Chapter – 5
Arithmetic Progressions

Exercise 5.1

Q. 1 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/ 4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv)The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8 % per annum.

Answer:

For a sequence to be AP, the difference of consecutive terms should remain constant and that is called the common difference of the AP

(i) Fare for 1st km = Rs. 15

Fare for 2nd km = Fare of first km + Additional fare for 1 km

= Rs. 15 + 8 = Rs 23

Fare for 3rd km = Fare of first km + Fare of additional second km + Fare of additional third km

= Rs. 23 + 8 = Rs 31

Fare of n km = 15 + 8 x (n - 1)

(We multiplied by n - 1 because first km was fixed and for rest we are adding additional fare.

In this, each subsequent term is obtained by adding a fixed number (8) to the previous term.

Hence, it is an AP

(ii) Let us take initial quantity of air = 1

Hence, quantity of air removed in first step = 1/4

Remaining quantity after 1st step

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Quantity removed after 2nd step = Quantity removed in first step x initial quantity

$$= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

Remaining quantity after 2nd step would be:

$$= \frac{3}{4} - \frac{3}{16} = \frac{12-3}{16} = \frac{9}{16}$$

After second step the difference between second and first and first and initial step is not the same, hence

Here a fixed number is not added to each subsequent term.

Hence, it is not an AP

(iii) Cost of digging of 1st meter = Rs 150

Cost of digging of second meter = cost of digging of first meter + cost of digging additional meter

Cost of digging of 2nd meter = 150 + 50

= Rs 200

Cost of digging of third meter = Cost of digging of first meter + cost of digging of second meter + cost of digging of third meter

Cost of digging of 3rd meter = 200 + 50

= Rs 250

Here, each subsequent term is obtained by adding a fixed number (50) to the previous term.

Hence, it is an AP.

(iv)Amount in the beginning = Rs. 10000

Interest at the end of 1st year at rate of 8%

$= 10000 \times 8\%$

$= 800$

Hence, amount at the end of 1styear

$= 10000 + 800$

$= 10800$

Now the interest will be made at the principal taken as amount of first year, Hence

Interest at the end of 2nd year at rate of 8%

$= 10800 \times 8\%$

$= 864$

Thus, amount at the end of 2ndyear

$= 10800 + 864 = 11664$

Since, each subsequent term is not obtained by adding a unique number to the previous term; hence, it is not an AP.

Q. 2 Write first four terms of the AP, when the first term a and the common difference d aregiven as follows:

(i) $a = 10$, $d = 10$

(ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$

(iv) $a = -1$, $d = 1/2$

(v) $a = -1.25$, $d = -0.25$

Answer: (i) Here, first term $a1 = 10$ and common difference, $d = 10$

Hence,

2nd term $a_2 = a_1 + d$

$= 10 + 10$

$= 20$

3rd term $a3 = a1 + 2d$

$= 10 + 2 \times 10$

$= 30$

4th term $a_4 = a_1 + 3d$

$= 10 + 30$

$= 40$

Therefore,

first four terms of the AP are:

10, 20, 30, 40, ……

(ii) Here,

First term $a = -2$ and Common difference $= 0$

Therefore, first four terms of the given AP are:

$a_1 = -2$, $a_2 = -2$, $a_3 = -2$ and $a_4 = -2$

(iii) Here, first term $a1 = 4$ and common difference $d = -3$

We know that $an = a + (n - 1)d$, where n = number of terms

Thus, second term $a_2 = a + (2 - 1)d$

$a_2 = 4 + (2-1)\times(-3)$

$= 4 - 3 = 1$

3rd term $a_3 = a + (3 - 1)d$

$= 4 + (3-1) \times (-3)$

$= 4 - 6$

$= -2$

4th term $a_4 = a + (4-1)d$

$= 4 + (4 - 1) \times (-3)$

$= 4 - 9 = -5$

Therefore,

First four terms of given AP are:

4, 1, - 2, - 5

(iv) We have,

1st term = - 1 and d = 1/2

Hence,

2nd term $a_2 = a1 + d$

$= -1 + 1/2$

$= - 1/2$

3rd term $a_3 = a1 + 2d$

$= -1 + 2 * 1/2$

$= 0$

4th term $a_4 = a1 + 3d$

$= -1 + 3 * 1/2$

$= 1/2$

Therefore,

The four terms of A.P. are -1, - 1/2, 0, 1/2

(v) We have

1st term = - 1.25 and d = - 0.25

2nd term $a_2 = a + d$

= -1.25 - 0.25

= -1.5

3rd term $a_3 = a + 2d$

= -1.25 + 2 × (-0.25)

= -1.25 - 0.5

= -1.75

4th term $a_4 = a + 3d$

= -1.25 + 3 × (-0.25)

= -2

Therefore, first four terms of the A.P. are: -1.25, -1.5, -1.75 and – 2.

Q. 3 For the following APs, write the first term and the common difference:

(i) 3, 1, -1, -3,….

(ii) –5, -1, 3, 7

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{5}, \frac{13}{3}, \ldots..$

(iv) 0.6, 1.7, 2.8, 3.9, ….

Answer: (i) Here, first term a = 3

Now,
The Common difference of the A.P. can be calculated as:
$a_4 - a_3$
= - 3 – (-1)
= - 3 + 1
= - 2
$a_3 - a_2$

$= -1 - 1$
$= -2$
$a_2 - a_1$
$= 1 - 3$
$= -2$
Now, here $a_{k+1} - a_k = -2$ for all values of k
Hence, first term = 3 and common difference = - 2

(ii) $a_4 - a_3$
$= 7 - 3$
$= 4$
$a_3 - a_2$
$= 3 - (-1)$
$= 3 + 1$
$= 4$
$a_2 - a_1$
$= -1 - (-5)$
$= -1 + 5$
$= 4$
Now, here, $a_{k+1} - a_k = 4$ for all values of k
Therefore, first term = - 5 and common difference = 4

(iii) From the question,
$a_4 - a_3$

$= \frac{13}{3} - \frac{9}{3}$
$= \frac{4}{3}$
$a_3 - a_2$

$= \frac{9}{3} - \frac{5}{3}$

$= \frac{4}{3}$
$a_2 - a_1$

$= \frac{4}{3}$

Now, $a_{k+1} - a_k = \frac{4}{3}$ for all values of k

Therefore,
First term = 1/3 and common difference = 4/3

(iv) $a_4 - a_3$
= 3.9 – 2.8

= 1.1
$a_3 - a_2$
= 2.8 – 1.7
= 1.1
$a_2 - a_1$
= 1.7 – 0.6

= 1.1
Now, here, $a_{k+1} - a_k = 1.1$ for all values of k
Therefore,
First term = 0.6 and common difference = 1.1

Q. 4 Which of the following are APs? If they form an AP, find the common difference d and write three more terms.
(i) 2, 4, 8, 16
(ii) 2, $\frac{5}{2}, 3, \frac{7}{2}$
(iii) -1.2, -3.2, =5.2, -78.2, ….
(iv) -10, -6, -2, 2, ….
(v) 3, $3 + \sqrt{2}, 3 + 2\sqrt{2},, 3 + 3\sqrt{2}$
(vi) 0.2, 0.22, 0.222, 0.2222,…
(vii) 0, -4, -8, -12, ..
(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$
(ix) 1, 3, 9, 27
(x) a, 2a, 3a, 4a,…
(xi) a, a^2, a^3, a^4,…

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$
(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$
(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$
(xv) $1^2, 5^2, 7^2, 73$

Answer:

For a sequence to be an AP, the difference between two consecutive terms remains the same, that is called the common difference of AP
(i) if $a_{k+1} - a_k$ is same for different values of k then the series is an AP.

We have, $a_1 = 2$, $a_2 = 4$, $a_3 = 8$ and $a_4 = 16$
$a_4 - a_3 = 16 - 8 = 8$
$a_3 - a_2 = 8 - 4 = 4$
$a_2 - a_1 = 4 - 2 = 2$
Here, $a_{k+1} - a_k$ is not same for all values of k.
Hence, the given series is not an AP.
(ii) As per the question:
$a_1 = 2$,
$a_2 = 5/2$,
$a_3 = 3$
And
$a_4 = 7/2$
$a_4 - a_3 = \frac{7}{2} - 3 = 1/2$

$a_3 - a_2 = 3 - \frac{5}{2} = 1/2$

$a_2 - a_1 = \frac{5}{2} - 2 = 1/2$
Now, we can observe that $a_{k+1} - a_k$ is same for all values of k.

