ ACD, show that Δ ADE ~ Δ ABC.
Answer:
To Prove: Δ ADE ~ Δ ABC
Given: ΔABE ≅ ΔACD
Proof: ΔABE ≅ ΔACD
∴ AB = AC (By CPCT) (i)
And,
AD = AE (By CPCT) (ii)

In ΔADE and ΔABC,

Dividing equation (ii) by (i)

$$\frac{AB}{AD} = \frac{AC}{AE}$$

∠A = ∠A (Common)
SAS Similarity: **Triangles** are **similar** if two sides in one **triangle** are in the same proportion to the corresponding sides in the other, and the included angle are equal.

Therefore,

ΔADE~ΔABC (By SAS similarity)
Hence, Proved.

Q. 7 In Fig. 6.38, altitudes AD and CE of Δ ABC intersect each other at the point P.
Show that:

(i) Δ AEP ~ Δ CDP
(ii) Δ ABD ~ Δ CBE
(iii) Δ AEP ~ Δ ADB
(iv) Δ PDC ~ Δ BEC

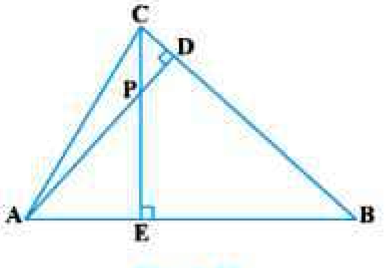


Fig. 6.38

(i) In ΔAEP and ΔCDP,

∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)
Hence, by using AA similarity,

$\Delta AEP \sim \Delta CDP$

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ (Each 90°)
$\angle ABD = \angle CBE$ (Common)
Hence, by using AA similarity,
$\Delta ABD \sim \Delta CBE$

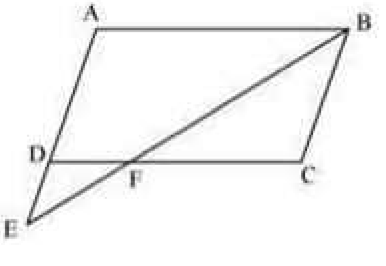
(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ (Each 90°)
$\angle PAE = \angle DAB$ (Common)
Hence, by using AA similarity,
$\Delta AEP \sim \Delta ADB$

(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ (Each 90°)
$\angle PCD = \angle BCE$ (Common angle)
Hence, by using AA similarity,
$\Delta PDC \sim \Delta BEC$

Q. 8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$.
Answer:



To Prove: $\Delta ABE \sim \Delta CFB$
Given: E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. As shown in the figure.
Proof:

In ΔABE and ΔCFB,

∠A = ∠C (Opposite angles of a parallelogram are equal)

∠AEB = ∠CBF (Alternate interior angles are equal because AE ∥ BC)

Therefore,

ΔABE ~ ΔCFB (By AA similarity)

Hence, Proved.

Q. 9 In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) Δ ABC ~ Δ AMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

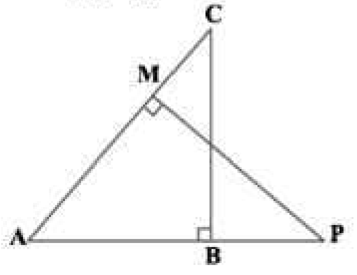


Fig. 6.39

Answer: (i) To Prove: Δ ABC ~ Δ AMP

Given: In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

Proof:

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ΔABC ~ ΔAMP (By AA similarity)

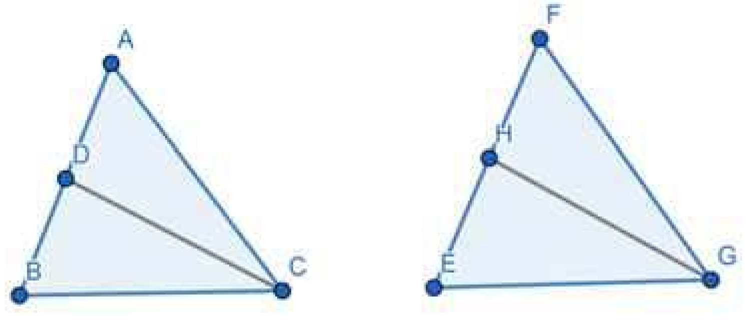
Hence, Proved.

(ii) **Δ ABC ~ Δ AMP**

Now we get that, Similarity Theorem - If the lengths of the corresponding sides of two triangles are proportional, then the triangles must be similar. And the converse is also true, so we have
$\frac{CA}{PA} = \frac{BC}{MP}$

Hence, Proved.

Q. 10 CD and GH are respectively the bisectors of ∠ ACB and ∠ EGF in such a way that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that:
(i) $\frac{CD}{GH} = \frac{AC}{FG}$
(ii) Δ DCB ~ Δ HGE
(iii) Δ DCA ~ Δ HGF
Answer:



Given, Δ ABC ~ Δ FEG …..eq(1)

⇒ corresponding angles of similar triangles

⇒ ∠ BAC = ∠ EFG ….eq(2)

And ∠ ABC = ∠ FEG …….eq(3)

⇒ ∠ ACB = ∠ FGE
⇒ $\frac{1}{2} < ACB = \frac{1}{2} < FGE$

⇒ ∠ ACD = ∠ FGH and ∠ BCD = ∠ EGH ……eq(4)

Consider Δ ACD and Δ FGH

⇒ From eq(2) we have

⇒ ∠ DAC = ∠ HFG

⇒ From eq(4) we have

⇒ ∠ ACD = ∠ EGH

Also, ∠ ADC = ∠ FGH

⇒ If the 2 angle of triangle are equal to the 2 angle of another triangle, then by angle sum property of triangle 3rd angle will also be equal.

⇒ by AAA similarity we have in two triangles if the angles are equal, then sides opposite to the equal angles are in the same ratio (or proportional) and hence the triangles are similar.

∴ Δ ADC ~ Δ FHG

⇒ By Converse proportionality theorem

⇒ $\frac{CD}{GH} = \frac{AC}{FG}$

Consider Δ DCB and Δ HGE

From eq(3) we have

⇒ ∠ DBC = ∠ HEG

⇒ From eq(4) we have

⇒ ∠ BCD = ∠ FGH

Also, ∠ BDC = ∠ EHG

∴ Δ DCB ~ ΔHGE

Hence proved.

Q. 11 In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF

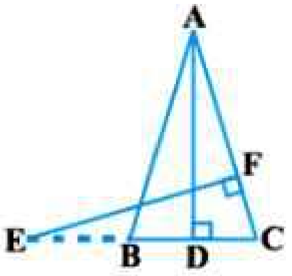
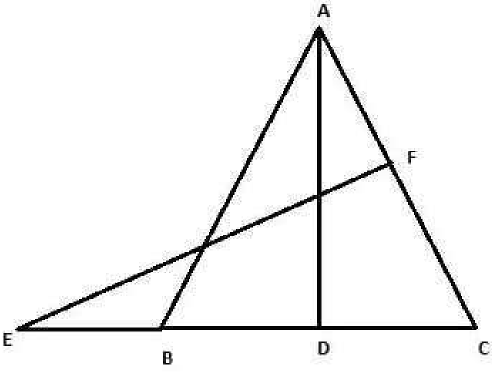


Fig. 6.40

Answer:



To Prove: Δ ABD ~ Δ ECF
Given: ABC is an isosceles triangle, AD is perpendicular to BC
BC is produced to E and EF is perpendicular to AC
Proof:
Given that ABC is an isosceles triangle

AB = AC

⇒ ∠ABD = ∠ECF

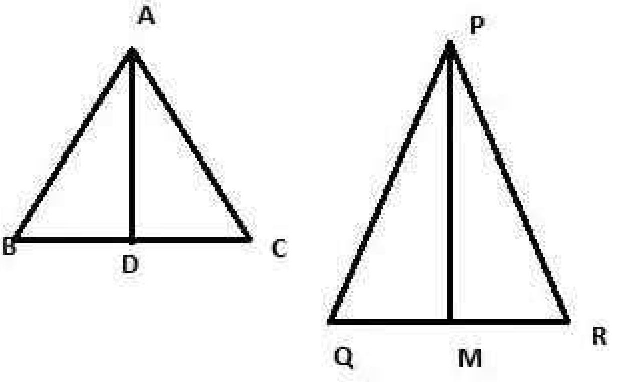
In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

∠ABD = ∠ECF (Proved above)

Therefore,
ΔABD ~ ΔECF (By using AA similarity criterion)
AA Criterion: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
Hence, Proved.

Q. 12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of Δ PQR (see Fig. 6.41). Show that Δ ABC ~ Δ PQR.
Answer:



To Prove: Δ ABC ~ Δ PQR
Given:
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Proof: Median divides the opposite side

$BD = \frac{BC}{2}$ and,

$QM = \frac{QR}{2}$

Now,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Multiplying and dividing by 2, we get

$\frac{AB}{PQ} = \frac{\frac{2}{2BC}}{\frac{1}{2QR}} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

In Δ ABD and Δ PQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Side-Side-Side (SSS) Similarity Theorem - If the lengths of the corresponding sides of two triangles are proportional, then the triangles must be similar.

