Chapter – 3
Pair of Linear Equations in Two Variables

Exercise 3.1

Q. 1 Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Given: Seven years ago, Aftab was seven times as old as his daughter and after 3 years Aftab will be 3 times as old as his daughter.

To Represent the situations algebraically and graphically we need to find the linear equations for these situations. Let present age of Aftab = x

Let present age of his daughter = y

Then, seven years ago the age of Aftab and his daughter must have been seven less than their present ages, Age of Aftab seven years ago = x – 7

Age of Daughter seven years ago = y – 7

According to the question,

Seven years ago, Aftab was seven times as old as his daughter, So

$x - 7 = 7(y - 7)$

$\Rightarrow x - 7 = 7y - 49$

$\Rightarrow x = 7y - 42$

Now for finding different points of this equation, we can either take different values of x and put them in the equation to obtain values of y or vice versa

Putting y = 5, 6 and 7 in equation (¡),

we get,

For y = 5,

$x = 7 \times 5 - 42$

$= 35 - 42 = -7$ For x =6,

$x = 7 \times 6 - 42$

$= 42 - 42 = 0$ For y = 7,

$x = 7 \times 7 - 42$

$= 49 - 42 = 7$

x	-7	0	7
y	5	6	7

Thus we got 3 points to plot on graph for this equation.

Three years from now,

Age of Aftab = x+3

Age of Daughter = y+3

According to the question,

$\Rightarrow x + 3 = 3(y + 3) \Rightarrow x + 3 = 3y + 9 \Rightarrow x = 3y + 6$

Now for finding different points of this equation, we can either take different values of x and put them in equation to obtain values of y or vice versa

Putting x = 0, 3 and 6

x	0	3	6
y	-2	-1	0

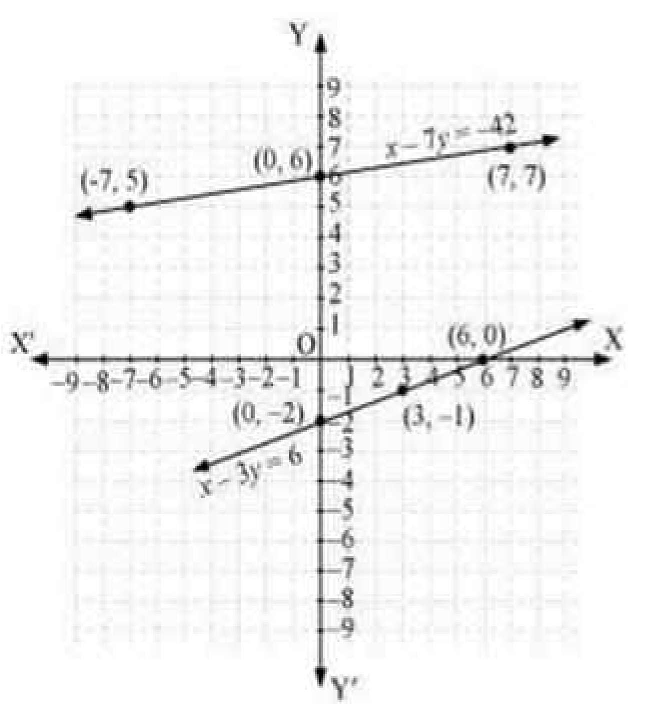
Thus we got 3 points to plot on graph for this equation.

Algebraic representation

x - 7y = -42 (i)

x - 3y = 6   (ii)

Graphical representation:



Q. 2 The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer:

We need to form linear equations for the situations.

Let cost of one bat = Rs. x

Let cost of one ball = Rs. y

In first case, three bats and 6 balls cost him 3900 rupees. Therefore our equation becomes

⇒ 3x + 6y = 3900 ....   (i)