Hence, it is an AP.

And, the common difference = 1/2

Next three terms of this series are:
$a_5 = a + 4d$
$= 2 + 4 * 1/2$

$= 4$
$a_6 = a + 5d$
$= 2 + 5 * 1/2$
$= \frac{9}{2}$
$a_7 = a + 6d$
$= 2 + 6 * 1/2$
$= 5$
Hence,
The next three terms of the AP are: 4, 9/2 and 5
(iii) $a_4 - a_3 = - 7.2 + 5.2 = - 2$
$a_3 - a_2 = - 5.2 + 3.2 = - 2$
$a_2 - a_1 = - 3.2 + 1.2 = - 2$

In this, $a_{k+1} - a_k$ is same for all values of k.
Hence, the given series is an AP.
Common difference = - 2
Next three terms of the series are:

$a_5 = a + 4d$

$= -1.2 + 4 \times (-2)$

$= -1.2 - 8 = -9.2$

$a_6 = a + 5d$

$= -1.2 + 5 \times (-2)$

$= -1.2 - 10 = -11.2$

$a_7 = a + 6d$

$= -1.2 + 6 \times (-2)$

$= -1.2 - 12 = -13.2$

Next three terms of AP are: - 9.2, - 11.2 and – 13.2

(iv)$a_4 - a_3 = 2 + 2 = 4$

$a_3 - a_2 = - 2 + 6 = 4$

$a_2 - a_1 = - 6 + 10 = 4$

Here, $a_{k+1} - a_k$ is same for all values of k

Hence, the given series is an AP.

Common difference $= 4$

Next three terms of the AP are:

$a_5 = a + 4d$

$= -10 + 4 \times 4$

$= -10 + 16 = 6$

$a_6 = a + 5d$

$= -10 + 5 \times 4$

$= -10 + 20 = 10$

$a_7 = a + 6d$

$= -10 + 6 \times 4$

$= -10 + 24 = 14$

Next three terms of AP are: 6, 10 and 14.

(v) $a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

Here, $a_{k+1} - a_k$ is same for all values of k

Hence, the given series is an AP.

Common difference $= \sqrt{2}$

Next three terms of the AP are

$a_6 = a + 5d = 3 + 5\sqrt{2}$

$a_7 = a + 6d = 3 + 6\sqrt{2}$

Next three terms of AP are: $3 + 4\sqrt{2}$, $3 + 5\sqrt{2}$ and $3 + 6\sqrt{2}$

(v)
$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

Here, $a_{k+1} - a_k$ is not same for all values of k

Hence, the given series is not an AP.
(vii) Here; $a_4 - a_3 = -12 + 8 = -4$

$a_3 - a_2 = -8 + 4 = -4$

$a_2 - a_1 = - 4 - 0 = - 4$

Since $a_{k+1} - a_k$ is same for all values of k.

Hence, this is an AP.

The next three terms can be calculated as follows:

$a_5 = a + 4d = 0 + 4(- 4) = - 16$

$a_6 = a + 5d = 0 + 5(- 4) = - 20$

$a_7 = a + 6d = 0 + 6(- 4) = - 24$

Thus, next three terms are; - 16, - 20 and – 24

(viii) Here, it is clear that $d = 0$

Since $a_{k+1} - a_k$ is same for all values of k.

Hence, it is an AP.

The next three terms will be same, i.e. – 1/2

(ix) $a_4 - a_3 = 27 - 9 = 18$

$a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 = 3 - 1 = 2$

Since $a_{k+1} - a_k$ is not same for all values of k.

Hence, it is not an AP.

(x) $a_4 - a_3 = 4a - 3a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_2 - a_1 = 2a - a = a$

Since $a_{k+1} - a_k$ is same for all values of k.

Hence, it is an AP.

Next three terms are:

$a_5 = a + 4d = a + 4a = 5a$

$a_6 = a + 5d = a + 5a = 6a$

$a_7 = a + 6d = a + 6a = 7a$

Next three terms are; 5a, 6a and 7a.

(xi) Here, the exponent is increasing in each subsequent term.
$a_4 = a^4$, $a_3 = a^3$, $a_2 = a^2$, $a_1 = a$
$a_4 - a_3 = a^4 - a^3$
$a_3 - a_2 = a^3 - a^2$

since, the difference is not same,
Since $a_{k+1} – a_k$ is not same for all values of k.

Hence, it is not an AP.

(xii) Different terms of this AP can also be written as follows:

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\ldots\ldots..$

$a_4 – a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$a_3 – a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_2 – a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Since ak+1 – ak is same for all values of k.

Hence, it is an AP.

Next three terms can be calculated as follows:

$a_5 = a + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$

$a_6 = a + 5d = \sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$

$a_7 = a + 6d = \sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$

Next three terms are; $5\sqrt{2}$, $6\sqrt{2}$ and $7\sqrt{2}$

(xiii) $a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$
$(\sqrt{12} = \sqrt{2 \times 2 \times 3} = 2\sqrt{3})$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$
$a_2 - a_1 = \sqrt{6} - \sqrt{3}$
Since $a_{k+1} - a_k$ is not same for all values of k.

Hence, it is not an AP

(xiv) The given terms can be written as follows:
1, 9, 25, 49, …
Here, $a_4 - a_3 = 49 - 25 = 24$
$a_3 - a_2 = 25 - 9 = 16$
$a_2 - a_1 = 9 - 1 = 8$
Since $a_{k+1} - a_k$ is not same for all values of k.
Hence, it is not an AP.
(xv) $1^2, 5^2, 7^2, 73$.....
$a = 1d = 5^2 - 1 = 25 - 1 = 24$
$d = 7^2 - 5^2 = 49 - 25 = 24$
$d = 74 - 49 = 24$As, common difference is same. The sequence is in A.P.

Exercise 5.2

Q. 1 Fill in the blanks in the following table, given that a_n is the first term, d the common difference and the nth term of the AP:

	a	d	n	a_n
(i)	7	3	8	….
(ii)	-18	….	10 18	0
(iii)	…	-3	….	-5
(iv)	-18.9	2.5	105	3.6
(v)	3.5	0		…

Answer: (i) Given: a = 7, d = 3 and n = 8,

a_n = ?
We know:
$a_n = a + (n – 1)d$
Thus, $a_n = 7 + (8 – 1)3 = 7 + 21 = 28$
(ii) Given $a = - 18$, $n = 10$, $a_n = 0$, d = ?
We know that $a_n = a + (n – 1)d$
Thus, $0 = - 18 + (10 – 1)d$
$0 = - 18 + 9d$
Or, $9d = 18$
Or, $d = 18/9 = 2$
(iii) Given $d = - 3$, $n = 18$, $a_n = - 5$, a = ?
We know that, $a_n = a + (n – 1)d$
Or, $- 5 = a + (18 – 1) (- 3)$
Or, $- 5 = a – 51$
Or, $a = - 5 + 51 = 46$
(iv) Given $a = - 18.9$, $d = 2.5$, $a_n = 3.6$, n = ?

We know that, $a_n = a + (n – 1)d$

Or, $3.6 = – 18.9 + (n – 1)2.5$
Or, $2.5(n – 1) = 3.6 + 18.9 = 22.5$
$n – 1 = 22.5/2.5 = 9$
$n = 9 + 1 = 10$
(v) Given that $a = 3.5$, $d = 0$, $n = 105$, a_n= ?
We know:,
$a_n = a + (n – 1)d$
$a_n = 3.5 + (105 – 1)0$
$a_n = 3.5 + 0$
$= 3.5$

Q. 2 Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4, . . . , is
A. 97 **B**. 77
C. –77 **D**. – 87
(ii) 11th term of the AP: -3 , $-\frac{1}{2}$, 2,…., is
A. 28 **B.** 22
C. –38 **D.** – 48

Answer:

(i) 30th term of the AP: 10, 7, 4, . . . , is

Here, $a = 10$, $d = – 3$ and $n = 30$

We know:
$a_n = a + (n – 1)d$

or, $a_{30} = 10 + (30 – 1)(- 3)$

$= 10 + 29(-3)$
$= 10 – 87$
$= - 77$
Hence,

Answer (C) – 77

(ii) 11th term of the AP: -3 , $-\frac{1}{2}$, 2,…., is

Here, a = - 3, d = 5/2 and n = 11

We know that, $a_n = a + (n – 1)d$

Or, $a_{11} = - 3 + (11 – 1)\ 5/2$

= - 3 + 10 × (5/2)

= -3 + (5)(5)= - 3 + 25= 22

Hence,

Answer (B) 22

Q. 3 In the following APs, find the missing terms in the boxes :

(i) 2 , $\boxed{\ }$,26

(ii) $\boxed{\ }$, 13, $\boxed{\ }$, 3

(iii) 5 , $\boxed{\ }$, $\boxed{\ }$, 9 1/2

(iv) -4 , $\boxed{\ }$, $\boxed{\ }$, $\boxed{\ }$, $\boxed{\ }$, 6

(v) $\boxed{\ }$, 38 , $\boxed{\ }$, $\boxed{\ }$, $\boxed{\ }$,-22

Answer:

In mathematics, an arithmetic progression (AP) or arithmetic sequence is a sequence of numbers such that the difference between the consecutive terms is constant.