ΔABD ~ ΔPQM (By SSS similarity)

∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

$\frac{AB}{PQ} = \frac{BC}{QR}$

The SAS Similarity Theorem states that if two sides in one triangle are proportional to two sides in another triangle and the included angle in both are **congruent**, then the two triangles are similar.

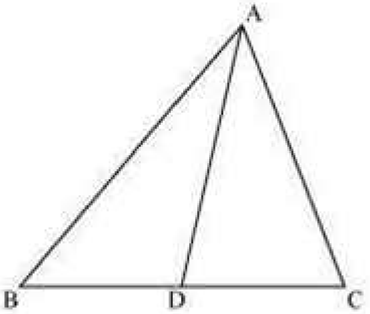
ΔABC ~ ΔPQR (By SAS similarity)

Hence, Proved.

Q. 13 D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that $CA^2 = CB.CD$

Answer:

In ΔADC and ΔBAC,

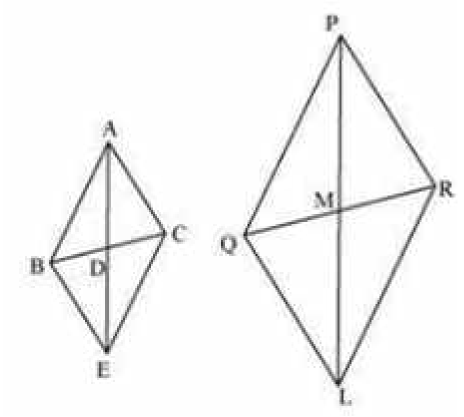


To Prove: CA^2 = CB . CD
Given: ∠ADC = ∠BAC
Proof: Now In ΔADC and ΔBAC,
∠ADC = ∠BAC

∠ACD = ∠BCA (Common angle)
According to AA similarity, if two corresponding angles of two triangles are equal then the triangles are similar
ΔADC ~ ΔBAC (By AA similarity)

We know that corresponding sides of similar triangles are in proportion

Hence in ΔADC and ΔBAC,
$$\frac{CA}{CB} = \frac{CD}{CA}$$
$$CA^2 = CB \times CD$$
Hence Proved.
Q. 14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR.
Show that Δ ABC ~ Δ PQR
Answer: To Prove: Δ ABC ~ Δ PQR
Given:
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$
Proof



Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML.
Then, join B to E, C to

E, Q to L, and R to L

We know that medians divide opposite sides.

Hence, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC,

Diagonals AE and BC bisect each other at point D.

Therefore,

Quadrilateral ABEC is a parallelogram.

AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given in the question that,

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AR}{PL}$

ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∠BAE = ∠QPL ..... (i)

Similarly, it can be proved that

ΔAEC ~ ΔPLR and

∠CAE = ∠RPL ..... (ii)

Adding equation (i) and (ii), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒∠CAB = ∠RPQ .... (iii)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

∠CAB = ∠RPQ [Using equation (iii)]

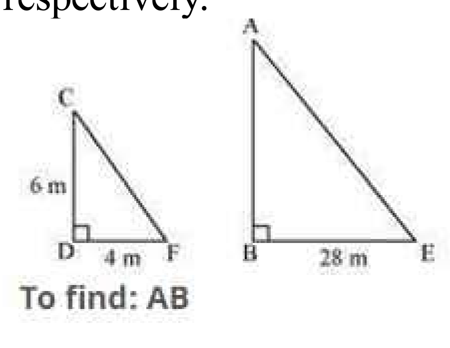
ΔABC ~ ΔPQR (By SAS similarity criterion)

Hence, Proved.

Q. 15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower

Answer: Let AB and CD be a tower and a pole respectively

And, the shadow of BE and DF be the shadow of AB and CD respectively.



At the same time, the light rays from the sun will fall on the tower and the pole at the same angle

Therefore,

∠DCF = ∠BAE

And,

∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

ΔABE ~ ΔCDF (AAA similarity)

Hence, By the properties of similar triangles that if two triangles are similar, their corresponding sides will be proportional.

$\frac{AB}{CD} = \frac{BF}{DF}$

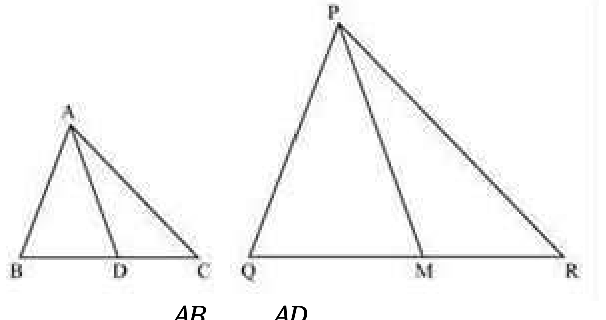
$\frac{AB}{6} = \frac{28}{4}$

AB = 42 m

Height of the Tower = 42 m

Q. 16 If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer: It is given that ΔABC is similar to ΔPQR



To Prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Given: Δ ABC ~ Δ PQR

AD and PM are medians

We know that the corresponding sides of similar triangles are in proportion

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ … eq(i)

And also the corresponding angles are equal

∠A = ∠P

∠B = ∠Q

∠C = ∠R ....eq(ii)

Since AD and PM are medians, they divide their opposite sides in two equal parts

$BD = \frac{BC}{2}$ and

$QM = \frac{QR}{2}$ … eq. (iii)

From (i) and (iii), we get

$\frac{AB}{PQ} = \frac{BD}{QM}$ (iv)

In ΔABD and ΔPQM,

∠B = ∠Q [Using (ii)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using (iv)]

ΔABD ~ ΔPQM (Since two sides are proportional and one angle is equal then by SAS similarity)

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

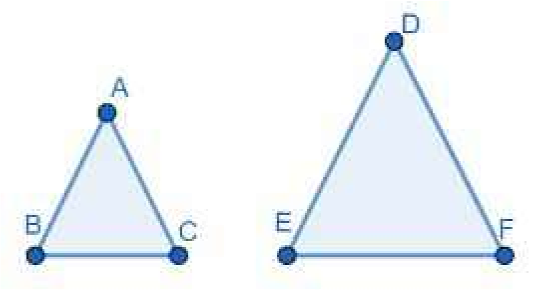
Hence, Proved.

Exercise 6.4

Q.1 Let Δ ABC ~ Δ DEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC

Answer: It is given that,

Δ ABC ~ Δ DEF



Therefore,

$$\frac{ar\,(\Delta ABC)}{ar\,(DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given:

EF = 15.4 cm

ar (ΔABC) = 64cm^2

ar (ΔDEF) = 121 cm^2

Hence,

$$\frac{ar\,(\Delta ABC)}{ar\,(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{64}{121} = \frac{BC \times BC}{15.4 \times 15.4}$$

Taking square root on both of the sides

$$\frac{BC}{15.4} = \frac{8}{11}$$

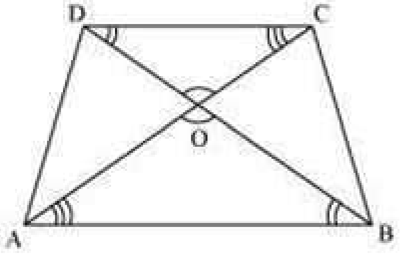
BC = (8 × 15.4) / 11

BC = 8 × 1.4 = 11.2 cm

Q. 2 Diagonals of a trapezium ABCD with AB ∥ DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD

Since AB ∥ CD,

∴∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)



n ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

ΔAOB ~ΔCOD (By AAA similarity)

When two triangles are similar, the reduced ratio of any two corresponding sides is called the **scale factor** of the similar triangles. If two similar triangles have a scale factor of $a : b$, then the ratio of their areas is $a^2 : b^2$.

$$\frac{ar\,(\Delta AOB)}{ar\,(\Delta COD)} = \frac{AB}{CD} \times \frac{AB}{CD}$$

Since, AB = 2 CD (Given)

Therefore,

$$\frac{ar\,(\Delta AOB)}{ar\,(\Delta COD)} = \frac{2CD \times 2CD}{CD \times CD}$$

$$\frac{ar\,(\Delta AOB)}{ar\,(\Delta COD)} = \frac{4}{1} = 4:1$$

Therefore, the ratio of the areas of triangles AOB and COD is 4:1.

