Chapter – 4
Quadratic Equations

Exercise 4.1

Q. 1 Check whether the following are quadratic equations :

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Answer : For a equation to be quadratic equation, degree of the equation(highest power of the variable in the equation) should be 2
Thus a quadratic equation is of the form, $ax^2 + bx + c = 0$, where $a \neq 0$
(i) $(x + 1)^2 = 2(x - 3)$

We know $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

Degree (highest power) of the equation is 2

Equation is in the form of $ax^2+bx+c = 0$, with $a \neq 0$

So, we can say that the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x + 6 - 2x = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Equation is of the form $ax^2+bx+c = 0$ with $a \neq 0$
Hence, the given equation is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$
$x(x + 1) - 2(x + 1) = x(x + 3) - 1(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$
$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$
$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$
$\Rightarrow -3x + 1 = 0$
$\Rightarrow 3x - 1 = 0$
Degree (highest power) of the equation is 1

Equation is not in the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow$ x(2x + 1)-3(2x + 1) = x^2 + 5x
$\Rightarrow$ $2x^2 + x - 6x - 3 = x^2 + 5x$
$\Rightarrow$ $2x^2 – 5x – 3 = x^2 + 5x$
$\Rightarrow$ $2x^2 – 5x – 3 - x^2 - 5x = 0$
$\Rightarrow$ $x^2 – 10x – 3 = 0$
Degree of the equation is 2
Equation is in the form $ax^2 + bx + c = 0$, with $a \neq 0$
Hence, the given equation is a quadratic equation.

(v) (2x – 1)(x – 3) = (x + 5)(x – 1)
$\Rightarrow$ 2x(x - 3) -1(x - 3) = x(x - 1) + 5(x - 1)
$\Rightarrow$ $2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$
$\Rightarrow$ $2x^2 – 7x + 3 = x^2 + 4x – 5$
$\Rightarrow$ $2x^2 – 7x + 3 - x^2 - 4x + 5 = 0$
$\Rightarrow$ $x^2 – 11x + 8 = 0$
Degree of the equation is 2
Equation is in the form $ax^2+bx+c = 0$, with $a \neq 0$

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x – 2)^2$

$\Rightarrow$ $x^2 + 3x + 1 = x^2 + 4 – 4x$

$\Rightarrow$ $x^2 + 3x + 1 - x^2 - 4 + 4x = 0$
$\Rightarrow$ 7x – 3 = 0
Degree of the equation is 1
It is not of the form $ax^2 + bx + c = 0$.
Hence, the given equation is not a quadratic equation.

(vii) $(x + 2)^3 = 2x (x^2 - 1)$
Apply the formula $(a+b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow$ $x^3 + 8 + 6x^2 + 12x = 2x^3 – 2x$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$
$\Rightarrow -x^3 + 8 + 6x^2 + 14x = 0$
$\Rightarrow x^3 - 14x - 6x^2 - 8 = 0$
Degree of the equation is 3
It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$
$(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$
$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 8 + 6x^2 - 12x = 0$
$\Rightarrow 2x^2 - 13x + 9 = 0$
Degree of the equation is 2 and
It is of the form $ax^2+bx+c = 0$, with $a \neq 0$
Hence, the given equation is a quadratic equation.

Q. 2 Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years)3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Answer: (i)The area of a rectangular plot is 528 m2.

Let the breadth of the plot be x m.

The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Thus, the length of the plot is $(2x + 1)$ m.

Area of a rectangle = Length × Breadth

$\therefore 528 = x(2x + 1)$

$\Rightarrow 2x^2 + x - 528 = 0$ (required quadratic form)

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

Let the consecutive integers be x and $x + 1$.

It is given that their product is 306.

$\therefore x(x+1) = 306$

$\Rightarrow x^2 + x - 306 = 0$ (required quadratic form)

(iii) Let Rohan's present age be x.

Given, Rohan's Mother is 26 years older than him

Hence, his mother's age = $x+26$

3 years hence,

Rohan's age = $x + 3$

Mother's age = $x + 26 + 3$

$= x + 29$

It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29) = 360$

$x^2 + 3x + 29x + 87 = 360$

⇒ $x^2 + 32x - 273 = 0$ (required quadratic form)

(iv) Let the speed of train be x km/h.

As speed = distance / time

⇒ Time taken for travel 480 km $= \frac{480}{x} hrs$

In the second condition,

speed of train = (x – 8) km/h

Given that the train will take 3 hours more to cover the same distance.