Let three terms a, b and c are in A.PTherefore, b - a = c - bRearranging we get,

$$b = \frac{a+c}{2}$$

So, the middle term is the average of two numbers of an A.P

And similarly,

a = 2b – c and

c = 2b - a

And also nth term of an AP is given by $a_n = a + (n - 1)d$ where, a = first term n = number of terms d = common difference(i)

We know: In AP, middle term is average of the other two terms

Hence, middle term = (2 + 26)/2 = 28/2 = 14

Thus, above AP can be written as 2, 14, 26

(ii) We know: In AP, middle term is average of the other two terms

The middle term between 13 and 3 will be;

(13 + 3)/2 = 16/2 = 8

Now, $a_4 - a_3 = 3 - 8 = -5$

$a_3 - a_2 = 8 - 13 = -5$

Thus, $a_2 - a_1 = -5$

Or, $13 - a_1 = -5$

Or, $a_1 = 13 + 5 = 18$

Thus, above AP can be written as 18, 13, 8, 3.

We have, a = 5 and $a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively.

Now common difference:

$a_4 = a + 3d$

$\frac{19}{2} = 5 + 3d$

$3d = \frac{19}{2} - 5$

$3d = \frac{9}{2}$

$d = \frac{3}{2}$

Hence, using d, 2nd term and 3rd term can be calculated as:

$a_2 = a + d$

$= 5 + \frac{3}{2}$

$= \frac{13}{2}$

$= 6\frac{1}{2}$

$a_3 = a + 2d$

$= 5 + 2\times\frac{3}{2}$
$= 8$

Therefore, the A.P. can be written as: $5, 6\frac{1}{2}, 8, 9\frac{1}{2}$

(iv) Here, $a = -4$ and $a_6 = 6$
We know, nth term of an AP is
$a_n = a + (n - 1)d$
where a and d are first term and common difference respectively.

Common difference:
$a_6 = a + 5d$
$6 = -4 + 5d$
$5d = 6 + 4 = 10$
$d = 2$
The second, third, fourth and fifth terms of this AP are:
$a_2 = a + d = -4 + 2 = -2$
$a_3 = a + 2d = -4 + 4 = 0$
$a_4 = a + 3d = -4 + 6 = 2$
$a_5 = a + 4d = -4 + 8 = 4$
Thus, the given AP can be written as: - 4, - 2, 0, 2, 4, 6

(v) Given: Second term = 38 and sixth term = -22So,a + d = 38.......(1)a = 38 - da + 5d = -22.....(2)Putting the value of a in equation 2, we get,38 - d + 5d = -2238 + 4d = -224d = -22 - 384d = -60d = -15Putting the value of d in equation 1, we get,a - 15 = 38a = 38 + 15 = 53Therefore, the series is53, 38, 23, 8, -7, -22.

Q. 4 Which term of the AP : 3, 8, 13, 18, . . . ,is 78?
Answer:

Given,
First term, a = 3, Common difference, $d = a_2 - a_1 = 8 - 3 = 5$,
nth term, $a_n = 78$
To find: n =?

We know that nth term of an A.P is given by:
an = a + (n – 1)d

78 = 3 + (n – 1)5
(n – 1) 5 = 78 – 3 = 75
5n - 5 = 75
5n = 80n = 80/5n = 16
Thus, 78 is the 16th term of given AP.

Q. 5 Find the number of terms in each of the following APs :

(i) 7, 13, 19, . . . , 205
(ii) 18, $15\frac{1}{2}$, 13, …., -47
Answer:
To find: Number of terms, n

(i) Given: a = 7, d = 6, $a_n = 205$, n = ?

We know that nth term of an AP is given by, $a_n = a + (n - 1)d$

205 = 7 + (n – 1)6

(n – 1)6 = 205 – 7 = 198

$$n - 1 = \frac{198}{6}$$

n – 1 = 33

n = 34

Thus, 205 is the 34th of term of this AP.

(ii) Given: a = 18, d = 15.5 – 18 = - 2.5, a_n = - 47

We know that nth term of an AP is given by, $a_n = a + (n - 1)d$

Or, - 47 = 18 + (n – 1)(- 2.5)

Or, (n – 1)(- 2.5) = - 47 – 18 = - 65

$$n - 1 = \frac{-65}{-2.5}$$

Or, n – 1 = 26

Or, n = 27

Thus, - 47 is the 27th term of this AP.

Q. 6 Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Answer: To find: Whether -150 is term of the AP

Given: a = 11, d = 8 – 11 = - 3, a_n = - 150, n = ?

We know that $a_n = a + (n - 1)d$

where a_n = nth term of AP

a = first terms of AP

n = no. of terms of AP

d = common difference of APApplying the formula, we get

Or, - 150 = 11 + (n – 1)(- 3)

Or, (n – 1)(-3) = - 150 – 11 = - 161

Or, $n - 1 = 161/3$

$n - 1 = 53.67$

It is clear that 161 is not divisible by three and we shall get a fraction as a result. But number of term cannot be a fraction.

Hence, - 150 is not a term of the given AP.
Q. 7 Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73

Answer: To Find : 31^{st} term.

Given: 11th term of AP, $a_{11} = 38$ and 16th term of AP, $a_{16} = 73$

We know that $a_n = a + (n - 1)d$

where, a_n = nth terms of AP
a = first term of APn = number of terms d = common difference
Hence,
$a_{11} = a + (11 - 1)d$
$a_{11} = a + 10d = 38$ ....eq(i)
And,
$a_{16} = a + (16 - 1)d$
$a_{16} = a + 15d = 73$ ....eq(ii)

Subtracting eq(i) from eq(ii), we get following:
$a + 15d - (a + 10d) = 73 - 38$
$a + 15d - a - 10 d = 35$
Or, $5d = 35$
Or, $d = 7$
Substituting the value of d in eq(i) we get;
$a + 10 \times 7 = 38$
Or, $a + 70 = 38$
Or, $a = 38 - 70 = - 32$

Now 31st term can be calculated as follows:
$a_{31} = a + (31 - 1)d$

$a_{31} = a + 30d$

$= - 32 + 30 \times 7$

$= - 32 + 210 = 178$

So, 31st term is 178.

Q. 8 An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29^{th}term.

Answer: Given, $a_3 = 12$ and $a_{50} = 106$
We know that nth term of an AP is given by
$a_n = a + (n - 1)d$
where, a_n = nth term of the AP
a = first term of APn = number of terms of APd = common difference of AP. Applying the formula for 3rd and 50th term we get,
$a_3 = a + 2d = 12$
$a_{50} = a + 49d = 106$
Subtracting 3rd term from 50th term, we get;
$a + 49d – a – 2d = 106 – 12$
$47d = 94$
$d = 2$
Substituting the value of d in 3rd term, we get;

$a + 2 \times 2 = 12$

Or, $a + 4 = 12$

Or, $a = 8$

Now, 29th term can be calculated as follows:

$a_{29} = a + 28d$

$= 8 + 28 x 2$

$= 8 + 56 = 64$

$a_{29} = 64$

Q. 9 If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP iszero?

Answer:

Given, $a_3 = 4$ and $a_9 = -8$

$a_3 = a + 2d = 4$

$a_9 = a + 8d = -8$

Subtracting 3rd term from 9th term, we get;

$a + 8d - a - 2d = -8 - 4 = -12$

$6d = -12$

$d = -2$

Substituting the value of d in 3rd term, we get;

$a + 2(-2) = 4$

$a - 4 = 4$

$a = 8$

Now; $0 = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$(n - 1)(-2) = -8$

$n - 1 = 4$

$n = 5$

Thus, 5th term of this AP is zero.