Dividing equation by 3 both side

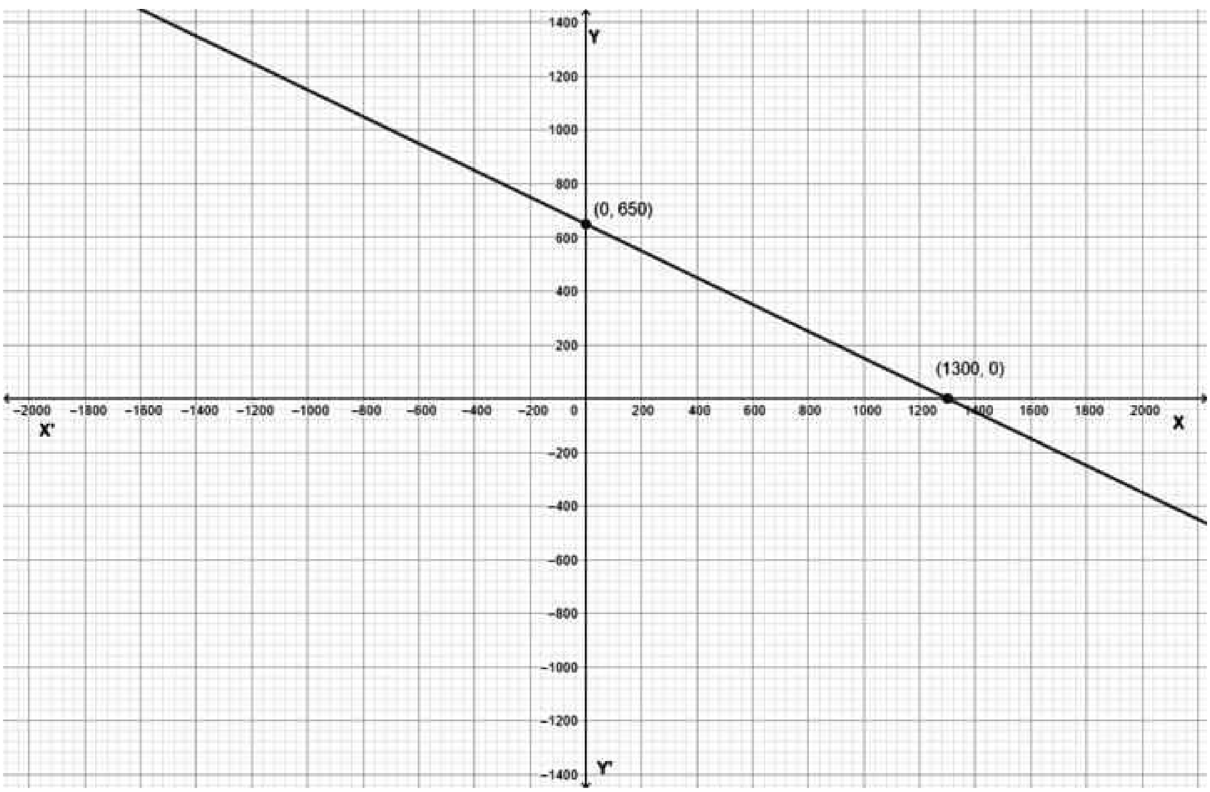
⇒ x + 2y = 1300

⇒ x = 1300 - 2y

For plotting the equation of graph, take different values of y and obtain the value of x from equation or you can do vice versa. At y = 0⇒ x = 1300 - 2(0) ⇒ x = 1300Now finding the value at x = 0⇒ 0 = 1300 - 2y⇒ 2y = 1300⇒ y = 650

X	0	1300
Y	650	0

From above points, we make the graph as follows [Graphic representation]



In second case he buys one bat and 3 balls for 1300, therefore,

⇒ x + 3y = 1300 ...... (ii) [Algebraic representation]

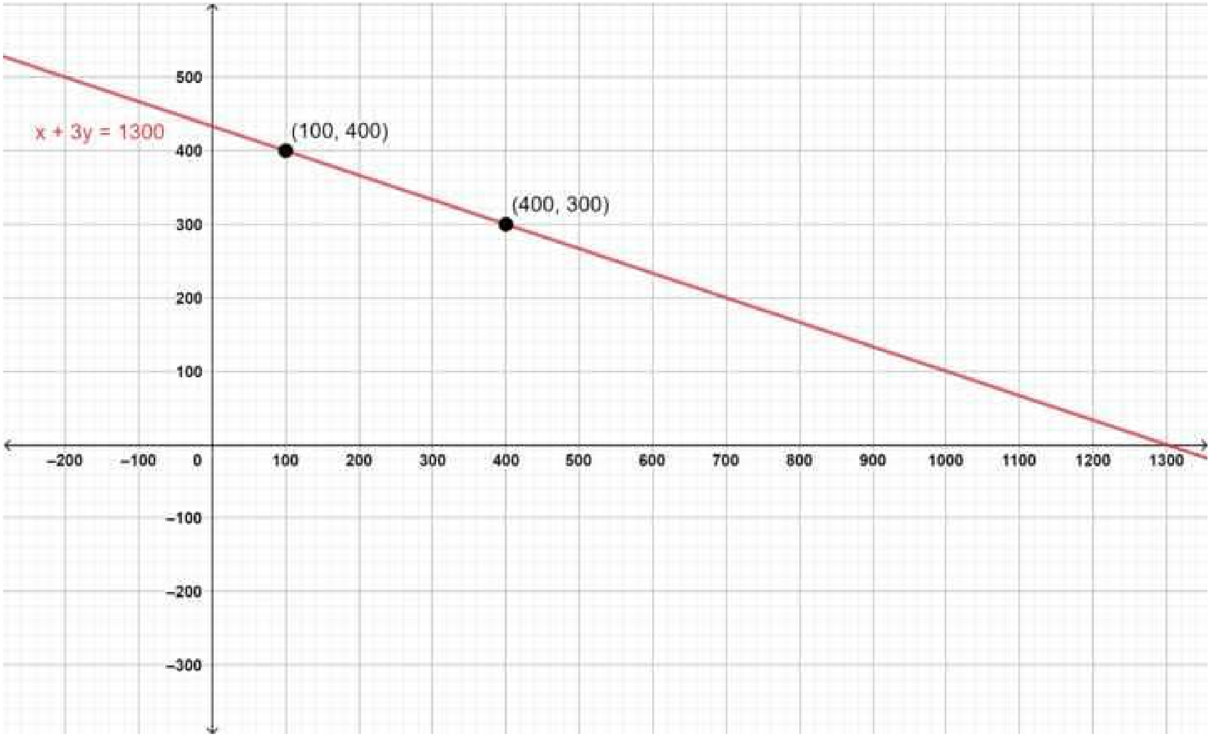
⇒ x = 1300 - 3y

At y = 400

⇒ x = 1300 - 3(400) ⇒ x = 100At y = 300⇒ x = 1300 - 3(300) ⇒ x = 400

x	100	400
y	400	300

From above points, we make the graph as follows, [Graphical representation]