Therefore, Time taken for traveling 480 km = $\left(\frac{480}{x} + 3\right)$ hrs

Speed × Time = Distance

$$(x - 8)\left(\frac{480}{x} + 3\right) = 480$$

$$\Rightarrow 480 + 3x - \frac{3840}{x} - 24 = 480$$

$$\Rightarrow 3x - \frac{3840}{x} = 24$$

⇒ $3x^2 - 3840 = 24x$

⇒ $3x^2 - 24x - 3840 = 0$

⇒ $x^2 - 8x - 1280 = 0$ (required quadratic form)

Exercise 4.2

Q. 1 Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100\,x^2 - 20x + 1 = 0$

Answer:

(i) $X^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$
$= x(x - 5) + 2(x - 5)$
$= (x - 5)(x+2)$
Roots of this equation are the values for which $(x - 5)(x+2) = 0$
$\therefore$ x - 5 = 0 or x + 2 = 0
i.e., x = 5 or x = -2

(ii) $2\,x^2 + x - 6$

$= 2x^2 + 4x - 3x - 6$

$= 2x\,(x+2) - 3(x+2)$

$= (x+2)(2x - 3)$

Roots of this equation are the values for which $(x+2)(2x - 3) = 0$

$\therefore$ x+ 2 = 0 or 2x – 3 = 0

$x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2}$
$= x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5)$

$(\sqrt{2}x + 5)(x + \sqrt{2})$

Roots of this equation are the values for which $(\sqrt{2}x + 5)(x + \sqrt{2})$ =0

$\sqrt{2}x + 5 = 0 \; or \; x + \sqrt{2} = 0$

$x = \frac{-5}{\sqrt{2}x}, x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$= \frac{1}{8}(16x^2 - 8x + 1)$

$= \frac{1}{8}(16x^2 - 4x - 4x + 1)$

$= \frac{1}{8}(4x(4x - 1) - 1(4x - 1))$

$= \frac{1}{8}(4x - 1)^2$

Roots of this equation are the values for which $(4x – 1)^2 = 0$

Therefore,

(4x – 1) = 0 or (4x – 1) = 0

i.e., $x = \frac{1}{4} \; r \; x = \frac{1}{4}$

(v) $100x^2 – 20x +1$

$= 100x^2 – 10x – 10x +1$

=10x(10x – 1) – 1 (10x – 1)

$=(10x – 1)^2$

Roots of this equation are the values for which $(10x – 1)^2 = 0$

Therefore,
(10x – 1) = 0 or (10x – 1) = 0
i.e. $x = \frac{1}{10}$ or $x = \frac{1}{10}$

Q. 2 Solve the problems given in Example 1.

Answer: (i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = 45 − x

After losing 5 marbles,

Number of John's marbles = x − 5

Number of Jivanti's marbles = 45 − x − 5 = 40 − x

Given that the product of their marbles is 124.

∴ (x – 5) (40 – x) = 124

⇒ $x^2 – 45x +324 = 0$

Now, to factorize this equation, we need to take numbers such that their product is 324 and sum is 45

⇒ $x^2 – 36x – 9x +324 = 0$

⇒ x(x – 36) – 9 (x – 36) = 0

⇒ (x – 36) (x – 9) = 0

Either x – 36 = 0 or x − 9 = 0

i.e., x = 36 or x = 9

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

$\therefore$ Cost of production of each toy = Rs $(55 - x)$

It is given that, total production of the toys = Rs 750

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

Now to factorize this equation we have to find two numbers such that their product is 750 and sum is 55

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Either $x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

Q. 3 Find two numbers whose sum is 27 and product is 182.

Answer: Let the first number be x and the second number is $27 - x$.

[As the sum of both the numbers is 27]

Therefore, their product = $x(27 - x)$

It is given that the product of these numbers is 182.

$x(27 - x) = 182$

$-x^2 + 27x - 182 = 0$

Changing the signs on both sides we get, $x^2 - 27x + 182 = 0$

Factorizing we get , 13 and 14 are the numbers whose sum is 27 and product is 182

$x^2 - 13x - 14x + 182 = 0$

$= x(x - 13) - 14(x - 13) = 0$

$= (x - 13)(x - 14) = 0$

Either $x - 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13

Therefore, the numbers are 13 and 14.