Q. 3 In Fig. 6.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar\,(ABC)}{ar\,(DBC)} = \frac{AO}{DO}$.

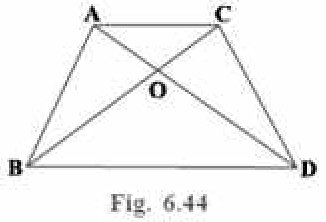
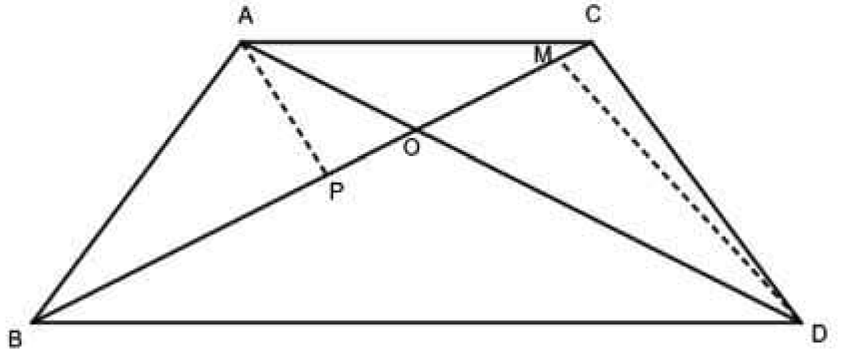


Fig. 6.44

Answer:



Construction: Draw two perpendiculars AP and DM on line BC and AB

To Prove $= \frac{area\ of\ \Delta ABC}{area\ of\ \Delta DBC} = \frac{AO}{DO}$

Area of a triangle = 1/2 × Base × Height

Therefore,

$\frac{ar\,(\Delta ABC)}{ar\,(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times BC \times DM}$

$= \frac{AP}{DM}$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity)

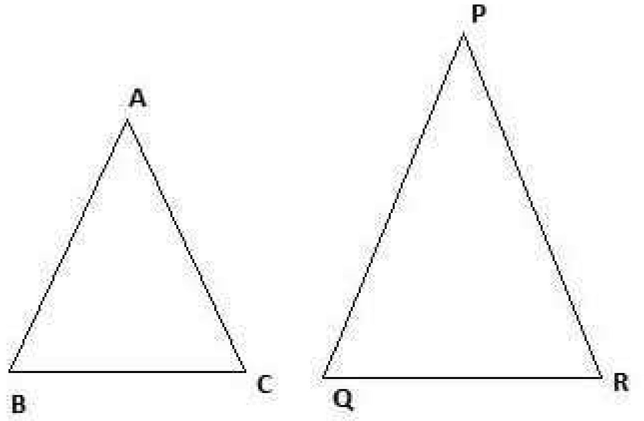
As we know in similar triangles the sides are proportional to each other.

$\frac{AP}{DM} = \frac{AO}{DO}$

$\frac{ar\,(\Delta ABC)}{ar\,(\Delta DBC)} = \frac{AO}{DO}$

Hence, proved.

Q. 4 If the areas of two similar triangles are equal, prove that they are congruent



Let us consider two similar triangles as ΔABC ~ ΔPQR (Given)
When two triangles are similar, the reduced ratio of any two corresponding sides is called the **scale factor** of the similar triangles.
If two similar triangles have a scale factor of *a* : *b*, then the ratio of their areas is $a^2 : b^2$.

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta PQR)} = \left(\frac{\Delta AB}{\Delta PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad \text{.... (i)}$$

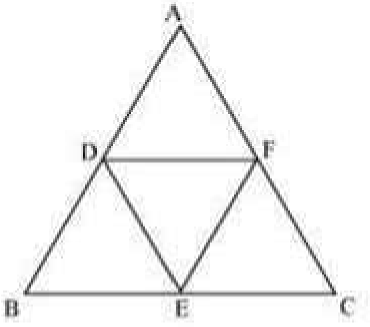
Given that,
ar(Δ ABC) = ar(Δ PQR)

Therefore putting in equation (i) we get,

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

AB = PQ
BC = QR
And,
AC = PR
Therefore,
ΔABC ≅ΔPQR (By SSS rule)
Hence, Proved.

Q. 5 D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of ΔDEF and Δ ABC.
Answer:



Given: D, E and F are the mid points of sides AB, BC and CA respectively.

Because D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC,

Midpoint Theorem: The line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is congruent to one half of the third side.
Therefore, From mid-point theorem,
DE ∥ AC and DE = $\frac{1}{2}$ AC

DF ∥ BC and DF = $\frac{1}{2}$ BC

EF ∥ AB and EF = $\frac{1}{2}$ AB
Now, In ΔBED and ΔBCA

∠BED =∠ BCA (Corresponding angles)

∠BDE = ∠BAC (Corresponding angles)

∠EBD = ∠CBA (Common angles)

Therefore,
If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).

ΔBED ~ ΔBCA (From the AAA similarity)

Theorem 6.6: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{ar\,(\Delta BED)}{ar\,(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\frac{ar\,(\Delta BED)}{ar\,(\Delta BCA)} = \left(\frac{DE}{2DE}\right)^2$$

$$\frac{ar\,(\Delta BED)}{ar\,(\Delta BCA)} = \frac{1}{4}$$

$$ar(\Delta BED) = \frac{1}{4}ar(\Delta BCA)$$

Similarly,

$$ar(\Delta CFE) = \frac{1}{4}ar(\Delta CBA)$$

And,

$$ar(\Delta ADF) = \frac{1}{4}ar(\Delta ABC)$$

Also,

$$ar(\Delta DEF) = ar(\Delta ABC) - [ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)]$$

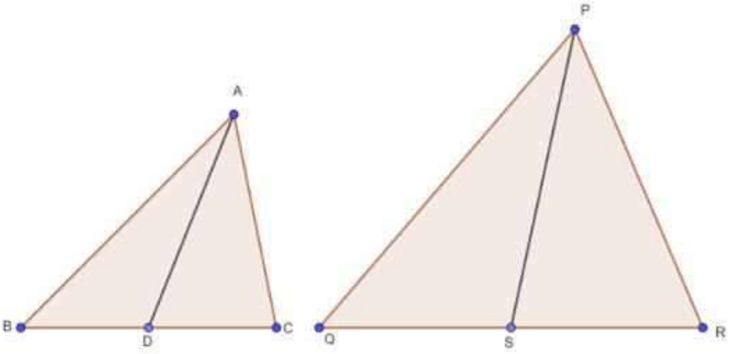
$$ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4}\,ar(\Delta ABC)$$

$$= \frac{1}{4}\,ar(\Delta ABC)$$

$$\frac{ar\,(\Delta DEF)}{ar\,(\Delta ABC)} = \frac{1}{4}$$

Q. 6 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer:



Let us assume two similar triangles as ΔABC ~ ΔPQR.

Let AD and PS be the medians of these triangles

Then, because ΔABC ~ΔPQR

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \qquad \text{.... (i)}$$

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ....(ii)

Since AD and PS are medians,

BD = DC =BC/2

And, QS = SR =QR/2

Equation (i) becomes,

$$\frac{AB}{PQ} = \frac{BD}{Qs} = \frac{AC}{PR} \qquad \text{..... (iii)}$$

In ΔABD and ΔPQS,

∠B = ∠Q [From (ii)]

And

$$\frac{AB}{PQ} = \frac{BD}{Qs} \quad \text{[From (iii)]}$$

ΔABD ~ ΔPQS (SAS similarity)

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{Qs} = \frac{AD}{PS} \qquad \text{.... (iv)}$$

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \left(\frac{\Delta AB}{\Delta PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From (i) and (iv), we get

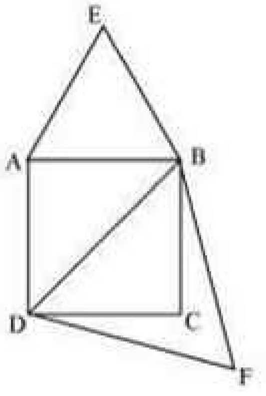
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

And hence,

$$\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \left(\frac{\Delta AD}{\Delta PS}\right)^2$$

Q.7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer: Let ABCD be a square of side a



To Prove = Area of ΔABE = 1/2 Area of ΔADB
Proof:
Let the side of square = aTo find the length of the diagonal of the square,
Pythagoras Theorem : It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.
Applying pythagoras theorem in ΔADB
$AD^2 + AB^2 = BD^2$
$BD = \sqrt{2}\, a$
Two desired equilateral triangles are formed as ΔABE and ΔDBF

Side of an equilateral triangle, ΔABE, described on one of its sides = a

Side of an equilateral triangle, ΔDBF, described on one of its diagonals = $\sqrt{2}\, a$

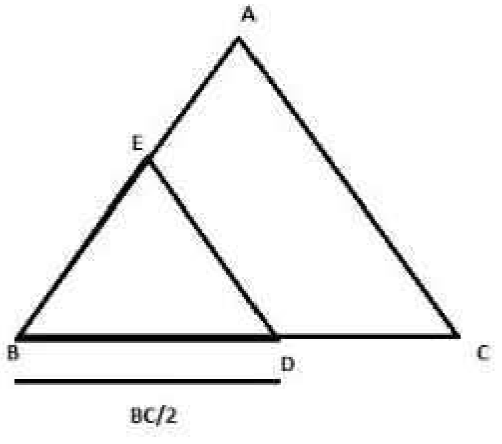
Area of an equilateral triangle $= \frac{\sqrt{3}}{4}\, side^2$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{\sqrt{3}}{4}(\sqrt{2}a)^2}$$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{a^2}{2a^2}$$

$Area\ of\ \Delta DBF = 2\ Area\ of\ \Delta\ ABE$
Therefore, Area of equilateral triangle on one of the side of the square is half of the area of equilateral triangle on diagonal.