Q. 3 The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer: Let the cost of each kg of apples = Rs. X

Let the cost of each kg of grapes = Rs. Y

According to the question,

Cost of 2 kg of Apples = 2 x

Cost of 1 kg of grapes = y

2x + y = 160 ….. (i)

2x = 160 – y

$$x = \frac{16-y}{2}$$

Putting y = 20, 40 and 60 we get,

x = (160 - 20)/2 = 70

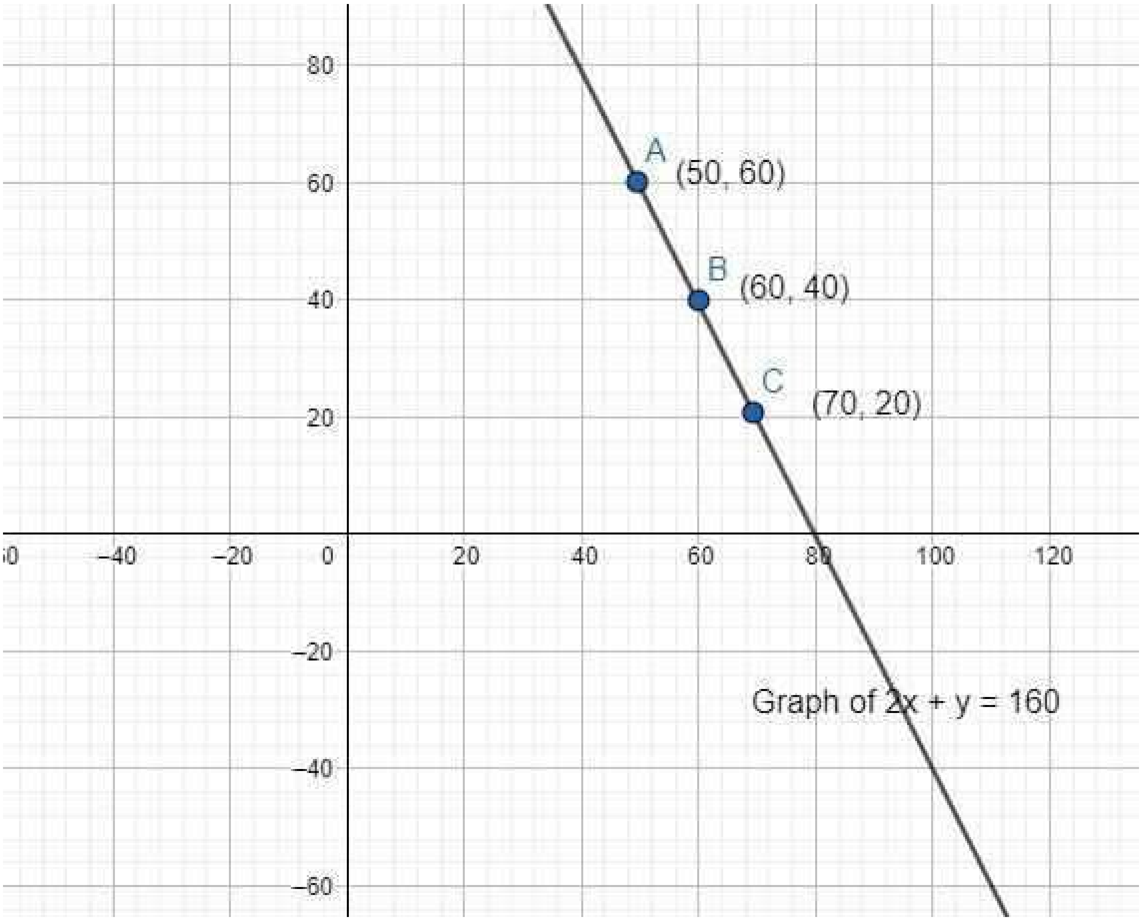
x = (160 - 40))/2 = 60

x = (160 - 60)/2 = 50

X	50	60	70
Y	60	40	20

Algebraic Representation: 2 x + y = 160

Graphical Representation:



Now taking another case

Cost of 4 kg of apples and 2 kg of grapes is Rs.300……………………………(Given)

So, $4x + 2y = 300$ …… (ii)