Q. 4 Find two consecutive positive integers, sum of whose squares is 365.

Answer:

To Find: Consecutive integers sum of whose square is 365 Consecutive integers mean that the difference between the integers is of 1Let the consecutive positive integers be x and $x + 1$.

Given that $x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

Now to factorize the above quadratic equation, we need to chose numbers such that their product is 182 and difference is 1

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$(x+14)(x - 13) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

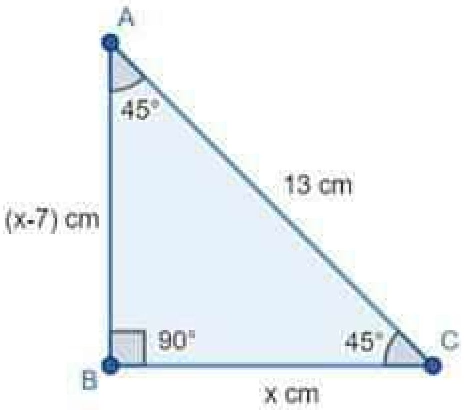
Since the question ask for positive integers, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Q. 5 The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer: Consider a right angle ΔABC



Let the base of the Δ be x

$\Rightarrow$ Altitude = x - 7

In the right-angled triangle

According to the Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$

$\Rightarrow 13^2 = x^2 + (x - 7)^2$

$\Rightarrow 169 – x^2 – x^2 - 49 + 14x = 0$

$\Rightarrow 2x^2 - 14x – 120 = 0$

$\Rightarrow x^2 – 7x - 60 = 0$

$\Rightarrow x^2 – 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x – 12) = 0$

$\Rightarrow (x + 5)(x - 12) = 0$

$\Rightarrow x = - 5$ or 12

But base cannot be negative, so $x = 12$ cm

$\Rightarrow$ altitude $= 12 - 7 = 5$ cm

Hence 5 cm and 12 cm are the two sides of the given triangle.

Q. 6 A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.
Answer: To find: Number of articles produced and cost of each article.

Let the number of articles produced be x.
Therefore, cost of production of each article = Rs $(2x + 3)$
The total cost of production = Total quantity produced x cost of one article

It is given that the total cost of production is Rs 90.
$\therefore x(2x + 3) = 90$
$\Rightarrow 2x^2 + 3x = 90$
$\Rightarrow 2x^2 + 3x – 90 = 0$
$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$
$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$
$\Rightarrow (2x + 15)(x - 6) = 0$
Either $2x + 15 = 0$ or $x - 6 = 0$,
i.e., $x = \frac{-15}{2}$ or $x = 6$
As the number of articles produced can only be a positive integer,
Therefore, x can only be 6.

Hence, the number of articles produced = 6
Cost of each article = $2 \times 6 + 3$

$= 12 + 3 =$ Rs 15.

Exercise 4.3

Q. 1 Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Answer: $2x^2 - 7x + 3 = 0$

Dividing by coefficient of x^2, we get

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Adding and subtracting the square of $\left(\frac{b}{2}\right) = \frac{7}{4}$, we get

$$x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + \frac{3}{2} = 0$$

Use the formula $(a-b)^2 = a^2 + b^2 - 2ab$ to get,

$$\left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$$

$$\left(x - \frac{7}{4}\right)^2 + \frac{24-49}{16} = 0$$

$$\left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$$

$$\left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

So,

$$\left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$$

When $x - \frac{7}{4} = -\frac{5}{4}$

$$x = -\frac{5}{4} + \frac{7}{4} = \frac{12}{4} = 3$$

Hence the value of x is 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

Dividing by coefficient of x^2

$x^2 + \frac{x}{2} - 2 = 0$

Adding and subtracting the square of $\frac{b}{2} = \frac{1}{4}$, we get

$x^2 + 2(x) \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - 2 = 0$

Use the formula $(a+b)^2 = a^2 + b^2 + 2ab$ *to get*,

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{16} + 2\right) = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

$\Rightarrow x + \frac{1}{4} = \pm \frac{\sqrt{33}}{16}$

$\Rightarrow x = \frac{-1}{4} \pm \frac{\sqrt{33}}{4}$

$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$

$\Rightarrow x = \frac{-1 + \sqrt{33}}{4}$

Or $x = \frac{-1 - \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Divide the whole equation by 4 to get, $\frac{4x^2 + 4\sqrt{3}x + 3}{4} = \frac{0}{4}$