Q. 8 ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is
A. 2 : 1 B. 1 : 2
C. 4 : 1 D. 1 : 4
Answer:



Given: D is mid point of BC
We know:

Equilateral triangles have all its angles as 60° and all its sides are of the same length. Therefore, all equilateral triangles are similar to each other.

Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore,
Side of $\Delta BDE = \frac{x}{2}$
Therefore,

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta BDE} = \left(\frac{Side\ of\ \Delta ABC}{Side\ of\ \Delta BDE}\right)^2 = \frac{\left(\frac{x}{x}\right)^2}{2}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta BDE} = 2^2 : 1$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta BDE} = 4 : 1$$

Hence, the correct answer is C.

Q. 9 Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio

A. 2 : 3 B. 4 : 9
C. 81 : 16 D. 16 : 81
Answer: If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.
It is given that the sides are in the ratio 4:9
Therefore,
Ratio between areas of these triangles $= \left(\frac{4}{9}\right)^2$

$= \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5

Q. 1 Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm
Answer: (i) Given: sides of the triangle are 7 cm, 24 cm, and 25 cm

Squaring the lengths of these sides, we get: 49, 576, and 625.

$49 + 576 = 625$
Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle satisfy Pythagoras theorem
Hence, it is a right triangle

We know that the longest side of a right triangle is the hypotenuse

Therefore, the length of the hypotenuse of this triangle is 25 cm

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm

Squaring the lengths of these sides, we will obtain 9, 64, and 36

However, $9 + 36 \neq 64$

Or, $32 + 62 \neq 82$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side

Therefore, the given triangle is not satisfying Pythagoras theorem

Hence, it is not a right triangle

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.
Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.
And, $2500 + 6400 \neq 10000$
Or, $502 + 802 \neq 1002$

Now, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side

Therefore, the given triangle is not satisfying Pythagoras theorem

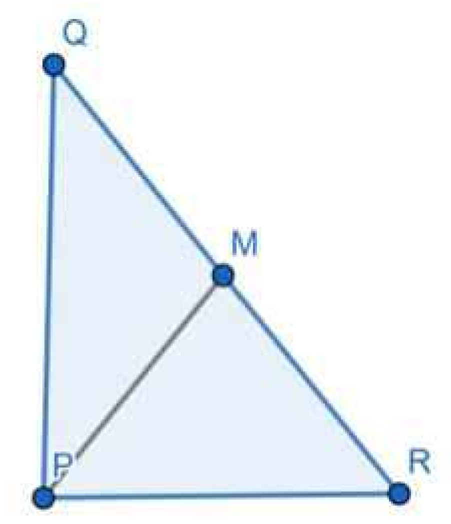
Hence, it is not a right triangle

(iv) Given: Sides are 13 cm, 12 cm, and 5 cm

Squaring the lengths of these sides, we get 169, 144, and 25.
Clearly, $144 + 25 = 169$
Or, $12^2 + 5^2 = 13^2$
The sides of the given triangle are satisfying Pythagoras theorem
Therefore, it is a right triangle
We know that the longest side of a right triangle is the hypotenuse
Therefore, the length of the hypotenuse of this triangle is 13 cm.

Q. 2 PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

Answer:



⇒ Let ∠MPR = x

⇒ In Δ MPR, ∠MRP = 180-90-x
⇒ ∠MRP = 90-x
Similarly in Δ MPQ,
∠MPQ = 90-∠MPR = 90-x
⇒ ∠MQP = 180-90-(90-x)
⇒ ∠MQP = x
In Δ QMP and Δ PMR
⇒ ∠MPQ = ∠MRP
⇒ ∠PMQ = ∠RMP
⇒ ∠MQP = ∠MPR
⇒ Δ QMP ~ Δ PMR
⇒ $\frac{QM}{PM} = \frac{MP}{MR}$
⇒ PM^2 = MR × QM
Hence proved.

Q. 3 In Fig. 6.53, ABD is a triangle right angled at A and AC ⊥ BD. Show that:

(i) AB^2 = BC. BD
(ii) AC^2 = BC. DC
(iii) AD^2 = BD. CD

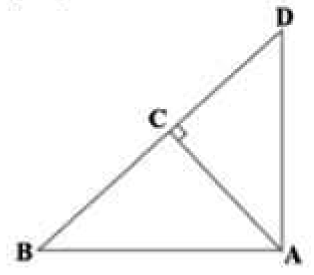


Fig. 6.53

Answer :

(i) In ΔADB and ΔCAB, we have

∠DAB = ∠ACB (Each of 90^o)

∠ABD = ∠CBA (Common angle)
Therefore,
ΔADB ~ ΔCAB (AA similarity)
$\frac{AB}{CB} = \frac{BD}{AB}$
$AB^2 = CB \times BD$
(ii) Let ∠CAB = x
In ΔCBA,
∠CBA + ∠CAB + ∠ACB = 180^o
As, ∠ACB = 90^o, we have
∠CBA = $180^o - 90^o - x$
∠CBA = $90^o - x$
Similarly, in ΔCAD
∠CAD = 90^o - ∠CBA
= $90^o - x$
∠CDA = $180^o - 90^o - (90^o - x)$
∠CDA = x
In triangle CBA and CAD, we have
∠CBA = ∠CAD
∠CAB = ∠CDA
∠ACB = ∠DCA (Each 90^o)
Therefore,
ΔCBA ~ ΔCAD (By AAA similarity)
$\frac{AC}{DC} = \frac{BC}{AC}$

$AC^2 = DC * BC$

(iii) In ΔDCA and ΔDAB, we have

∠DCA = ∠DAB (Each 90^o)

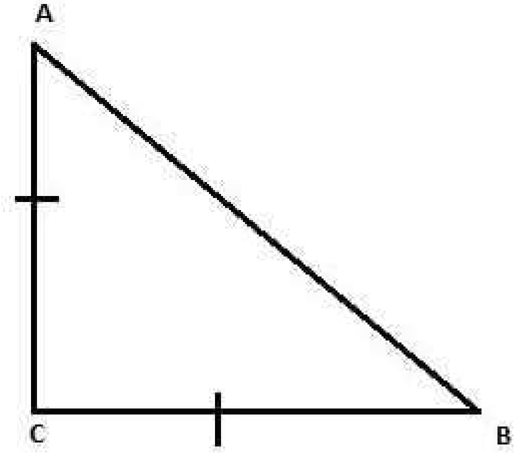
∠CDA = ∠ADB (Common angle)
Therefore,
ΔDCA ~ΔDAB (By AA similarity)

$$\frac{DC}{DA} = \frac{DA}{BD}$$

$AD^2 = BD \times CD$

Q. 4 ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer:



To Prove: $2\ AC^2 = AB^2$

Given: ΔABC is an isosceles triangle

Proof:

AC = CB (Two sides of an isosceles triangle are equal, as the side opposite to right angle is largest, rest of the two sides are equal)
Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

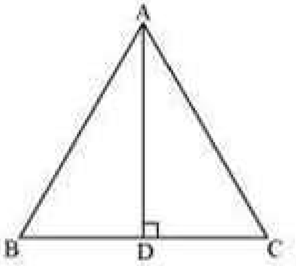
Using Pythagoras theorem in ΔABC (i.e., right-angled at point C), we get

$AC^2 + CB^2 = AB^2$

$AC^2 + AC^2 = AB^2$ (AC = CB)
So,
$2AC^2 = AB^2$
Hence, Proved.

Q. 5 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.
Answer: To Prove: ABC is a right angled triangle
Given: $AB^2 = 2AC^2$
Now 2 AC^2 can be split into two parts
$AB^2 = AC^2 + AC^2$
in an isosceles triangle ABC two sides are equal, and it is given that AC = BC. So,
$AB^2 = AC^2 + BC^2$ (As, AC = BC)
Now According to pythagoras theorem, in a right angled triangle, square of one side equals to the sum of squares of other two sides
And clearly above equation satisfies it. Thus, the equation satisfies pythagoras theorem and the triangle should be right angled for that.
Therefore, the given triangle is a right-angled triangle.
Hence, Proved.