Dividing the equation by 2, we get,

$2x + y = 150$

$y = 150 – 2x$

Putting x = 70, 75 and 80 we get,

$y = 150 – 2×70 = 10$

$y = 150 – 2(75) = 0$

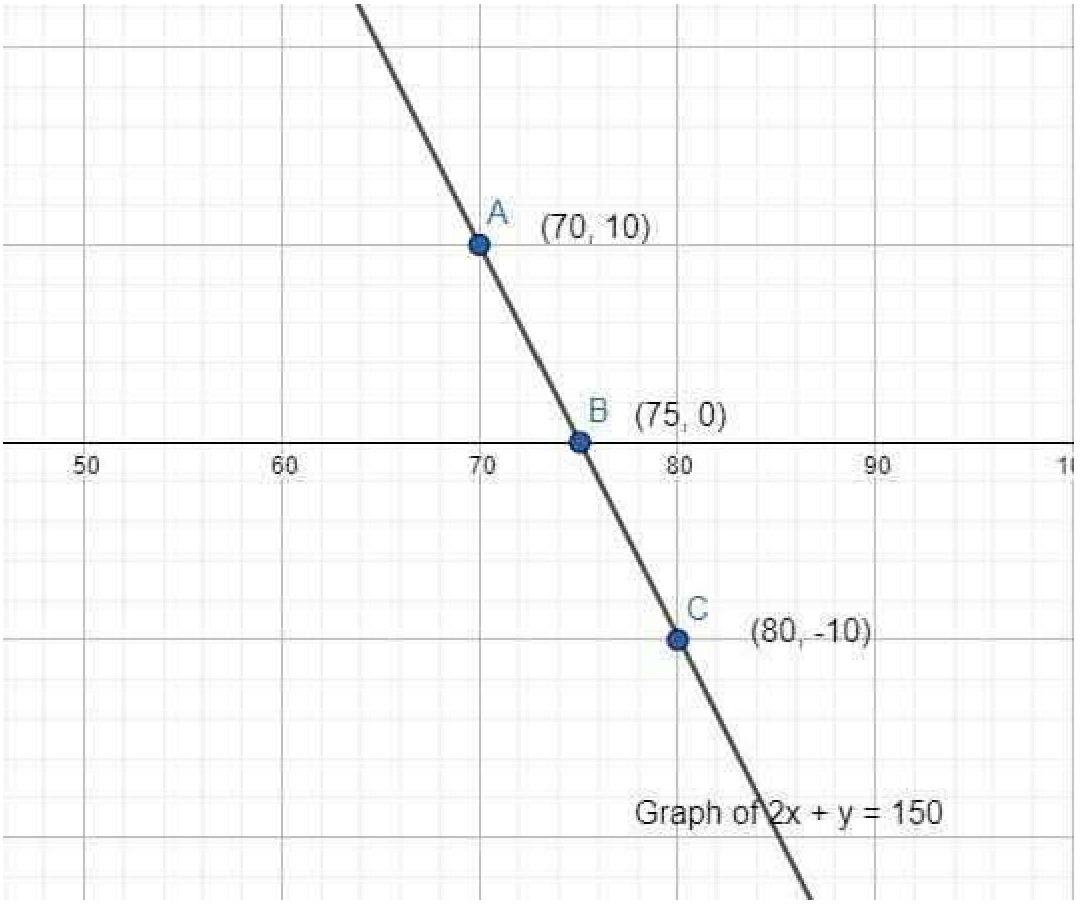
y = 150 – 2(80) = 150 – 160 = -10

X	70	75	80
Y	10	0	-10

Algebraic representation:

4 x + 2 y = 300

Graphical representation:



Exercise 3.2

Q. 1 Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer:

For representing the situation graphically and algebraically, we need to form linear equations

(i) Let number of girls = x

Let number of boys = y

According to the question, Total no of students is equal to 10,

$x + y = 10$

$\Rightarrow$ $x = 10 - y$ ....eq(i)

Now we will find different points to plot the equation. We can take any value of y and put in eq (i) to obtain the value of x at that point

Putting y = 4, 5 and 6. we get,

at x = 4

X = 10 – 4 = 6

at x = 5

X = 10 – 5 = 5

at x = 6

X = 10 – 6 = 4

x	4	5	6
y	6	5	4

Number of girls is 4 more than number of boys ………..Given

So,

x = y + 4

⇒ y = x - 4 .... (ii)

Now for plotting the points on graph, take any values of x and put them in eq (ii) to obtain values of y

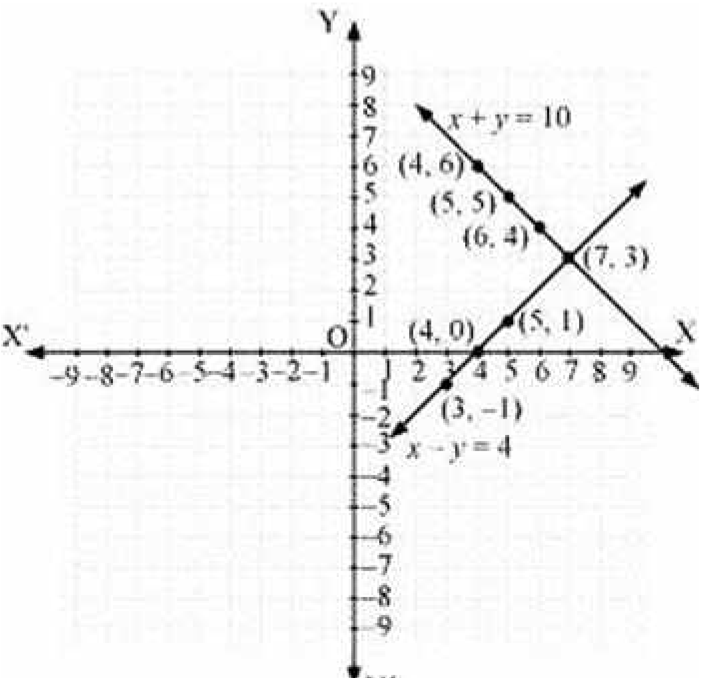
Putting x = 3,5 and 7 we get

at x = 3y = 3 - 4 = -1at x = 4y = 5 - 4 = 1at x = 7y = 7 - 4 = 3

x	3	5	7
Y	-1	1	3

Graphical representation:

Plotting the points obtain on graph we get,



As, both lines intersect each other at (7, 3)

Solution of this pair of equation is (7, 3)i.e. No of girls, x = 7No of boys, y = 3

(ii) Let cost of one pencil = Rs. X

Let cost of one pen = Rs. Y

According to the question, 5 pencils and 7 pens together cost Rs 50

5x + 7y = 50

⇒5x = 50 - 7y

$\Rightarrow x = 10 - \frac{7}{5}y$

Putting value of y = 0, 5, 10 we get,

x = 10 – 0 = 10

$x = \frac{50-35}{5} = \frac{15}{5} = 3$

$x = \frac{50-70}{5} = \frac{-20}{5} = -4$

x	10	3	-4
Y	0	5	10

Now,

7 pencils and 5 pens together cost Rs. 46

7x + 5y = 46

⇒ 5y = 46 – 7x

$y = \frac{46-7x}{5}$

Putting x = -2, 3, 8 we get

$y = \frac{46-(7\times-2)}{5} = \frac{46+14}{5} = \frac{60}{5} = 12$

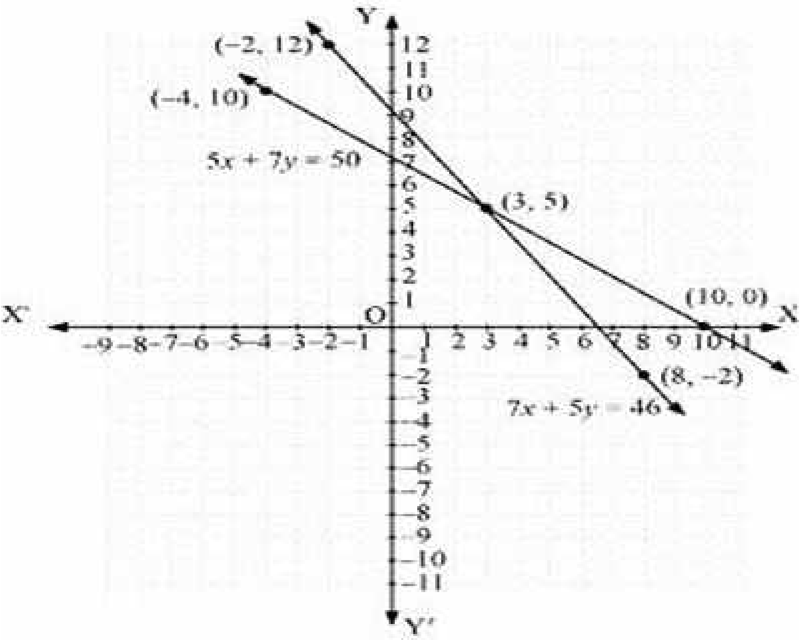
$y = \frac{46-7\times3}{5} = \frac{46-21}{5} = \frac{25}{5} = 5$

$y = \frac{46-7\times8}{5} = \frac{46-56}{5} = \frac{-10}{5} = -2$

x	-2	3	8
y	12	5	-2

Graphical Representation:

Plotting the points obtain on graph we get,



As, both lines intersect each other at (3, 5)

Solution of this pair of equation is (3, 5) i.e. cost of pencil, x = 3cost of pen, y = 5.