Add and subtract the square of half of the coefficient of x,

$\Rightarrow x = x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$\Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$

Use the formula $(a+b)^2 = a^2 + b^2 + 2ab$ to get,

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} - \frac{3}{4} = 0$

$\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = \pm 0$

$\Rightarrow x + \frac{\sqrt{3}}{2} = 0$ and $x + \frac{\sqrt{3}}{2} = -0$

$\Rightarrow$ x = $0 - \frac{\sqrt{3}}{2}$ and x = $0 - \frac{\sqrt{3}}{2}$

$\Rightarrow$ x = $-\frac{\sqrt{3}}{2}$ and x = $-\frac{\sqrt{3}}{2}$

$\therefore$ Roots of equation are = x = $-\frac{\sqrt{3}}{2}$, $-\frac{\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

Dividing by coefficient of x^2 we get,

$x^2 + \frac{x}{2} - 2 = 0$

Adding and subtracting the square of $\frac{b}{2} = \frac{1}{4}$, we get

$\Longrightarrow x^2 + \frac{x}{2} + 2 + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 0$

$x^2 + \frac{x}{2} + 2 = 0$

Adding and subtracting the square of coefficient of x we get,

$\Longrightarrow x^2 + \frac{x}{2} + 2 + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 0$

Use the formula $(a+b)^2 = a^2 + b^2 + 2ab$ to get,

$x^2 + \frac{x}{2} + 2 = 0$

$\Longrightarrow \left(x + \frac{1}{4}\right)^2 + 2 - \left(\frac{1}{4}\right)^2 = 0$

$\Longrightarrow \left(x + \frac{1}{4}\right)^2 + 2 - \frac{1}{16} = 0$

$\Longrightarrow \left(x + \frac{1}{4}\right)^2 + \frac{32-1}{16} = 0$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 + \frac{31}{16} = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = - \frac{31}{16}$$

Square of a number cannot be negative, Hence roots of the given equation do not exist.

Q. 4 **The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$ Find his present age.**

Answer :

Let the present age of Rehman be x years.

Three years ago, his age was $(x - 3)$ years.

Five years hence, his age will be $(x + 5)$ years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $\frac{1}{3}$.

$$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow 3(2x + 2) = (x - 3)(x + 5)$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x - 7) + 3(x - 7) = 0$

$\Rightarrow x = 7, -3$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Q. 5 In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer:

Given : In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

To find: her marks in two subjects.

Solution:

Let the marks obtained in Mathematics by Shefali be 'a'.

Given, sum of the marks obtained by Shefali in Mathematics and English is 30.

Marks obtained in english = 30 – a

Also, she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210.

$\Rightarrow (a + 2)(30 - a - 3) = 210$

$\Rightarrow (a + 2)(27 - a) = 210$

$\Rightarrow 27a - a^2 + 54 - 2a = 210$

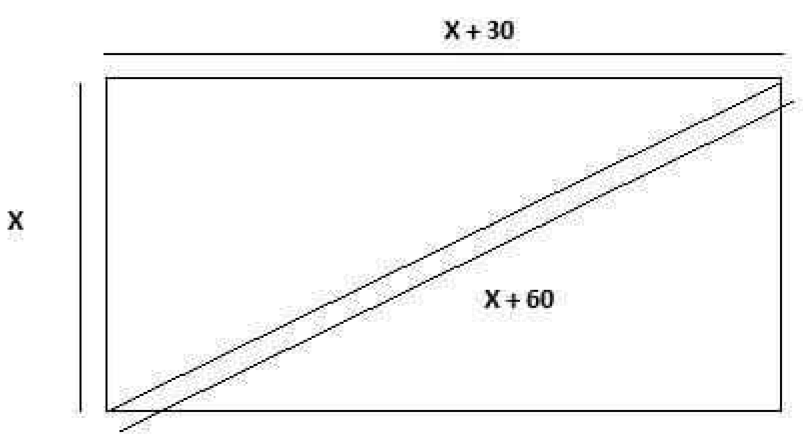
$\Rightarrow -a^2 + 25a + 54 = 210$

$\Rightarrow -a^2 + 25a + 54 - 210 = 0$

$\Rightarrow -a^2 + 25a - 156 = 0$
$\Rightarrow a^2 - 25a + 156 = 0$
In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.
$\Rightarrow a^2 - 13a - 12a + 156 = 0$
$\Rightarrow a(a - 13) - 12(a - 13) = 0$
$\Rightarrow (a - 12)(a - 13) = 0$
$\Rightarrow a = 12$ or 13
If marks in mathematics is 12
marks in english is 30 - a = 30 - 12 = 18

If marks in mathematics is 13marks in english is 30 - a = 30 - 13 = 17
Hence
Marks in Mathematics = 12, Marks in English = 18
Or
Marks in Mathematics = 13, Marks in English = 17

Q. 6 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.



To find: length and breadth of the field
Let the shorter side of the rectangle be x m.