Q. 6 ABC is an equilateral triangle of side 2a. Find each of its altitudes
Answer: Let AD be the altitude of the given equilateral triangle, ΔABC
We know that altitude bisects the opposite side
BD = DC = a

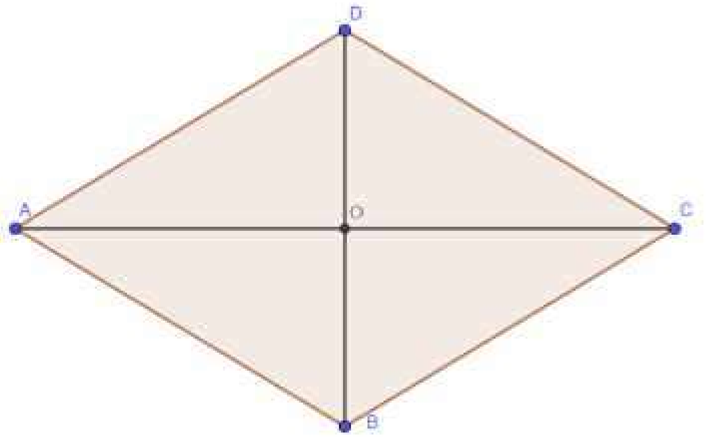


In triangle ADB,

$\angle ADB = 90^{O}$

Using Pythagoras theorem, we get
$AD^2 + DB^2 = AB^2$
$AD^2 + a^2 = (2a)^2$
$AD^2 + a^2 = 4a^2$
$AD^2 = 3a^2$
$AD = a\sqrt{3}$
In an equilateral triangle, all the altitudes are equal in length.
Hence, the length of each altitude will be $\sqrt{3}a$.

Q. 7 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
Answer: In Rhombus ABCD,
AB, BC, CD and AD are the sides of the rhombus.BD and AC are the diagonals.
To prove: $AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$
Proof:
The figure is shown below:



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$AB^2 = AO^2 + OB^2$ ....eq(i)
$BC^2 = BO^2 + OC^2$ ....eq (ii)
$CD^2 = CO^2 + OD^2$ ....eq(iii)
$AD^2 = AO^2 + OD^2$ ....eq (iv)
Now after adding all equations, we get,

$AB^2 + BC^2 + CD^2 + AD^2 = 2\,(AO^2 + OB^2 + OC^2 + OD^2)$
Diagonals of a rhombus bisect each other,
Thus AO = AC/2, OB = BD/2, OC=AC/2, and OD= BD/2

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$$

$$= 4\left[\frac{AC^2}{4} + \frac{BD^2}{4}\right]$$

$=(AC)^2 + (BD)^2$

Hence Sum of squares of sides of a rhombus equals to sum of squares of diagonals of rhombus.

Q. 8 In Fig. 6.54, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

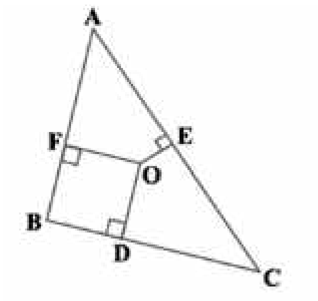
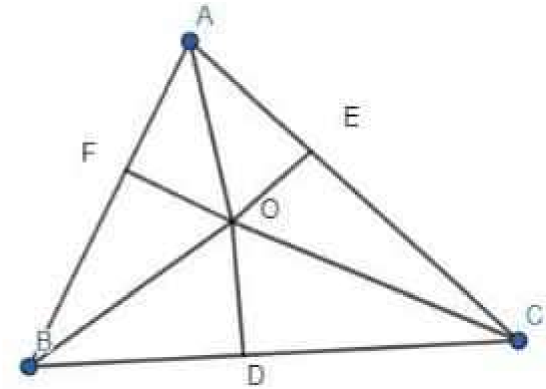


Fig. 6.54

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer: (i)



To Prove : $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$
Given: OD, OE and OF are perpendiculars on sides BC, AC and AB respectively
Construction : Join OA, OB and OC
Now according to pythagoras theorem, In a right angled triangle, $(\text{hypotenuse})^2 = (\text{altitude})^2 + (\text{base})^2$

Applying Pythagoras theorem in ΔAOF, we obtain

$OA^2 = OF^2 + AF^2$ ....eq(i)

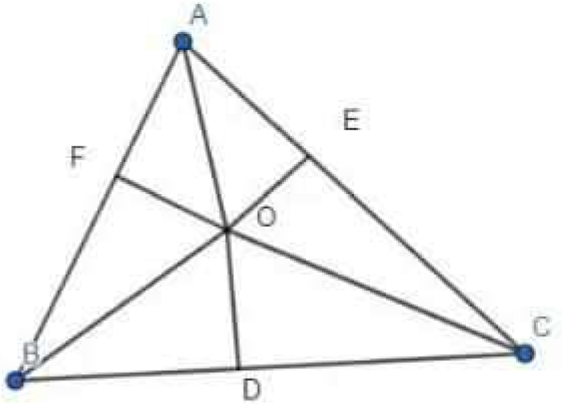
Similarly, in ΔBOD,

$OB^2 = OD^2 + BD^2$ ....eq(ii)

Similarly, in ΔCOE,

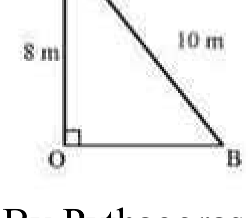
$OC^2 = OE^2 + EC^2$ ....eq(iii)

Adding these equations, we get
$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$
Rearranging the equations we get,
$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$
Hence, Proved
(ii)



To Prove: $AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$
From the above given result from (i),
$AF^2 + BD^2 + EC^2 = (AO^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$
and from eq(i), (ii) and (iii)
$AO^2 - OE^2 = AE^2$, $OC^2 - OD^2 = CD^2$, $OB^2 - OF^2 - BF^2$
Putting these values in above equation we get,
$AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$
Hence, Proved

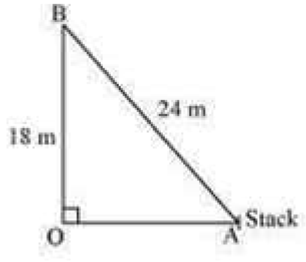
Q. 9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall
Answer: Let OA be the wall and AB be the ladder



By Pythagoras theorem,
$AB^2 = OA^2 + BO^2$

$(10)^2 = (8)^2 + OB^2$
$100 = 64 + OB^2$
$OB^2 = 36$
$OB = 6$ cm
Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

Q. 10 A wire attached to a vertical pole of height 18 m is 24 m long and has a stack attached to the other end. How far from the base of the pole should the stack be driven so that the wire will be taut?
Answer:
To find: OA
Let OB be the pole and AB be the wire
By Pythagoras theorem,
Pythagoras Theorem: the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.



$AB^2 = OB^2 + OA^2$

$(24)^2 = (18)^2 + OA^2$

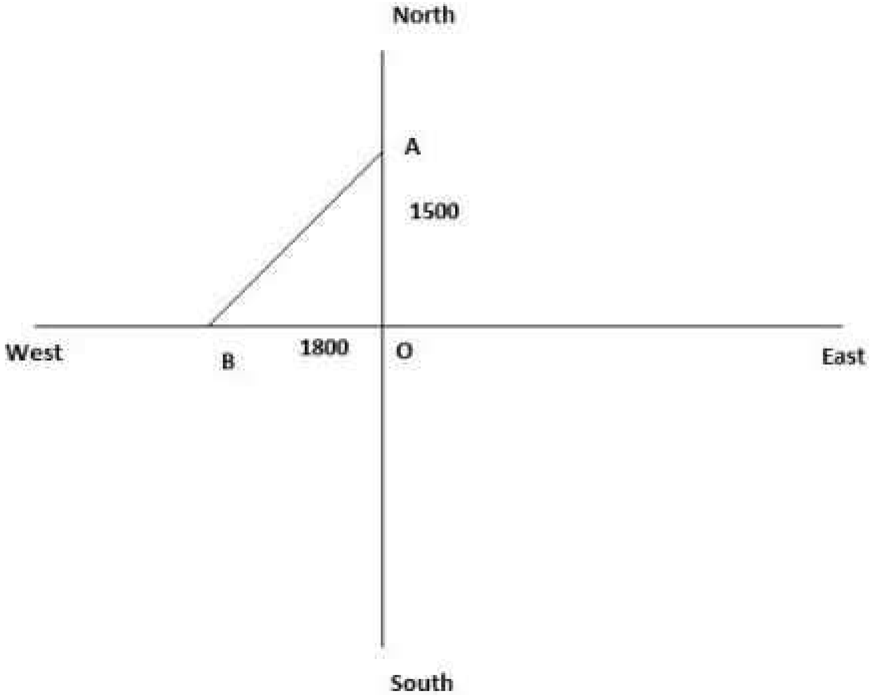
$OA^2 = (576 - 324)$

$OA^2 = 252$

$OA = 6\sqrt{7}$ m

Q. 11 An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer:



we know, Distance = speed × time Distance traveled by the plane flying towards north in $1\frac{1}{2} hrs = 1{,}000 \times 1\frac{1}{2}$
= 1,500 km

Similarly, distance traveled by the plane flying towards west in $1\frac{1}{2} hrs = 1{,}200 \times 1\frac{1}{2}$
= 1,800 km

Let these distances be represented by OA and OB respectively.
Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.
Applying Pythagoras theorem,
$AB^2 = OA^2 + OB^2$

$AB = \sqrt{OA^2 + OB^2}$

$AB = \sqrt{(1500)^2 + (1800)^2}$

$AB = \sqrt{2250000 + 3240000}$

$AB = \sqrt{5490000}$

$AB = 300\sqrt{61}$

Distances between planes is 300√61 km.