Q. 10 The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Answer: To find: Common Difference, d

nth term of an AP is given by, $a_n = a + (n - 1) d$ where, a is the first term, n is the number of terms in an AP and d is the common difference. Tenth and seventeenth terms of this AP can be given as follows:

$a_{10} = a + 9d$

$a_{17} = a + 16d$

It is give that 17th term is 7 more than the tenth term, So

Subtracting 10th term from 17th term, we get;

$a + 16d – a – 9d = 7$

$7d = 7$

$d = 1$

Hence common difference of the A.P is 1.

Q. 11 Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Answer: To find : n such that $a_n = a_{54} + 132$

Given: $a = 3, d = 15 – 3 = 12$

nth term of an AP is given by:

$a_n = a + (n - 1) d$

where,a = first term of APn = no. of terms of APd = common difference of AP
54th term can be given as follows:
$a_{54} = a + (54 - 1)d$

$= 3 + 53 \times 12$

$= 3 + 636$

$= 639$
In question we have to find the term which is 132 more than 639 i.e. 771
Now $a_n = 771, n = ?$

Again applying the formula of nth term

$771 = a + (n – 1)d$

$771 = 3 + (n - 1)12$

$(n – 1)12 = 771 – 3 = 768$

$n - 1 = 768/12$

$n – 1 = 64$

$n = 65$

Thus, the required term is 65th term.

Q. 12 Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Answer: Let the common difference of two AP's be d, their first terms as a and a'

nth term of both the AP's will be given by

$a_n = a + (n - 1) d$

$a'_n = a' + (n - 1) d$

Now 100th term of 1st AP will be given by: a_{100}

$= a + (100 - 1) d = a + 99d$

100th term of second AP will be given by: a'_{100}

$= a' + (100 - 1) d = a' + 99d$

Given, $a_{100} - a'_{100} = (a + 99 d) - (a' + 99 d)$

$\Rightarrow a_{100} - a'_{100} = (a - a')$

So, difference does not depend on number of terms. Thus, $a_{1000} - a'_{1000} = 100 = a_{100} - a'_{100}$

So the difference between their 1000th terms is 100.

Q. 13 How many three-digit numbers are divisible by 7?

Answer:

Since, 100 is the smallest three digit number and it gives a reminder of 2 when divided by 7, therefore, 105 is the smallest three digit number which is divisible by 7

Since, 999 is greatest three digit number, and it gives a remainder of 5, thus 999 – 5 = 994 will be the greatest three digit number which is divisible by 7

Therefore, here we have,
First term (a) = 105,
The last term (a_n) = 994
The common difference = 7
We know that, $a_n = a + (n – 1)d$
Or, $994 = 105 + (n – 1)7$
Or, $(n – 1)7 = 994 – 105 = 889$
Or, $n – 1 = 127$
Or, $n = 128$
Thus, there are 128 three digit numbers which are divisible by 7.
Q. 14 How many multiples of 4 lie between 10 and 250?
Answer: 12 is the first number after 10 which is divisible by 4

Since, 250 gives a remainder of 2 when divided by 4, thus 250 – 2 = 248 is the greatest number less than 250 which is divisible by 4.

Here, we have first term (a) = 12, last term (n) = 248 and common difference (d) = 4
Thus, number of terms (n) =?
We know that, an = a + (n -1)d

Or, $248 = 12 + (n – 1)4$
Or, $(n – 1)4 = 248 – 12 = 236$
Or, $n – 1 = 59$
Or, $n = 60$
Thus, there are 60 numbers between 10 and 250 that are divisible by 4.
Q. 15 For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?
Answer: In first AP: a = 63, d = 2
In second AP: a = 3, d = 7
As per question:
nth term of first AP = nth term of second AP
$63 + (n – 1) 2 = 3 + (n – 1) 7$

$\Rightarrow 63 - 3 + (n - 1) 2 = (n - 1) 7$

$\Rightarrow 60 + 2n - 2 = 7n - 7$

$\Rightarrow 2n + 58 = 7n - 7$

$\Rightarrow 2n + 58 + 7 = 7n$
$\Rightarrow 2n + 65 = 7n$
$\Rightarrow 7n - 2n = 65$
$\Rightarrow 5n = 65$
$\Rightarrow n = 65/5 = 13$
Thus, for the 13 value of n, nth term of given two APs will be equal.

Q. 16 Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12
Answer:
To Find: A.P
Given: $a_3 = 16$, $a_7 - a_5 = 12$
For this question, We need to find third, fifth and seventh term of an AP by formula of nth term. We know that, nth term of an AP is given by :$a_n = a + (n - 1) d$
So, Given $a_3 = 16$ and $a_7 - a_5 = 12$
where a_3 = third term of the AP, and so on
$a_3 = a + 2d = 16$ ....eq(i)
$a_5 = a + 4d$
$a_7 = a + 6d$
As per question;

7th term exceeds the fifth term by 12, So the difference of seventh and fifth term will be 12
$a + 6d - a - 4d = 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = 6$
Substituting the value of d in eq (i), we get;
$a + 2 \times 6 = 16$
Or, $a + 12 = 16$
Or, $a = 16 - 12 = 4$

Thus, the AP can be given as follows:

a, a+d, a+2d, a+3d, a+4d..... and thus,

4, 10, 16, 22, 28, …

Q. 17 Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253

Answer: To find: 20th term from the last

In the given AP First term, a = 3common difference, d = 5

we know, nth term of an AP is $a_n = a + (n - 1)d$

Now, Let the no of terms in given AP is n

last term = 253

$\Rightarrow 253 = a + (n - 1)\, d$

$\Rightarrow 253 = 3 + (n - 1) \times 5$

$\Rightarrow 253 = 3 + 5n - 5 = -2$

$\Rightarrow 5n = 253 + 2 = 255$

$\Rightarrow n = 255/5 = 51$

Total number of terms in AP is 51

If there are n terms in an AP then some mth term from end will be equal to n - m + 1 term from beginning

Therefore, 20th term from the last term will be 32th term from starting (as, 51 - 20 + 1 = 32)

$a_{32} = a + 31d$

$= 3 + 31 \times 5$

$= 3 + 155 = 158$

Thus, required term is 158.

Q. 18 The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is44. Find the first three terms of the AP.

Answer: Given, $a_8 + a_4 = 24$ and $a_{10} + a_6 = 44$

we know, nth term of an AP is $a + (n - 1)d$

where a is first term and d is common difference. Therefore, $a_8 = a + 7d$

$a_4 = a + 3d$

As per question:

$a + 7d + a + 3d = 24$

Or, $2a + 10d = 24$

Or, $a + 5d = 12$ …………(1)

Also,

$a_{10} = a + 9d$

$a_6 = a + 5d$

Acc. to question :

$a + 9d + a + 5d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$……….(2)

Subtracting equation (1) from equation (2);

$a + 7d – a – 5d = 22 – 12$

$\Rightarrow 2d = 10 \Rightarrow d = 5$

Putting the value of d in equation (1), we get$a + 5(5) = 12 \Rightarrow a + 25 = 12 \Rightarrow a = -13$As, first three terms of any AP are a, a + d, a + 2d.

First three terms of given AP are -13, -13 + 5 = -8, -13 + 2(5) = -3.

Q. 19 SubbaRao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer: Here, a = 5000, d = 200 and an = 7000

We know, an = a + (n – 1)d

7000 = 5000 + (n – 1)200

(n -1)200 = 7000 – 5000

(n – 1)200 = 2000

n – 1 = 10

n = 11

i.e. after 11 years, starting from 1995, his salary will reach to 7000, so we have to add 10 in 1995, because these numbers are in years.

Thus, 1995 + 10 = 2005

Hence, his salary reached at Rs. 7000 in 2005.

Q. 20 Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n**.**

Answer: Ramkali's saving in First week = Rs. 5

Ramkali's saving in second week = Ramkali's saving of first week + Ramkali's saving of second week

= 5 + 1.75 = Rs. 6.75

Rampkali's saving in third week = 6.75 + 1.75 = Rs. 8.50And thus an AP will be formed as 5, 6.75. 8.50,......... with common difference of 1.75Now, Saving in nth week = Rs. 20.75Therefore, a = 5, d = 1.75 and a_n = 20.75

We know, a_n = a + (n -1)d

$20.75 = 5 + (n - 1)1.75$

$(n - 1)\ 1.75 = 20.75 - 5$

$(n - 1)\ 1.75 = 15.75$

$n - 1 = 15.75/1.75 = 9$

$n = 10$

So in 10 weeks her savings will be Rs. 20.75.