Then, larger side of the rectangle = $(x + 30)$ m

we know,

By pythagoras theorem,$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$
Diagonal of a rectangle is $\sqrt{[(\text{length})^2 + (\text{breadth})^2]}$

Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$
It is given that the diagonal of the rectangle is 60 m more than the shorter side.
$\sqrt{x^2 + (x + 30)^2} = x + 60$
Squaring both sides, we get,

$\Rightarrow x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$

$\Rightarrow x^2 - 60x - 2700 = 0$
Now for solving this quadratic equation, we need to factorize 60 in such a way that the product is 2700 and the difference is 60
$\Rightarrow x(x - 90) + 30(x - 90) = 0$

$\Rightarrow (x - 90)(x + 30) = 0$

$\Rightarrow x = 90, -30$

However, side cannot be negative. Therefore, **the length of the shorter side will be 90 m.**

Hence, **length of the larger side will be (90 + 30) m = 120 m**

Q. 7 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
Answer : Assumption: Let the larger and smaller number be x and y respectively.
Given:
According to the given question difference of squares of two numbers is 180. and the square of smaller number is 8 times square of the larger number.
So,

$\Rightarrow x^2 - y^2 = 180$ eq(i)

$\Rightarrow y^2 = 8x$ eq(ii)

putting value of eq(ii) in eq(i)

$\Rightarrow x^2 - 8x = 180$

$\Rightarrow x^2 - 8x - 180 = 0$

For factorizing this quadratic equation, the product of numbers should be 180 and their difference should be 8
$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$\Rightarrow (x - 18)(x+10) = 0$

$\Rightarrow x = 18, -10$
The larger number cannot be negative as it makes the square of smaller number negative, which is not possible.

Therefore the larger number will be 18 only.

$x = 18$
$\therefore y^2 = 8x$
$= 8\times18 = 144$
$= y = \pm\sqrt{144} = \pm12$
$\therefore$ smaller number $= \pm12$

Therefore, the numbers are 18 and 12 or 18 and -12.
Note: As we need to solve this problem using one variable, so the variable y is written in terms of variable x.

Q. 8 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer: To find: Speed of the train

Let the speed of the train be x km/hr.

Time taken to cover 360 km = $\frac{360}{x}$ hr, As

$time = \frac{distance}{speed}$

Now, given that if the speed would be 5 km/hr more, the same distance would be covered in 1 hour less, i.e. if speed = x + 5, and $time = \left(\frac{360}{x} - 1\right)$

then, using distance = speed x time, we have

$(x + 5)\left(\frac{360}{x} - 1\right) = 360$

Now we can form the quadratic equation from this equation

$360 - x + \frac{1800}{x} - 5 = 360$

$\frac{360x - x^2 + 1800 - 5x}{x} = 360$

Now, cross multiplying we get⇒ 360 x - x^2 + 1800 - 5 x = 360 x

⇒ x^2 + 5 x – 1800 = 0

Now we have to factorize in such a way that the product of the two numbers is 1800 and the difference is 5⇒ x^2 + 45 x – 40 x – 1800 = 0

⇒ x(x + 45) – 40(x+45) = 0

⇒ (x+45)(x – 40) = 0

⇒ x = - 45, 40

since, the speed of train can't be negative, so, speed will be 40 km/hour.

Q. 9 Two water taps together can fill a tank in hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer: Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = (x − 10) hr

Part of the tank filled by a smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by the larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

So 75/8 hours, multiplied by the sum of parts filled with both pipes in one hour equal to complete work i.e 1.

$$\frac{75}{8}\left(\frac{1}{x} + \frac{1}{x-10}\right) = 1$$
$$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$
$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)} = \frac{8}{75}$$

$\Rightarrow 75(2x - 10) = 8x^2 - 80x$

$\Rightarrow 150x - 750 = 8x^2 - 80x$

$\Rightarrow 8x^2 - 230x + 750 = 0$

Now for factorizing the above quadratic equation, two numbers are to be found such that their product is equal to 750 x 8 and their sum is equal to 230

$\Rightarrow 8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow 8x(x - 25) - 30(x - 25) = 0$

⇒ (x – 25)(8x – 30) = 0

⇒ x = 25, $\frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8}$= 3.75 hours.