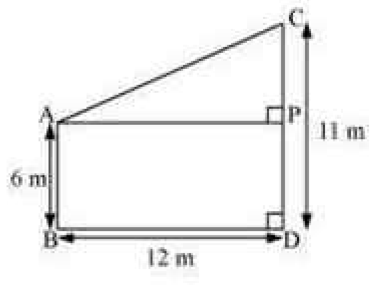
Q. 12 Two poles of heights 6 m and 11 m stand on aplane ground. If the distance between the feetof the poles is 12 m, find the distance between their tops

Answer: Let CD and AB be the poles of height 11 m and 6 m

Therefore, CP = 11 - 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain



$AP^2 + PC^2 = AC^2$

$(12)^2 + (5)^2 = AC^2$

$AC^2 = (144 + 25)$

$AC^2 = 169$

AC = 13 m

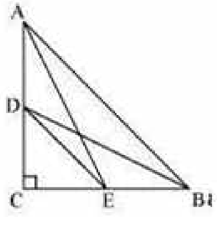
Therefore, the distance between their tops is 13 m

Q. 13 D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Answer: To Prove: $AE^2 + BD^2 = AB^2 + DE^2$

Given: D and E are midpoints of AD and CB and ABC is right angled at C

Applying Pythagoras theorem in ΔACE, we obtain



Pythagoras theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

$AC^2 + CE^2 = AE^2$ ....eqn(i)
Applying Pythagoras theorem in triangle BCD, we get
$BC^2 + CD^2 = BD^2$ ....eqn(ii)

Adding equations (i) and (ii), we get

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ ....eqn (iii)
Applying Pythagoras theorem in triangle CDE, we get

$DE^2 = CD^2 + CE^2$

Applying Pythagoras in triangle ABC, we get
$AB^2 = AC^2 + CB^2$
Putting these values in eqn(iii), we get

$DE^2 + AB^2 = AE^2 + BD^2$
Hence, Proved.
Q. 14 The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD(see Fig. 6.55). Prove that $2AB^2 = 2AC^2 + BC^2$

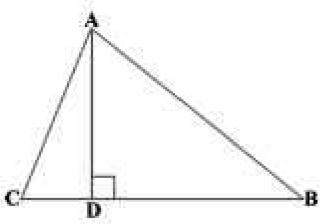


Fig. 6.55

Answer :

We have two right angled triangles now ΔACD and ΔABD

Applying Pythagoras theorem for ΔACD, we obtain
$AC^2 = AD^2 + DC^2$
$AD^2 = AC^2 - DC^2$ ....eq(i)

Applying Pythagoras theorem in ΔABD, we obtain
$AB^2 = AD^2 + DB^2$
$AD^2 = AB^2 - DB^2$ ....eq(ii)
Now we can see from equation i and equation ii that LHS is same.
Thus,

From (i) and (ii), we get
$AC^2 - DC^2 = AB^2 - DB^2$ (iii)
It is given that 3DC = DB
Therefore,
DC + DB = BC
DC + 3DC = BC
4 DC = BC ....eq(iv) and also,DC = DB/3putting this in eq (iii)DB = $\frac{3BC}{4}$
So,
$DC = \frac{BC}{4} and\ DB = \frac{3BC}{4}$

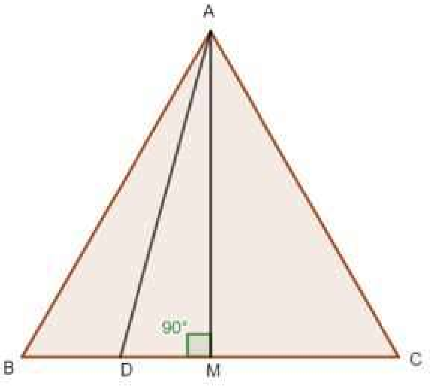
Putting these values in (iii), we get

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$
$$AC^2 - BC \times \frac{BC}{16} = AB^2 - \frac{9 \times BC \times BC}{16}$$
$16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$16AB^2 - 16AC^2 = 8BC^2$

$2AB^2 = 2AC^2 + BC^2$

Q. 15 In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$ Prove that $9\ AD^2 = 7\ AB^2$
Answer: The figure is given below:



Given: BD = BC/3

To Prove: 9 AD^2 = 7 AB^2

Proof:
Let the side of the equilateral triangle be a, and AM be the altitude of ΔABC

BM = MC = BC/2 = a/2 [Altitude of an equilateral triangle bisect the side]

And, then, in ΔABM, by pythagoras theorem we write,
Pythagoras Theorem : Square of the Hypotenuse equals to the sum of the squares of other two sides.

$AM^2 = AB^2 - BM^2$

or $AM^2 = a^2 - a^2/4$

$$AM^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$$

$$AM = \frac{a\sqrt{3}}{2}$$

BD = a/3 [BC = a]

DM = BM – BD

= a/2 – a/3

= a/6

According to pythagoras theorem in a right angled triangle,(hypotenuse)^2 = (altitude)^2 + (base)^2

Applying Pythagoras theorem in ΔADM, we obtain

$AD^2 = AM^2 + DM^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$AD^2 = \frac{3a^2}{4} + \frac{a^2}{36}$$

$$AD^2 = \frac{27a^2 + a^2}{36}$$

$$AD^2 = \frac{28a^2}{36}$$

Now, a = AB or $a^2 = AB^2$

$$AD^2 = \frac{28AB^2}{36}$$

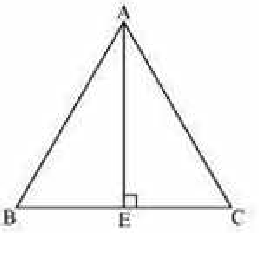
$36\ AD^2 = 28\ AB^2$

$9\ AD^2 = 7\ AB^2$

Hence, Proved.

Q. 16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes

Answer: Let the side of the equilateral triangle be a, and AE be the altitude of ΔABC



To Prove: 4 × (Square of altitude) = 3 × (Square of one side)

Proof:

Altitude of equilateral triangle divides the side in two equal parts. Therefore,

$$BE = EC = \frac{BC}{2} = \frac{a}{2}$$

Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \frac{a^2}{4}$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

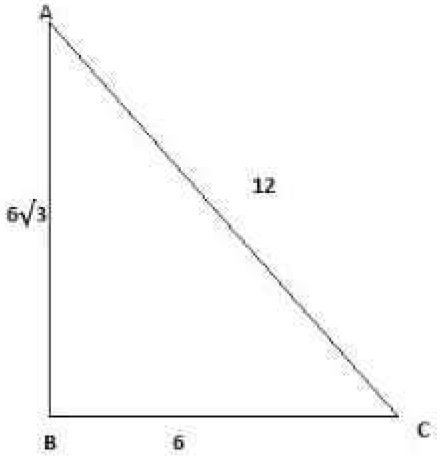
$$AE = \frac{\sqrt{3}a}{2}$$

$$4 \times AE^2 = 3 \times a^2$$

4 × (Square of altitude) = 3 × (Square of one side)

Q. 17 Tick the correct answer and justify:
In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm, the angle B is:
A. 120° B. 60°
C. 90° D. 45°

Answer:



Given that, AB = $6\sqrt{3}$ cm,
AC = 12 cm,

And BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.
ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B

$\angle B = 90°$

Hence, the correct answer is (C).

Exercise 6.6

Q. 1 In Fig. 6.56, PS is the bisector of ∠QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$

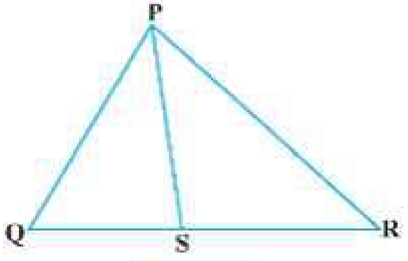
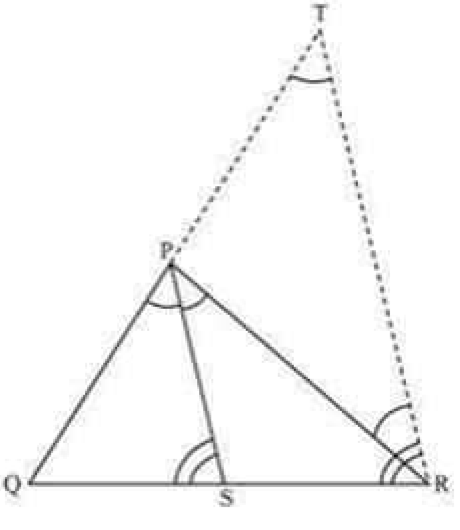


Fig. 6.56

Answer: Construct a line segment RT parallel to SP which intersects the extended line segment QP at point T



Given: PS is the angle bisector of ∠QPR.