Exercise 5.3

Q. 1 Find the sum of the following APs:

(i) 2, 7, 12, . . ., to 10 terms.

(ii) –37, –33, –29, . . ., to 12 terms.

(iii) 0.6, 1.7, 2.8, . . ., to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots, to$ 11 terms

Answer: (i) Here, a = 2, d = 5 and n = 10

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$

= 5(4 + 45)

= 5 × 49

= 245

thus, sum of the 10 terms of given AP (S_n)=245

(ii) Here, a = - 37, d = 4 and n = 12

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2 \times (-37) + (11)4]$

= 6(-74 + 44)

= 6 × (-30)

=-180

Thus, sum of the 12 terms of given AP (S_n)= -180

(iii) Here, a = 0.6, d = 1.1 and n = 100

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n - 1)d]$

$S_{100} = \frac{100}{2}\ [2 \times (0.6) + (99).1]$

50(1.2 + 108.9)

= 50 × (110.1)

=5505

Thus, sum of the 100 terms of given AP (S100) = 5505

(iv) Here,

$a = \frac{1}{15}, n = 11$

$d = \frac{1}{12} - \frac{1}{5} = \frac{1}{60}$

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n - 1)d]$

$S_{11} = \frac{11}{2}\left[2 \times \left(\frac{1}{15}\right) + (10)\frac{1}{60}\right]$

$= \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right)$

$= \frac{11}{2} \times \left(\frac{4+5}{30}\right)$

$= \frac{33}{20}$

Thus, sum of the 100 terms of given AP (S_n) = $\frac{33}{20}$

Q. 2 Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

(ii) 34 + 32 + 30 +… +10

(iii) -5 +(-8)+(-11)+… +(-230)

Answer: (i) Here, a = 7, d = 3.5 and last term = 84

Number of terms can be calculated as follows;

an =a + (n-1)d

Or, 84 = 7 + (n-1)3.5

Or, (n-1)3.5 = 84-7

Or, n - 1 = 77/3.5 = 22

Or, n = 23

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{23} = \frac{23}{2}[2 \times (7) + (22).5]$$

$$= \frac{23}{2}(14 + 77)$$

$$= \frac{23}{2} \times (91)$$

$$= 1046\frac{1}{2}$$

(ii) Here, a = 34, d = - 2 and last term = 10

Number of terms can be calculated as follows:

an = a + (n – 1)d

Or, 10 = 34 + (n – 1)(- 2)

Or, 10 = 34 – (n – 1)(2)

Or, (n – 1)2 = 34 – 10 = 24

Or, n – 1 = 12

Or, n = 13

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

$S_{13} = \frac{13}{2}[2 \times (34) + (12)(-2)]$

$= \frac{13}{2}[68 + (-24)]$

$= \frac{13}{2} \times (44)$

$= 286$

Thus, sum of the given AP (Sn) =

(iii) Here, a = - 5, d = - 3 and last term = - 230

Number of terms can be calculated as follows:

an = a + (n – 1)d

Or, - 230 = - 5 + (n – 1)(- 3)

Or, - 230 = - 5 – (n – 1)3

Or, (n – 1)3 = - 5 + 230 = 225

Or, n – 1 = 75

Or, n = 76

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

$S_{23} = \frac{76}{2}[2 \times (-5) + (-3)75]$

$= \frac{76}{2}(-10 - 225)$

$38 \times (-235)$

= - 8930.

Q. 3 In an AP:

(i) given a= 5, d = 3, a_n = 50, find n and s_n.

(ii) given $a = 7$, $a_{13} = 35$, find d and S_{13}.
(iii) given $d = 3$, $a_{12} = 37$ find a and S_{12}
(iv) given $a_3 = 15$, $S_{10} = 125$ find d and a_{10}.
(v) given $d = 5$, $S_9 = 75$, find a and a_9.
(vi) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.
(vii) given $a= 8$, $a_n = 62$, $S_n = 210$ find n and d.
(viii) given $a_n = 4$, $d = 2$, $S_n=-14$ find n and a.
(ix) given $a = 3$, $n = 8$, $S= 192$, find d
(x) given $l = 28$, $S= 144$, and there are total 9 terms. Find a.

Answer:

(i) To find: n, S_n

Given: $a_n = 50$

$a = 5d = 3$

We know, nth term of an AP is given by

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively and n are the number of terms of the A.P

Number of terms can be calculated as follows:

$a_n = a + (n – 1)d$

Or, $50 = 5 + (n – 1)3$

Or, $(n – 1)3 = 50 – 5 = 45$

Or, $n – 1 = 15$
Or, $\mathbf{n = 16}$
Sum of N terms can be given as follows:

$$\boldsymbol{S = \frac{N}{2}[2a + (n - 1)d]}$$

$\boldsymbol{S_{16} = \frac{16}{2}[2 \times (5) + (15)3]}$

$= 8(10 + 45)$

$= 8 \times 55$
$= 440$

$S_n = 440$

(ii) To find: d and S_{13}

Given:
$a = 7$

$a_{13} = 35$

We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively.

So, Common difference can be calculated as follows:

$a_n = a + (n – 1)d$

Or, $35 = 7 + 12d$

Or, $12d = 35 – 7 = 28$

Or, **$d = 7/3$**

Sum of n terms can be given as follows:
$S = \frac{N}{2}[2a + (n - 1)d]$

$S_{16} = \frac{13}{2}[2 \times (7) + (12)\frac{7}{3}]$

$= \frac{13}{2}(14 + 28)$

$= \frac{13}{2} \times (42)$

= 273

$S_{13} = 273$

(iii) To Find: a, S_{12}

Given:

d = 3

$a_{12} = 37$

We know, nth term of an AP is

an = a + (n - 1)d

where a and d are first term and common difference respectively.

Therefore, First term can be calculated as follows:

$a_n = a + (n – 1)d$

Or, 37 = a + 11 × 3

Or, a = 37 – 33 = 4

a = 4

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n - 1)d]$

$S_{12} = \frac{13}{2}[2 \times (4) + (11)3]$

= 6(8 + 33)

= 6 × 41

$S_{12} = 246$

(iv) To find: d, a_{10}

Given: $a_3 = 15$

$S_{10} = 125$

Sum of n terms can be given as follows:
$$S = \frac{N}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times (a) + (9)d]$$

$125 = 5(2a + 9d)$

$25 = 2a + 9d$ ....eq(i)
We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively.

According to the question; the 3rd term is 15, which means;

$a + 2d = 15$ ....eq(ii)
Now,

Subtracting equation 2 times eq(ii) from equation eq(i), we get;

$(2a + 9d) – 2(a + 2d) = 25 – 30$
Or, $2a + 9d - 2a - 4d = - 5$

Or, $5d = - 5$ **$d = -1$**

Now,

Substituting the value of d in equation (2), we get;

$a + 2(- 1) = 15$

Or, $a – 2 = 15$

Or, a = 17
10^{th} term can be calculated as follows;
$a_{10} = a + 9d$
$= 17 - 9 = 8$
$a_{10} = 8$
Thus, d = - 1 and 10th term = 8

(v) To find: a and a_9

Given:
d = 5

$S_9 = 75$

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$$75 = \frac{9}{2}[2 \times (a) + (8)5]$$

$$75 = \frac{9}{2}(2a + 40)$$

$$\frac{50}{3} = 2a \times 40$$

$$a = -\frac{35}{3}$$

Now,

We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively.

9^{th} term can be calculated as follows:
$a_9 = a + 8d$
$$= -\frac{35}{3} + 40$$

$= \frac{85}{3}$

(vi)

To find : n and an

Given:

a = 2

d = 8

Sn = 90

Sum of n terms can be given as follows

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$90 = \frac{N}{2}[2 \times (2) + (N-1)8]$$

$$90 = \frac{N}{2}(4 + 8N - 8)$$

90 = N(2 + 4N - 4)
$4N^2 - 2N - 90 = 0$
$2N^2 - N - 45 = 0$
For solving this quadratic equation, we have to factor 1 in such a way that product comes out to be 90 and the difference should be 1 (2N +9)(N -5)

Hence, n = - 9/2 and n = 5
Rejecting the negative value,

We have **n = 5**

We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are first term and common difference respectively.