Q. 2 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) 5x – 4y + 8 = 0

7x + 6y – 9 =0

(ii) 9x + 3y + 12 = 0

18x + 6y + 24 = 0

(iii) 6x – 3y + 10 = 0

2x – y + 9 = 0

Answer: (i) Comparing these equation with

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

We get

$a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

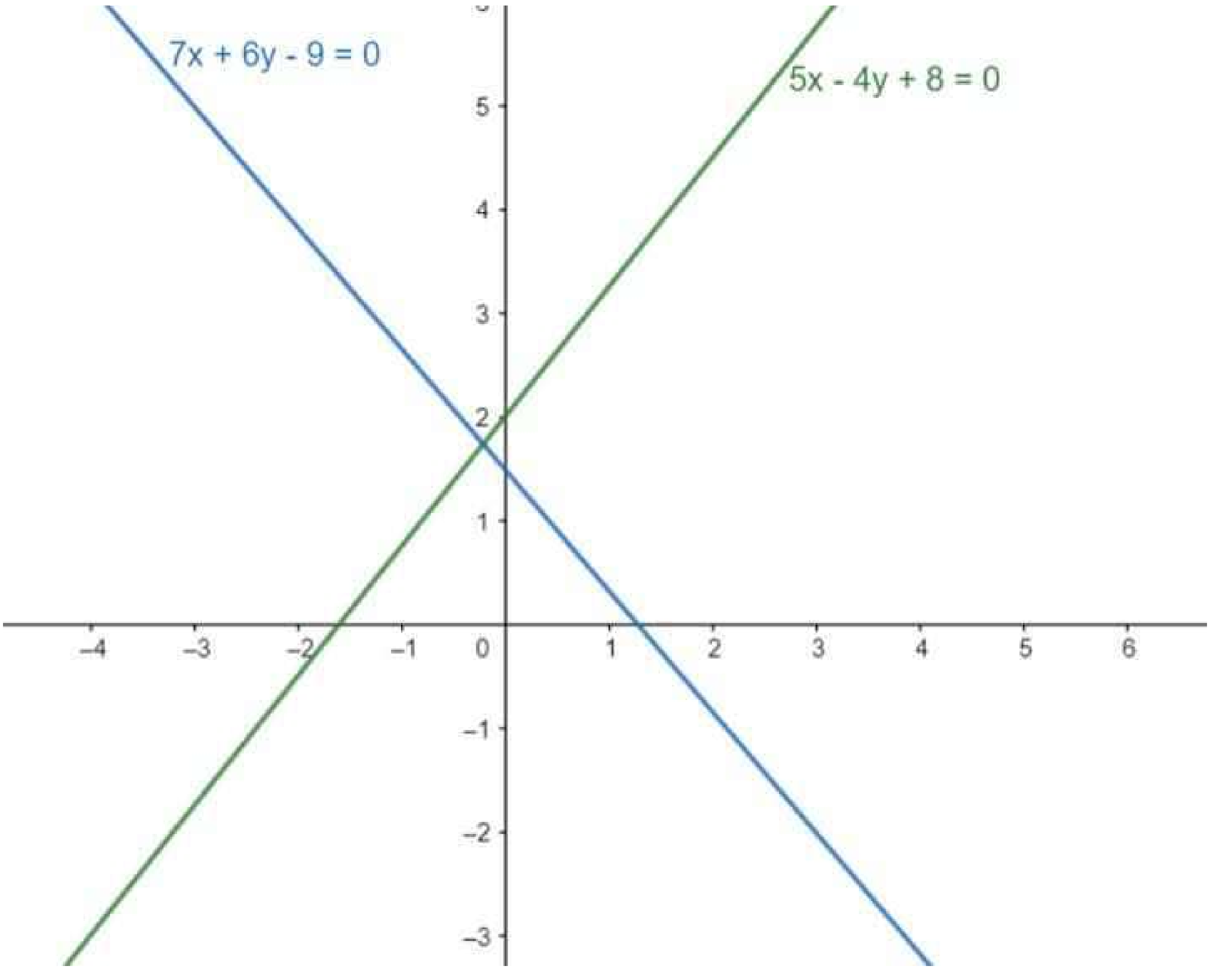
Hence,

$= \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = -\frac{4}{6}$ *and* $\frac{c_1}{c_2} = \frac{8}{-9}$

We find that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, both lines intersect at one point.

Graph of the lines look like below



ii) Comparing these equations with,

$a_1 x + b_1 y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

We get

$= a_1 = 9, b_1 = 3, c_1 = 12$

$= a_2 = 18, b_2 = 6, c_2 = 24$

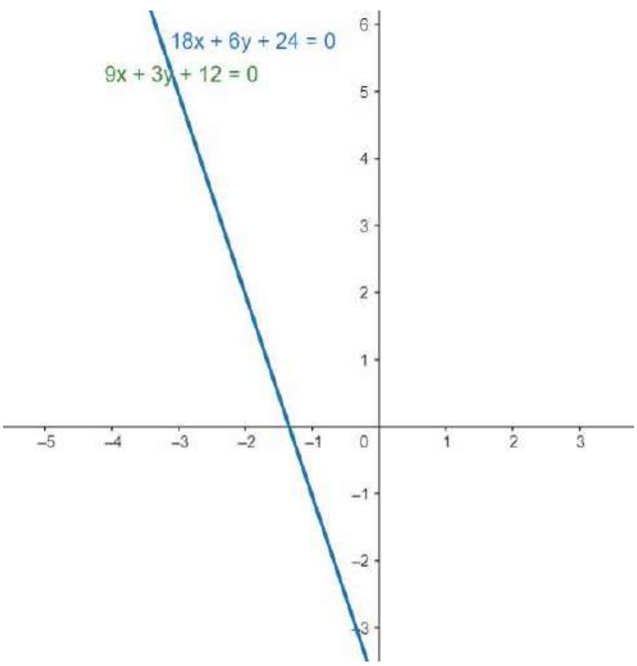
Hence,

$= \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ *and* $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

We find that

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, both lines are coincident.



iii) Comparing these equations with,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

We get

$= a_1 = 6, b_1 = -3, c_1 = 10$

$= a_2 = 2, b_2 = -1, c_2 = 9$

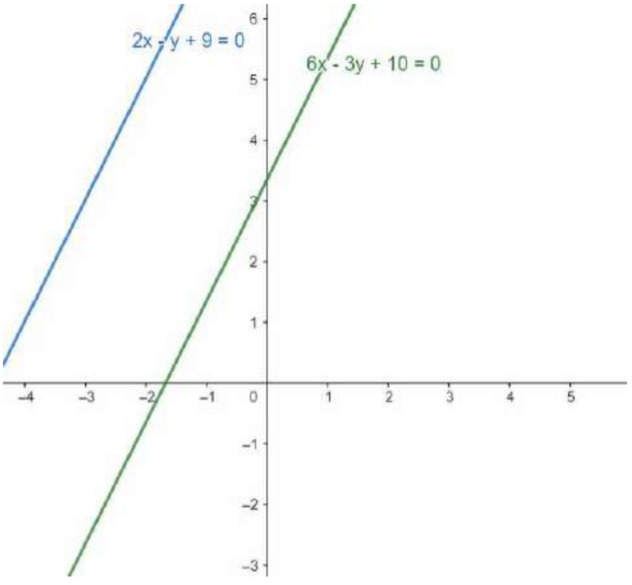
Hence,

$$= \frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = -\frac{3}{-1} = \ 3 \ and \ \frac{c_1}{c_2} = \frac{10}{9}$$