Proof:

∠QPS = ∠SPR (i)

By construction,

∠SPR = ∠PRT (As PS ∥ TR, By interior alternate angles) (ii)

∠QPS = ∠QTR (As PS ∥ TR, By interior alternate angles) (iii)

Using these equations, we get:

∠PRT = ∠QTR

PT = PR

By construction,

PS ∥ TR

Basic Proportionality Theorem: If a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides in proportion.
By using basic proportionality theorem for ΔQTR,

$$\frac{QS}{SR} = \frac{PQ}{PR}$$

Hence, Proved.

Q. 2 In Fig. 6.57, D is a point on hypotenuse AC of Δ ABC, DM ⊥ BC and DN ⊥ AB. Prove that:
(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$

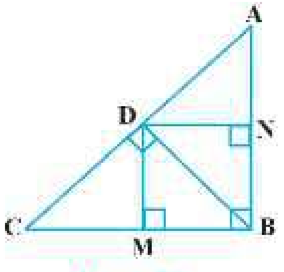


Fig. 6.57

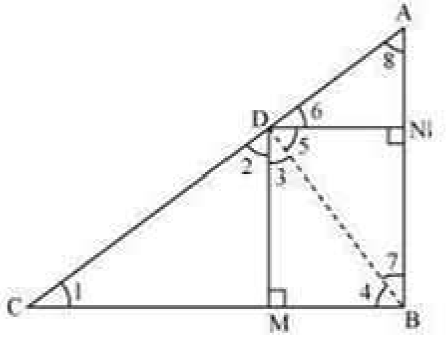
Answer: (i) To Prove: $DM^2 = DN.\ MC$
Construction: join DB

We have, DN ∥ CB,

DM ∥ AB,

And ∠B = 90° (Given)

As opposite sides are parallel and equal and also each angle is 90°, DMBN is a rectangle.



DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC

∠CDB = 90°
Now from the figure we can say that

∠2 + ∠3 = 90° ....eq(i)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180° **[Sum of angles of a triangle = 180°]**

∠1 + ∠2 = 90° ....eq(ii)

In Δ DMB,

∠3 + ∠DMB + ∠4 = 180° **[Sum of angles of a triangle = 180°]**

⇒∠3 + ∠4 = 90° ....eq(iii)

From (i) and (ii), we get

$\angle 1 = \angle 3$

From (i) and (iii), we get

$\angle 2 = \angle 4$

In ΔDCM and ΔBDM,

$\angle 1 = \angle 3$ (Proved above)

$\angle 2 = \angle 4$ (Proved above)

ΔDCM similar to ΔBDM (AA similarity)
(AA Similarity : When you have two triangles where one is a smaller version of the other, you are looking at two similar triangles.)

BM/DM = DM/MC
Cross multiplying we get,

DN/DM = DM/MC (BM = DN)

$DM^2 = DN \times MC$

Hence, Proved.

(ii) To Prove: $DN^2 = AN \times DM$
In right triangle DBN,

$\angle 5 + \angle 7 = 90°$ (iv)

In right triangle DAN,

$\angle 6 + \angle 8 = 90°$ (v)

D is the foot of the perpendicular drawn from B to AC

$\angle ADB = 90°$

∠5 + ∠6 = 90° (vi)

From equation (iv) and (vi), we obtain

∠6 = ∠7

From equation (v) and (vi), we obtain

∠8 = ∠5

In ΔDNA and ΔBND,

∠6 = ∠7 (Proved above)

∠8 = ∠5 (Proved above)

Hence,

ΔDNA similar to ΔBND (AA similarity criterion)
(AA similarity Criterion: When you have two triangles where one is a smaller version of the other, you are looking at two similar triangles.)

AN/DN = DN/NB

$DN^2 = AN \times NB$

$DN^2 = AN \times DM$ (As NB = DM)
Hence, Proved.

Q. 3 In Fig. 6.58, ABC is a triangle in which ∠ ABC > 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2\ BC.BD$.

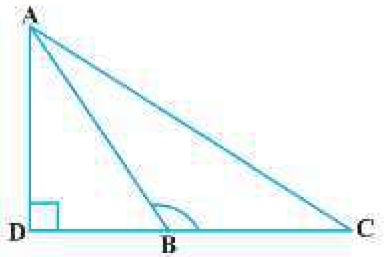


Fig. 6.58

Answer:

Using Pythagoras theorem in ΔADB, we get:

$AB^2 = AD^2 + DB^2$ (i)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (i)]

Q. 4 In Fig. 6.59, ABC is a triangle in which ∠ ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

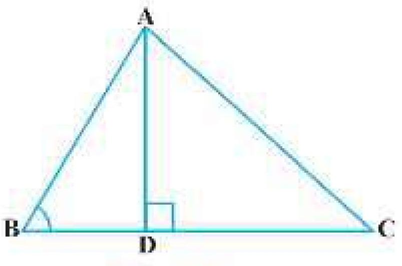
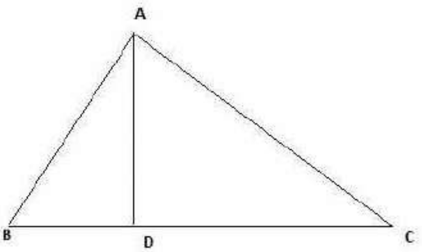


Fig. 6.59

Answer:



To Prove: $AC^2 = AB^2 + BC^2 - 2BC \times BD$
Given: AD is Perpendicular on BC and angle ABC < 90°
Proof:
Pythagoras Theorem: It states that the square of the hypotenuse (the

side opposite the right angle) is equal to the sum of the squares of the other two sides. Applying Pythagoras theorem in Δ ADB, we obtain

$AD^2 + DB^2 = AB^2$

$AD^2 = AB^2 – DB^2$ .....eq(i)

Applying Pythagoras theorem in Δ ADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 – BD^2 + DC^2 = AC^2$ [Using equation (i)]

$AB^2 – BD^2 + (BC - BD)^2 = AC^2$ [DC = BC - BD]

$AC^2 = AB^2 – BD^2 + BC^2 + BD^2 - 2BC \times BD$

$\mathbf{AC^2 = AB^2 + BC^2 - 2BC \times BD}$
Hence, Proved.

Q. 5 In Fig. 6.60, AD is a median of a triangle ABC and AM⊥ BC. Prove that:

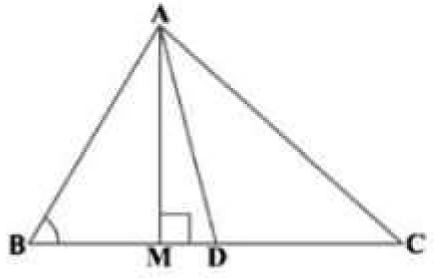


Fig. 6.60

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2\,AD^2 + \frac{1}{2}BC^2$

Answer:

(i) Using, Pythagoras theorem in ΔAMD, we get

$AM^2 + MD^2 = AD^2$ (i)

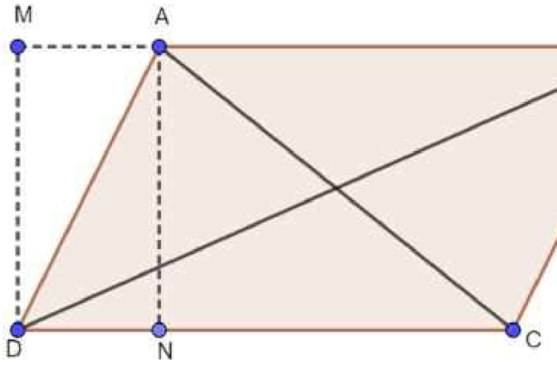
Applying Pythagoras theorem in ΔAMC, we obtain
$AM^2 + MC^2 = AC^2$
$AM^2 + (MD + DC)^2 = AC^2$
$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$
$AD^2 + DC^2 + 2MD.DC = AC^2$[Using equation (i)]
Using, $DC = BC/2$ we get
$AD^2 + (BC/2)^2 + 2MD * (BC/2) = AC^2$
$AD^2 + (BC/2)^2 + MD * BC = AC^2$