As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

Q. 10 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer: Let the average speed of passenger train be x km/h.

Average speed of express train = (x + 11) km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

As,

$tti\,me = \frac{di\,stance}{speed}$

Time taken by passenger train $= \frac{132}{x}$

Time taken by express train = $\frac{132}{x+11}$

Now Time taken by passenger train - Time taken by express train = 1

$\therefore\ \frac{132}{x} - \frac{132}{x+11} = 1$

$= 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$

$\Rightarrow\ \frac{132\times 11}{x(x+11)} = 1$

$\Rightarrow 132 \times 11 = x(x+11)$

$\Rightarrow x^2 + 11x - 1452 = 0$

Now we have to factorize this equation such that the product of numbers is 1452 and their difference is 11
$\Rightarrow x^2 + 44x - 33x - 1452 = 0$

$\Rightarrow x(x+44) - 33(x+44) = 0$

$\Rightarrow (x+44)(x - 33) = 0$

$\Rightarrow x = -44, 33$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Q. 11 Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.
Answer:

Let the sides of the two squares be x m and y m.

Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

It is given that $4x - 4y = 24$ [Difference of perimeter] or $x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$ [sum of squares is 468]

$(6+y)^2 + y^2 = 468$

$36 + y^2 + 12y + y^2 = 468$

$2y^2 + 12y - 432 = 0$

$y^2 + 6y - 216 = 0$

$y^2 + 18y - 12y - 216 = 0$

$y(y+18)(y - 12) = 0$

$y = -18$ or 12.

However, side of a square cannot be negative.
Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

Exercise 4.4

Q. 2 Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Answer: We know that if an equation $ax^2 + bx + c = 0$ has two equal roots,
its discriminant
$(b^2 - 4ac)$ will be 0.

(i) $2x^2 + kx + 3 = 0$
Comparing equation with $ax^2 + bx + c = 0$, we obtain,
$a = 2, b = k, c = 3$
Discriminate $= b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$
For equal roots,
Discriminant $= 0$
$k^2 - 24 = 0$
$k^2 = 24$
$= k = \pm\sqrt{24} = \pm 2\sqrt{6}$
(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain,
$a = k, b = -2k, c = 6$
Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$
For equal roots, $b^2 - 4ac = 0$
$= 4k^2 - 24k = 0$
$= 4k(k - 6) = 0$
Either $4k = 0$ or $k = 6$
$= k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Therefore, if this equation has two equal roots, k should be 6 only.

Q. 3 Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Answer: Let the breadth of mango grove be *l*.
Length of mango grove will be 2*l*.

Area of mango grove = $(2l)(l) = 2l^2$

$2l^2 = 800$

$\Rightarrow l^2 - 400 = 0$

$\Rightarrow l^2 = 400$

$l = \pm 20$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m
Length of mango grove = $2 \times 20 = 40$ m

Q. 4 Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer: Let the age of one friend be x years.
Age of the other friend will be $(20 - x)$ years.

4 years ago,

age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4) = (16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -20, c = 112$

Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48$

As $b^2 - 4ac < 0$,
Therefore, no real root is possible for this equation and hence, this situation is not possible.

Q.5 Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.
Answer: Let the length and breadth of the park be l and b.
Perimeter $= 2(l + b) = 80$
$l + b = 40$ Or, $b = 40 - l$
Area $= l \times b = l(40 - l)$
$= 40l - l^2 = 400$ Given
$l^2 - 40l + 400 = 0$
Comparing this equation with $al^2 + bl + c = 0$, we obtain
$a = 1, b = -40, c = 400$
Discriminant D $= b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$
As $b^2 - 4ac = 0$,
Therefore, this equation has equal real roots and hence, this situation is possible.
Root of this equation,
$$l = -\frac{b}{2a}$$
$$l = -\frac{(-40)}{2(1)} = \frac{40}{2}$$
Therefore, length of park, l = 20 m

And breadth of park, b = 40 − l = 40 − 20 = 20 m