We find that

$$= \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, both lines are parallel



Q. 3 On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent, or inconsistent.

(i) 3x + 2y = 5; 2x – 3y = 7

(ii) 2x – 3y = 8 ; 4x – 6y = 9

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

(iv) 5x – 3y = 11; -10x + 6y = -22

(v) $\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12

Answer:

We get

$$= \frac{a_1}{a_2} = \frac{3}{2}$$

$= \frac{b_1}{b_2} = -\frac{2}{3}$

Hence,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations will intersect at one point only and have only one possible solution and pair of linear equations is inconsistent.

(ii) We get,

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$= \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$

$= \frac{c_1}{c_2} = \frac{8}{9}$

Hence,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and have no possible solution.

And pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

9x – 10 y = 14

We get,

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{18} = \frac{1}{6}$

$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{-30} = -\frac{1}{6}$

$= \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$

Hence,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations will intersect each other at one point and have only one possible solution.

And, the pair of linear equations is consistent.

(iv) We get

$$= \frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}$$

$$= \frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}$$

$$= \frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$$

Hence,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore these pair of lines has the infinite number of solutions. and pair of linear equations is inconsistent

v) We get

$$= \frac{a_1}{a_2} = \frac{4}{6} = \frac{2}{3}$$

$$= \frac{b_1}{b_2} = \frac{2}{3}$$

$$= \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

Hence,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore these pair of lines have infinite number of solutions and pair of linear equation is consistent.

Q. 4 Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) x + y = 5; 2x + 2y = 10

(ii) x – y = 8, 3x – 3y = 16

(iii) 2x +y – 6 = 0, 4x -2y-4 = 0

(iv) 2x – 2y – 2 =0, 4x -4y – 5 = 0
Answer:

(i) We get

$= \frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$

Hence,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

x + y = 5

x = 5 – y

Putting y = 1, 2, 3 we get

x = 5 – 1 = 4

x = 5 – 2 = 3

x = 5 – 3 = 2

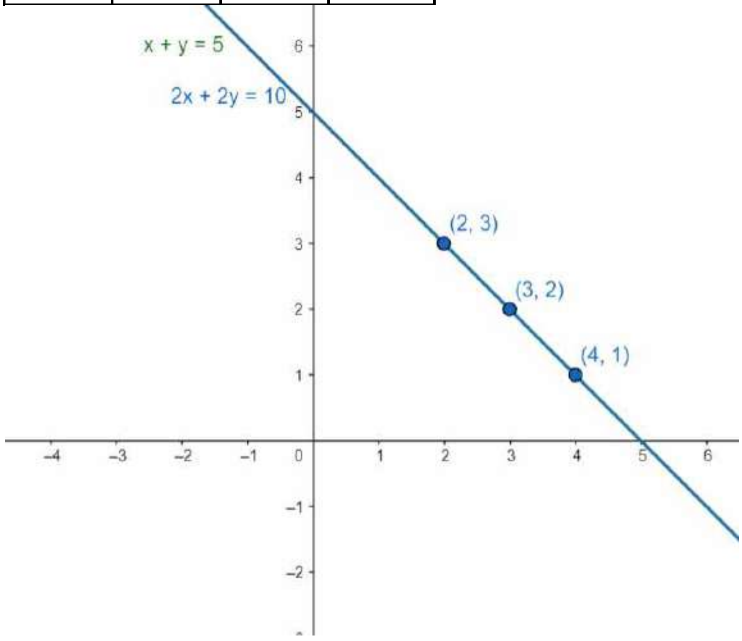
X	4	3	2
Y	1	2	3

And, 2x + 2y = 10

$X = \frac{10-2y}{2}$

X	4	3	2

Y	1	2	3



(ii) We get

$$= \frac{a_1}{a_2} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Hence,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, these linear equations are parallel to each other and have no possible solution,

Hence, the pair of linear equations is inconsistent.

(iii) We get,

$$= \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{-2}$$

$\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$

Hence,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution.

Hence, pair of linear equations is consistent.

$2x + y - 6 = 0$

$= y = 6 - 2x$

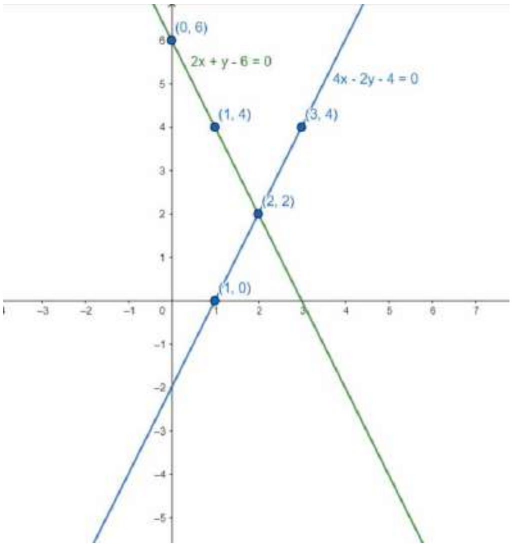
X	0	1	2
Y	6	4	2

And, $4x - 2y - 4 = 0$

$= y = \frac{4x-4}{2}$

X	1	2	3
Y	0	2	4

Graphical representation



(iii) We get,

$= \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{2}{5} s$

Hence,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and have no possible solution,

Hence, the pair of linear equations is inconsistent.