(ii) Using Pythagoras theorem in ΔABM, we obtain
$AB^2 = AM^2 + MB^2$
$= (AD^2 - DM^2) + MB^2$
$= (AD^2 - DM^2) + (BD - MD)^2$
$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$
$= AD^2 + BD^2 - 2BD \times MD$
$= AD^2 + (BC/2)^2 - 2 (BC/2) * MD$
$= AD^2 + (BC/2)^2 - BC * MD$
(iii)Using Pythagoras theorem in ΔABM, we obtain
$AM^2 + MB^2 = AB^2$ (1)
Applying Pythagoras theorem in ΔAMC, we obtain
$AM^2 + MC^2 = AC^2$(2)
Adding equations (1) and (2), we obtain
$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$2AM^2+BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD (- BD + DC) = AB^2 + AC^2$
$2 (AM^2 + MD^2) + (BC/2)^2 + (BC/2)^2 + 2MD (-BC/2 + BC/2) = AB^2 + AC^2$.
$2AD^2 + BC^2/2 = AB^2 + AC^2$

Q. 6 Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides

Answer: ABCD is a parallelogram in which AB = CD and AD = BC

Perpendicular AN is drawn on DC and perpendicular DM is drawn on AB extend up to M



In Δ AMD,

$AD^2 = DM^2 + AM^2$ ....eq(i)

In Δ BMD,

$BD^2 = DM^2 + (AM + AB)^2$

Or, $(AM + AB)^2 = AM^2 + AB^2 + 2\ AM \times AB$

$BD^2 = DM^2 + AM^2 + AB^2 + 2AM \times AB$ ....eq(ii)

Substituting the value of AM^2 from (i) in (ii), we get

$BD^2 = AD^2 + AB^2 + 2 \times AM \times AB$ ....eq(iii)

In Δ AND,

$AD^2 = AN^2 + DN^2$ ....eq(iv)

In Δ ANC,

$AC^2 = AN^2 + (DC - DN)^2$

Or, $AC^2 = AN^2 + DN^2 + DC^2 - 2 \times DC \times DN$ ....eq(v)

Substituting the value of AD^2 from (iv) in (v), we get

$AC^2 = AD^2 + DC^2 - 2 \times DC \times DN$ ....eq(vi)

We also have,

AM = DN and AB = CD

Substituting these values in (vi), we get

$AC^2 = AD^2 + DC^2 - 2 \times AM \times AB$ ....eq(vii)

Adding (iii) and (vii), we get

$AC^2 + BD^2 = AD^2 + AB^2 + 2 \times AM \times AB + AD^2 + DC^2 - 2 \times AM \times AB$

Or, $AC^2 + BD^2 = AB^2 + BC^2 + DC^2 + AD^2$

Hence, proved.

Q. 7 In Fig. 6.61, two chords AB and CD intersect each other at the point P. Prove that:

(i) Δ APC ~ Δ DPB

(ii) AP . PB = CP . DP

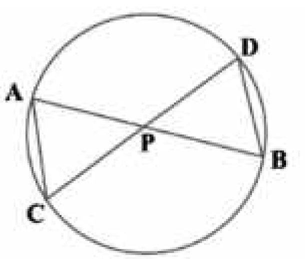


Fig. 6.61

Answer: (i) In triangle APC and DPB,

∠CAP = ∠BDP (Angles on the same side of a chord are equal)

∠APC = ∠DPB (Opposite angles)

Hence,

ΔAPC ~ Δ DPB (By AAA similarity)

(ii) Since, the two triangles are similar

Hence,

$\frac{AP}{CP} = \frac{DP}{PB}$

or $AP * PB = CP * DP$

Hence, proved.

Q. 8 In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P(when produced) outside the circle. Prove that

(i) Δ PAC ~ Δ PDB

(ii) PA . PB = PC. PD

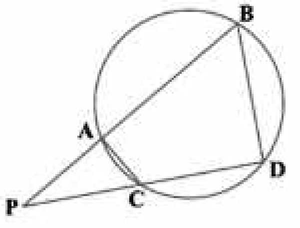


Fig. 6.62

Answer: (i) In triangle PAC and PDB

∠PAC + ∠CAB = 180o (Linear pair)

∠CAB + ∠BDC = 180O (Opposite angles of a cyclic quadrilateral are supplementary)

Hence,

∠PAC = ∠PDB

Similarly, ∠PCA = ∠PBD

Hence,

Δ PAC ~ Δ PDB

(ii) Since the two triangles are similar, so

$$\frac{PA}{PC} = \frac{PD}{PB}$$

or $PA * PB = PC * PD$

Hence, proved.

Q. 9 In Fig. 6.63, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$ Prove that AD is the bisector of $\angle$ BAC

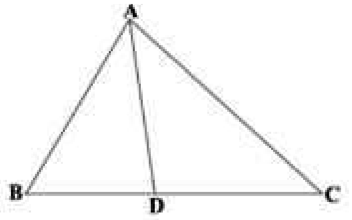
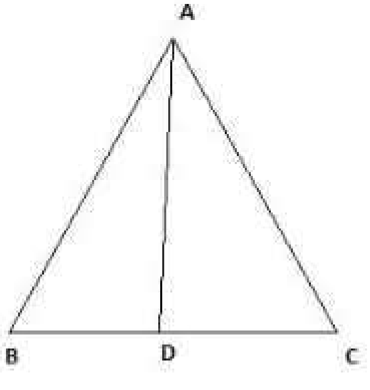


Fig. 6.63

Answer:



To Prove: AD bisects $\angle$BAC Given: $\frac{BC}{CD} = \frac{AB}{AC}$

Now from ΔABD and ΔADC, As it is given that $\frac{BC}{CD} = \frac{AB}{AC}$

And, AD is common to both triangles, Therefore, ΔABD ~ ΔADC (BY SSS theorem)Now by similarity $\angle$BAD = $\angle$DAC Hence, AD must be the bisector of the angle BAC.

Q. 10 Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string(from the tip of her rod to the fly) is taut, how much string does she have out(see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

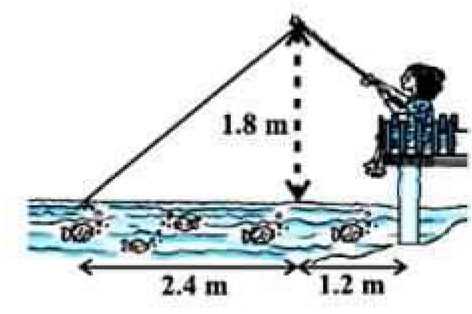
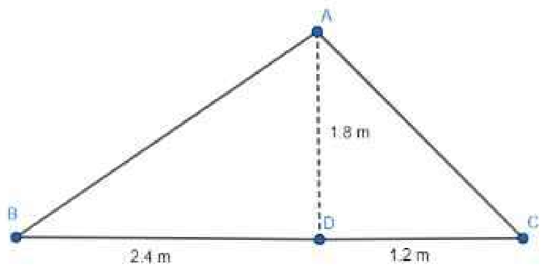


Fig. 6.64

Answer:



As per the question:

AD = 1.8 m
BD = 2.4 m
CD = 1.2 m
speed of string when she pulls in = 5 cm per second

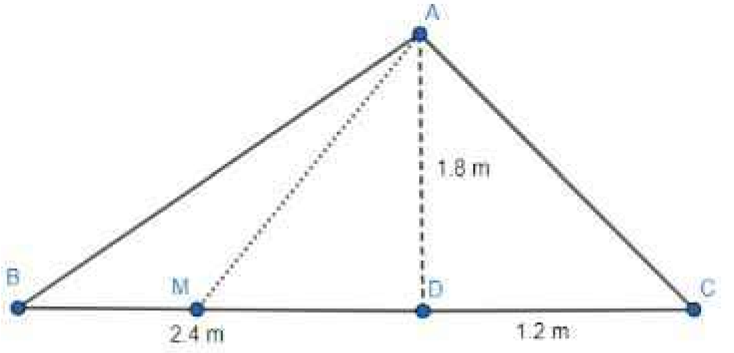
To find: Length of string, AB and

In triangle ABD, length of string i.e. AB can be calculated as follows [By Pythagoras theorem]

$AB^2 = AD^2 + BD^2$

$= (1.8)^2 + (2.4)^2$
$= 3.24 + 5.76$
$= 9$

Or, AB = 3 m



Let us assume that the string reaches at point M after 12 seconds

Now, To find: Distance of fly, from the girl.
Length of string pulled in, after 1 second = 5 cm

Length of string pulled in, after 12 seconds = 5 * 12 = 60 cm

= 0.6 m [As, 1 m = 100 cm]

Remaining length, AM = 3 – 0.6 = 2.4 m

In triangle AMD, we can find MD by using Pythagoras theorem,

$MD^2 = AM^2 - AD^2$
$= 2.4^2 - 1.8^2$
$= 5.76 - 3.24$
$= 2.52$ m

Or, MD = 1.58 m
Also,

Horizontal distance between the girl and the fly = CD + MD
= 1.2 + 1.58 = 2.78 m