Now, 5^{th} term will be:
$a_5 = a + (5 - 1) d$
$a_5 = a + 4d$
$= 2 + 4 \times 8$
$= 2 + 32 = 34$
$a_5 = 34$

(vii) To find: n and d
Given:
$a = 8$
$a_n = 62$
$S_n = 210$
Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$$210 = \frac{N}{2}[a + a + (n - 1)8]$$

Since, $a_n = a + (n - 1)d$, we have

$420 = n\ (8 + 62)$

$420 = n \times 70$
$n = 6$
Now, for calculating d:
we know that $a_n = a + (n - 1) d$

$a_6 = a + 5d$

Or, $62 = 8 + 5d$

Or, $5d = 62 – 8 = 54$

Or, $d = 54/5$

Common difference, $d = 54/5$

(viii) To find: n and a

Given: $a_n = 4$

$d = 2S_n = -14$

Sum of n terms can be given as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$-14 = \frac{n}{2}[a + a + (n-1)d]$$

$$-28 = n(a+4)$$

$n = \frac{-28}{a+4}$ ………………….eq (i)

We know;

We know, nth term of an AP is

$a_n = a + (n - 1)d$

where a and d are the first term and common difference respectively.

therefore,

$4 = a + (n - 1)2$

Or, $4 = a + 2n - 2$

Or, $a + 2n = 6$

Or, $2n = 6 - a$

Or, $n = (6 - a)/2$ ....eq(ii)

Using (i) and (ii):

$$\frac{-28}{a+4} = \frac{6-a}{2}$$

Cross multiplying, we get

$-56 = (6 - a)(a + 4)$

$24 + 2a - a^2 = -56$
$a^2 - 2a - 80 = 0$
Factorizing 2 in such a way that product of the two terms is 80 and their difference is 2

$(a + 8)(a - 10) = 0$

Therefore, $a = -8$ and $a = 10$

As a is smaller than 10 and d has positive value, hence we'll take $a = -8$
Now, we can find the number of terms as follows:
$a_n = a + (n - 1)d$
$4 = -8 + (n - 1)2$
$(n - 1)2 = 4 + 8 = 12$
$n - 1 = 6, n = 7$
Hence, **n = 7 and a = - 8**
(ix) To find: d

Given: $a = 3$

$n = 8 S_n = 192$

Sum of n terms can be given as follows:
$$S = \frac{N}{2}[2a + (n - 1)d]$$

$$192 = \frac{8}{2}[2 \times 3 + 7d]$$

$192 = 4(6 + 7d)$

$7d = 42$

$d = 6$

(x) To Find: a

Given: $l = 28, S = 144$

Sum of n terms can be given as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

$144 = \frac{9}{2}[a + a_n]$

288 = n (a + 28)

9a + 252 = 288

9a = 288 -252

9a = 36

a = 4

Q. 4 How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Answer: Given that, a = 9, d = 8 and S_n = 636

Let no of terms be N.

Now

We know:

$S = \frac{N}{2}[2a + (n-1)d]$

$636 = \frac{N}{2}[2 \times (9) + (N-1)8]$

$\Rightarrow 636 = \frac{N}{2}(18 + 8N - 8)$

$\Rightarrow$ 636 = N(9 +4N -4)

$\Rightarrow$ 636 = 9N + 4N^2 - 4N

$\Rightarrow$ 4N^2 + 5N -636 = 0

Now to factorize the above quadratic equation, we need to make the product (636 . 4 = 2544) and difference should be 5$\Rightarrow$ 4N^2 + 53N - 48N -636 = 0

⇒ N(4N + 53) - 12(4N + 53) = 0

⇒ (N - 12)(4N + 53) = 0

Now, N = - 53/4 and N = 12

Taking the integral value and rejecting the fractional value, **we have n = 12.**

Q. 5 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer: Sum of n terms of an AP is given by,

$S_n = \frac{n}{2}(a + a_n)$

where 'a' is first term of AP, n is the number of terms of AP and 'an' is last term of AP.

Given, First term, a = 5and last term, an = 45

Also, sum = 400Putting the values in the formula, we get $400 = \frac{n}{2}(5 + 45)$

$\Rightarrow 800 = 50n\, n = \frac{800}{50}$

⇒ n = 16. i.e. there are 16 terms in given AP.Now the nth term of an AP is given by the formula, a_n=a+(n-1)d where, a is the first term, n is the number of terms, a_n is the nth term and d is the common difference.
45 = 5 + (16 - 1) x d⇒15d = 40

⇒d = 40/15

Therefore, common difference of AP is 8/3.

Q. 6 The first and the last terms of an AP are 17 and 350 respectively. If the common differenceis 9, how many terms are there and what is their sum?

Answer: We have;

First term, a = 17, Last term, an = 350 and

Common difference, d = 9

We know, nth term formula:

$a_n = a + (n - 1)d$

$350 = 17 + (n - 1)9$

$(n - 1)9 = 350 - 17$

$n - 1 = 333/9 = 37$

$n = 38$

and we know, sum of 'n' terms of an AP if first and last term is given is

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{38}{2}[17 + 350]$$

$$= 19 * 367$$

$= 6973$

Q. 7 Find the sum of first 22 terms of an AP in which and 22nd term is 149.

Answer: We have; $n = 22$, $d = 7$ and $a_{22} = 149$

where n is number of terms and

d is common difference.

We know;

$a_n = a + (n - 1)d$

$\Rightarrow$ 149 = a + (22 - 1)7 $\Rightarrow$ 149 = a + 147 $\Rightarrow$ a = 149 - 147 $\Rightarrow$ a = 2
The sum can be calculated as follows:
$S = \frac{N}{2}[2a + (n-1)d]$

$= \frac{22}{2}[2 + 149]$

= 11*151

= 1661

Q. 8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
Answer: To Find: S_{51}

Given: $a_2 = 14$, $a_3 = 18$

We have; $a_2 = 14$, $a_3 = 18$ and n = 51
We know,
$d = a_3 - a_2 = 18 - 14 = 4$

Therefore, d = 4
$a_2 - a = 4$
$14 - a = 4$
a = 10
So, first term = 10

Now, sum can be calculated as follows:

$S = \frac{N}{2}[2a + (n-1)d]$

Where, a and d are first term and common difference respectively.
$= \frac{51}{2}[2 * 10 + 50 * 4]$

$= 51 \times (20 + 200)/2$

$= 51 \times 110$

= 5610

S_{51} = 5610

Q. 9 If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Given: $S_7 = 49$, $S_{17} = 289$

Sum of n terms of an A.P is given by the formula,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where, S = Sum, n = number of terms, a = first term and d = common difference.

$S_7 = 49$

Therefore, $49 = \frac{7}{2}(2a + (7-1)d)$

⇒ 49 = 7(a + 3 d)

⇒ 7= a + 3 d

⇒ a + 3 d = 7 [1]

Similarly,

$$S_{17} = \frac{17}{2}[a + (17-1)d]$$

289 = 17(a + 8 d)

17 = a + 8 d

a + 8 d = 17 [2]

Subtracting [1] from [2] we get;

a + 8 d – a – 3 d = 17 – 7

⇒ 5 d = 10

⇒ d = 2

Using the value of d in the equation [1], we can find 'a' as follows:

a + 3d =7

⇒ a + 6 = 7
⇒ a = 1

Using the values of a and d; we can find the sum of first n terms as follows:

$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$

$$S_n = \frac{n}{2}[2 + 2n - 2]$$

$$\Rightarrow S = n^2$$

Q. 10 Show that a_1, a_2, ..., a_n form an AP where a_n is defined as below :
(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.
Answer:
(i)Let us take different values for a, i.e. 1, 2, 3 and so on
a = 3 + 4 = 7
$a_2 = 3 + 4 \times 2 = 11$
$a_3 = 3 + 4 \times 3 = 15$
$a_4 = 3 + 4 \times 4 = 19$
Here; each subsequent member of the series is increasing by 4 and hence it is an AP.
The sum of "n" terms is given as:

$$Sum = \frac{n}{2}[2a + (n-1)d]$$

where, n = no. of terms
a= first term
d = common difference
∴ The sum of first 15 terms is given as:
= (15/2) [2 × 7 + (15-1)×4]
= (15/2) [14 + (14)×4]
= (15/2) [70]
= 525

(ii) Let us take values for a, i.e. 1, 2, 3 and so on

$a = 9 – 5 = 4$
$a_2 = 9 – 5 \times 2 = - 1$
$a_3 = 9 – 5 \times 3 = - 6$
$a_4 = 9 – 5 \times 4 = - 11$
Here; each subsequent member of the series is decreasing by 5 and hence it is an AP.
∴ The sum of first 15 terms is given as:
$= (15/2) [2 \times 4 + (15-1)\times-5]$
$= (15/2) [8 + (14)\times-5]$
$= (15/2) [8-70]$
$= 15(-31)$

$= -465$

Q. 11 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)?

What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.
Answer:
As the sum till first term will only contain first term, $S_1 = a_1$

We can find the first term as follows: $S_n = 4n - n^2$
So, by putting the values in place of 1 we will get the sum till that term
$S_1 = 4 \times 1 - 1^2 = 3$
Now; sum of first two terms can be calculated as follows:
$S_2 = 4 \times 2 - 2^2$
$= 8 - 4 = 4$
Hence, $S_2 = 4$

Now we know by the formula that: $S_n - S_{n-1} = a_n$

Hence; second term $= 4 – 3 = 1$

And second term - first term = common difference of AP

Therefore, common difference, $d = 1 - 3 = -2$
Now we have the AP as follows
First term, $a = 3$Common difference $= -2$So AP will look like : 3, 1, -1, -3, -5, ...And, nth term of an AP is given by the formula, $a_n = a + (n - 1)d$
Putting the value of a and d in above formula we get$a_n = 3 + (n - 1) -2$
$a_n = 3 - 2n + 2$
$a_n = 5 - 2n$

So, 3rd term can be can be calculated by just replacing n with 3
$a_3 = 5 - 2 \times 3$
$a_3 = -1$
Similarly $a_{10} = 5 - 2\times 10$
$a_{10} = -15$
And so on we can calculate any number of terms of this AP.

Q. 12 Find the sum of the first 40 positive integers divisible by 6
Answer: The smallest positive integer which is divisible by 6 is 6 itself and its 40th multiple will be $6 \times 40 = 240$

So, we have $a = 6$, $d = 6$, $n = 40$ and 40th term = 240.

Sum of first 40 positive integers divisible by 6 can be calculated as follows:

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$$= \frac{40}{2}[2 * 6 + (39)6]$$

$= 20(12 + 234)$

$= 20*246$

$= 4920$

Q. 13 Find the sum of the first 15 multiples of 8.
Answer:
As we have to calculate sum of first 15 multiples of 8, series will look like:
8, 16, 24, ... up to 15 terms
Given: First term, a = 8, Common Difference, d = 8 and nth term, n = 15
Sum of first 15 multiples of 8 can be calculated as:
Sum of n terms of an A.P is given by:
$S_n = \frac{N}{2}[2a + (n - 1)d]$
Applying the values from above
$= \frac{15}{2}[2 * 8 + (14)8]$

= 15 (8 + 56)

= 15 × 64 = 960
Thus sum of first 15 multiples of 8 is 960.
Q. 14 Find the sum of the odd numbers between 0 and 50.
Answer:
To Find: Sum of odd numbers from 0 and 50
Let us write these numbers1, 3, 5, ,................, 49As we can clearly see this forms an AP with first term, a = 1 and common difference, d = 2 and nth term, a_n = 49
Now, first we need to find number of terms, for that we have the formula of nth terms of an AP given by a_n = a + (n - 1) d
Putting the values we get49 = 1 + (n - 1) 248 = (n - 1) 2(n - 1) = 24n = 25So there are 25 odd numbers between 0 and 50And sum of these 25 numbers are given by using sum of
$S_n = \frac{n}{2}[2a + (n - 1)d]$
Putting the values of formula we get,
$S_{25} = \frac{25}{2}[2 \times 1 + (25 - 1) \times 2]$
Terms of an AP
$S_{25} = \frac{25}{2}[2 \times 1 + (25 - 1) \times 2]$
Terms of an AP

$$S_{25} = \frac{25}{2} \times [48 + 2]$$

$$S_{25} = 25 \times 25$$
$$S_{25} = 625$$
So, the sum of odd numbers between 0 and 50 is 625.

Q. 15 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Answer:

Given: a = 200, d = 50 and n = 30

We can find the penalty by using the sum of 30 terms:

$$S = \frac{N}{2}[2a + (n - 1)d]$$
$$= \frac{30}{2}[2 * 200 + (29)50]$$
= 15(400 + 1450)

= 15*1850

= 27750

Hence, he has to pay an amount of Rs. 27750 as penalty

Q. 16 A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer: As, each price is decreased by 20 rupees, we can consider amount given to each student in increasing order to be an AP.

Let the lowest price given be 'a' and then prize is increased by Rs 20. In that case, we have First term = a Common difference, d = - 20No of

terms, n = No of students getting prize = 7Sum of 'n' terms = Total prize money = 700
Now,
we know, sum of 'n' terms on AP is
$S = \frac{n}{2}[2a + (n - 1)d]$
$700 = \frac{7}{2}[2 \times a + 6 \times (-20)]$
⇒ 700 × 2 = 7[2a - 120]⇒ 200 = 2a - 120⇒ 2a = 200 + 120⇒ 2a = 320⇒ a = 160Hence, the divisions are Rs. 160, Rs. 140, Rs. 120, Rs. 100, Rs. 80, Rs. 60, Rs. 40.

Q. 17 In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?
Answer: First there are 12 classes and each class has 3 sections
Since each section of class 1 will plant 1 tree, so 3 trees will be planted by 3 sections of class 1.Thus every class will plant 3 times the number of their class
(for example class (iii) will plant = 3 x 3 = 9 plants)
Similarly,
No. of trees planted by 3 sections of class 1 = 3
No. of trees planted by 3 sections of class 2 = 6
No. of trees planted by 3 sections of class 3 = 9

No. of trees planted by 3 sections of class 4 = 12
Its clearly an AP with first term = Number of trees planted by class 1 = 3
We have; a = 3, d = 3 and n = 12
We can find the total number of trees as follows:
$S = \frac{N}{2}[2a + (n - 1)d]$
$= \frac{12}{2}[2 * 3 + (11)3]$
= 6 (6 + 33)
= 6 × 39

= 234

Total number of trees planted by students = 234.

Q. 18 A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles?

(Take $\pi = \frac{22}{7}$)

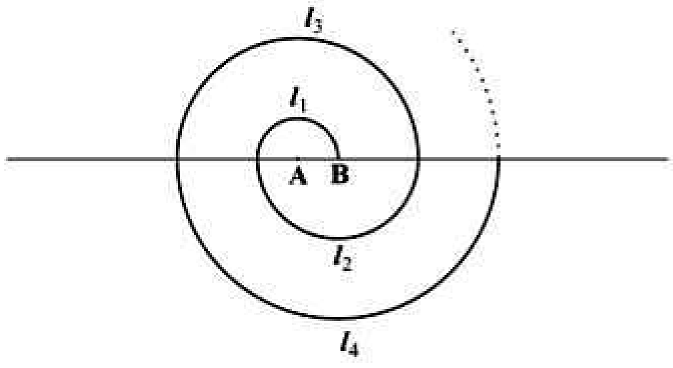

Fig. 5.4

[Hint : Length of successive semicircles is l_1, l_2, l_3, l_4 with centres at A, B, A, B, . . .,respectively.]

Answer: Circumference of 1st semicircle = πr = 0.5π

Circumference of 2nd semicircle = πr = 1π = π

Circumference of 3rd semicircle =πr = 1.5π

It is clear that a = 0.5 π, d = 0.5π and n = 13

Hence; the length of the spiral can be calculated as follows:

$S = \frac{N}{2}[2a + (n - 1)d]$

$= \frac{13}{2}[2 * 0.5\pi + (12).5\pi]$

$= \frac{13}{2}\ [\pi + 6\pi]$

$= \frac{13}{2}(7\pi)$

$= \frac{13}{2} \times 7 \times \frac{22}{7}$

= 143 cm.

Q. 19 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row,18 in the row next to it and so on (see Fig. 5.5). In how may rows are the 200 logs placed and how many logs are in the top row?

Answer : As, the rows are going up, the no of logs are decreasing, 20, 19, 18, ...,it's an AP Suppose 200 logs are arranged in 'n' rows, thenWe have; First term, a = 20,Common difference, d = - 1 and Sum of n terms, S_n = No of logs = 200

We know;

$$S = \frac{N}{2}[2a + (n - 1)d]$$

$$200 = \frac{N}{2}[2 \times 20 + (N - 1)(-1)]$$

400= N (40 - N +1)

400 = 41N - N2

N^2 -41N + 400 =0

(N -16)(N –25)

Thus, n = 16 and n = 25

If number of rows is 25 then;

a_{25} = 20 + 24 × (- 1)

= 20 – 24 = - 4

Since; negative value for number of logs is not possible hence; number of rows = 16

a_{16} = 20 + 15 × (- 1)

= 20 – 15 = 5

Thus, number of rows = 16 and number of logs in top rows = 5

Q. 20 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 5.6).

Fig. 5.6

A competitor starts from the bucket, picks up the nearest potato, runs back with it, deposit in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is [2× 5 + 2× (5+3)].

Answer: Distance covered in picking and dropping 1st potato = 2 × 5 = 10 m

Distance covered in picking and dropping 2nd potato = 2 (5+3) = 16 m

Distance covered in picking and dropping 3rd potato = 2(5 + 3 + 3) = 22 m

Therefore, a = 10, d = 6 and n = 10

Total distance can be calculated as follows:

$$S = \frac{N}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 * 10 + (9)6]$$

= 5(20 + 54)

= 5*74

= 370 m

Total distance run by the competitor = 370 m.

Exercise 5.4

Q. 1 Which term of the AP: 121, 117, 113, . . ., isits first negative term?

[Hint: Find n for $a_n < 0$]

Answer: Here, first term, a = 121common difference, $d = a_2 - a_1 = 117 - 121 = -4$

Let the first negative term be 'a_n'.

Also, we know that, nth term of an AP is given by $a_n = a + (n - 1)d$

We have to find least value of n, such that $a_n < 0$

$\Rightarrow a + (n - 1)d < 0$

$\Rightarrow 121 + (n - 1)(-4) < 0$
$\Rightarrow 121 - 4(n - 1) < 0$
$\Rightarrow 4(n - 1) > 121$
$\Rightarrow 4n - 4 > 121$
$\Rightarrow 4n > 125$
$\Rightarrow n > 31.25$
Therefore, n is 32 [least positive integer greater than 31.25 is 32]

Hence, the 32nd term of AP is first negative term. Also, $a_{32} = a + 31d$
$= 121 + 31(-4) \quad = 121 - 124 = -3$

Q. 2 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.
Answer: To find: S_{16}
Given $a_3 + a_7 = 6$
$a_3 \times a_7 = 8$
nth term of an AP is given by the formula $a_n = a + (n - 1) d$
where, $a_n =$ nth term
n = number of term d = common difference. So now its given that sum of third and seventh term is 6, thus we need to find 3rd and 7th term first,$a_3 = a + 2d$
$a_7 = a + 6d$
As per question;
$a_3 + a_7 = 6$
So now,

$a + 2d + a + 6d = 6$
$2a + 8d = 6$
$a + 4d = 3$
$a = 3 – 4d$ ....eq (i)
Similarly,
Product of third and seventh term is given as 8. So,

$(a + 2d)(a + 6d) = 8$
$a^2 + 6ad + 2ad + 12d^2 = 8$ ....eq(ii)
Substituting the value of a in equation (ii), we get;

$(3 - 4d)^2 + 8(3 - 4d)d + 12d^2 = 8$
$9 - 24d + 16d^2 + 24d - 32d^2 + 12d^2 = 8$
$9 - 4d^2 = 8$
$2d = 1$
$d = \pm 1/2$
Using the value of d in equation (1), we get;
$a = 3 - 4d$
$a = 3 - 4 \times \frac{1}{2}$
or, $a = 3 - 2 = 1$

Sum of first 16 terms is calculated as follows:

$S = \frac{n}{2}[2a + (n - 1)d]$
$S_{16} = \frac{16}{2}[2 \times 1 + (16 - 1) \times \frac{1}{2}]$

$S_{16} = 8[2 + (15/2)]$

$= 4 \times 19$
$S_{16} = 76$
Thus, sum of first 16 terms of this AP is 76.

Now by taking $d = -1/2$, we get, $a = 3 - 4(-1/2)a = 3 + 2 = 5$

$S = \frac{16}{2}[2 \times 5 + (16 - 1)\frac{-1}{2}]$
$S = 8[10 - 15/2]S = 4[20 - 15]S = 4[5] = 20$So, another possible value of sum is 20.

Q. 3 A ladder has rungs 25 cm apart.(see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are$2\frac{1}{2}$m apart, what is the length of the wood required for the rungs?
[Hint: Number of rungs = $\frac{250}{25}$]

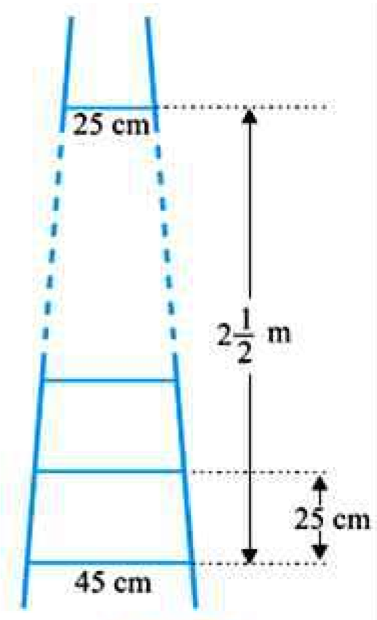

Fig. 5.7

Answer: Total distance between top and bottom rung = 2m 50 cm

$$\text{Number of rungs} = \frac{250}{25} + 1$$

Distance between any two rungs = 25

Number of rungs = 11

And it is also given that bottom most rungs is of 45 cm length and top most is of 25 cm length. As it is given that the length of rungs decrease uniformly, it will for an AP with a = 25, a_{11} = 45 and n = 11

d can be calculated as follows:
nth term of an AP is given by

$a_n = a + (n - 1)d$
$a_{11} = a + 10d$
$45 = 25 + 10d$
$10d = 45 - 25 = 20$
$d = 2$

Total length of wood will be equal to the sum of 11 terms:

$S = \frac{N}{2}[2 + (n - 1)d$

$= \frac{11}{2}[2 * 25 + 10 * 2]$

$= 11[\ 25 + (10)]$

$= 11 \times 35$

$= 385$ cm

Therefore, total wood required for rungs is equal to 385 m

Q. 4 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [Hint: $S_{x-1} = S_{49} - S_x$]

Answer: The AP in the above problem is

1, 2, 3, - - - , 49

With first term, a = 1

Common difference, d = 1

nth term of AP = a + (n - 1)d

$a_n = 1 + (n - 1)1$

$a_n = n$ [1]

Suppose there exist a mth term such that, (m<49)

Sum of first m - 1 terms of AP = Sum of terms following the mth term

Sum of first m - 1 terms of AP = Sum of whole AP - Sum of first m terms of AP

As we know sum of first n terms of an AP is,

$S_n = \frac{n}{2}[a + a_n]$ if last term a_n is given

$$\frac{m-1}{2}(a + a_{m-1}) = \frac{49}{2}(a + a_{49}) - \frac{m}{2}(a + a_m)$$

(m - 1)(1 + m - 1) = 49(1 + 49) - m(1+m) [using 1]

(m - 1)m = 2450 - m(1 + m)

$m^2 - m = 2450 - m + m^2$

$2m^2 = 2450$

$m^2 = 1225$

m = 35 or m = - 35 [not possible as no of terms can't be negative.]

and $a_m = m = 35$ [using 1]

So, sum of no of houses preceding the house no 35 is equal to the sum of no of houses following the house no 35.

Q. 5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long andbuilt of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50\ m^3$]

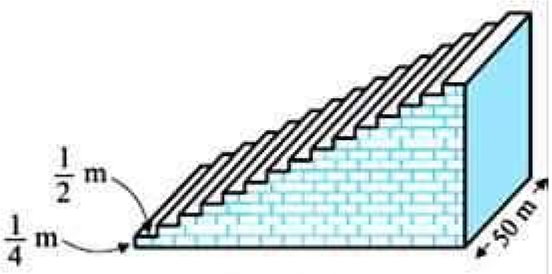

Fig. 5.8

Dimensions of 1st step = 50 m × 0.25 m × 0.5 m

Volume of 1st step = 6.25 m^3

Dimensions of 2nd step = 50 m × 0.5 m × 0.5 m

Volume of 2ndstep = 12.5 m^3

Dimensions of 3rdstep = 50 m × 0.75 m × 0.5 m

Volume of 3rdstep = 18.75 m^3

Clearly, volumes of respective steps are in AP
Now, we have a = 6.25, d = 6.25 and n = 15

Sum of 15 terms can be calculated as follows:

$$S_{15} = \frac{15}{2}[2 * 6.25 + (14)6.25]$$

$$= \frac{15}{2}(100)$$

= 750

Hence, the volume of concrete will be $750m^3$.