Q. 5 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer: Let the length and breadth of garden be 'x' and 'y' respectively.

Given, Half the perimeter is 36 m We know perimeter of Rectangle = 2(length + breadth) Half the perimeter = x + y ⇒ x + y = 36 ⇒ x = 36 - y [1] Also, Length is 4 m more than width ⇒ x = y + 4 [2] From [1] and [2], we have 36 - y = y + 4 ⇒ 2y = 32 ⇒ y = 16 Putting in [1], we have ⇒ x = 36 - 16 = 20 Hence, Length and Breadth of garden are 20 m and 16 m respectively.

Q. 6 Given the linear equation 2x + 3y – 8= 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

(ii) Parallel lines

(iii) Coincident lines

Answer:

(i) Intersecting lines: For this condition,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The second line such that it is intersecting the given line is

2x + 4y - 6 = 0

As

$\frac{a_1}{a_2} = \frac{2}{2} = 1$

$\frac{b_1}{b_2} = \frac{3}{4}$ *and* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Parallel lines:

For this condition

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the second line can be 4x + 6y - 8 = 0

As $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$,

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-8}{-8} = 1$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) Coincident lines: For coincident lines,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence,

the second line can be 6x + 9y - 24 = 0

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

So,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q. 7 Draw the graphs of the equations x – y + 1 = 0 and 3x +2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Equation: 1

x - y + 1 = 0

x = y – 1

Let us find the coordinates satisfying the above equation,

X	0	1	2	-1
Y	1	2	3	0

Equation 2:

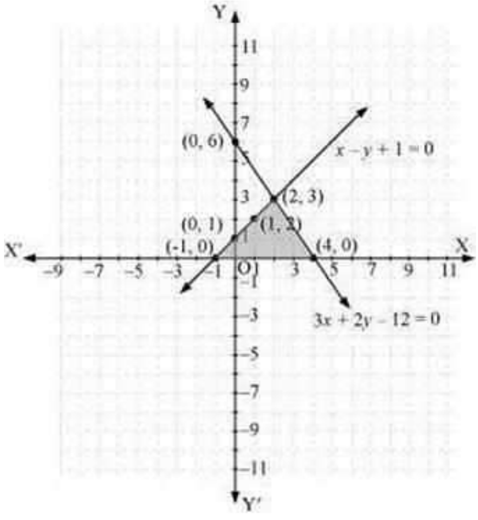
3x + 2y - 12 = 0

$x = \frac{12-2y}{3}$

X	4	2	0
Y	0	3	6

By taking, coordinates we can plot the both equations in graph.

Hence, the graphic representation is as follows.


Y
X'
X
Y'
O
(0, 6)
(2, 3)
(0, 1)
(1, 2)
(-1, 0)
(4, 0)
x − y + 1 = 0
3x + 2y − 12 = 0
11
9
7
3
1
−1
−3
−5
−7
−9
−11
−9
−7
−5
−3
−1
1
3
5
7
9
11

Exercise 3.3

Q. 1 Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$
$x - y = 4$

(ii) $s - t = 3$
$\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$
$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$
$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$
$\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{2} = -2$
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution

i) $x + y = 14$...............(i)

$x - y = 4$....................(ii)

From equation (i)
take x on one side and when we take y to the other side its sign changes and we get,

$x = 14 - y$ .... (iii)

Putting value of x in equation (ii) we get,

$(14 - y) - y = 4$

$14 - 2y = 4$

$2y = 10$

$y = \frac{10}{2} = 5$

Putting value of y in equation (iii) we get,

x = 14 - 5 = 9

Hence, x = 9 and y = 5

ii) s - t = 3......................(i)

and,

$\frac{s}{3} + \frac{t}{2} = 6$ ....(ii)

From equation (i) we get,

taking t to the other side, the sign of t changes to positive

s = t + 3..............(iii)

Putting value of x from (iii) to (ii)

⇒ $\frac{t+3}{3} + \frac{t}{2} = 6$

⇒ 2t + 6 + 3t = 36

⇒ 5t = 30

⇒ t = $\frac{30}{5} = 6$

Putting value of t in equation (iii) , we get,

s = 6 + 3 = 9

Hence, s = 9, t = 6

iii) 3x - y = 3 .... (i)

$9x - 3y = 9$ .... (ii)

Comparing with general pair of equations i.e. $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3$, $b_1 = -1$, $c_1 = -3$

$a_2 = 9$, $b_2 = -2$ and $c_2 = -9$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = 3$

and In this case, the system of linear equation is consistent and has infinite solutions.

iv) $0.2\ x + 0.3\ y = 1.3$ .... (i)

$0.4\ x + 0.5\ y = 2.3$ .... (ii)

From equation (i) we get,

$$0.2\ x = 1.3 - 0.3\ y$$
$$x = \frac{1.3 - 0.3y}{0.2}$$
$$x = \frac{1.3}{0.2} - \frac{0.3}{0.2}y$$
$$x = 6.5 - 1.5\ y$$

Putting value of x in equation (ii) we get,

$$(6.5 - 1.5\ y) \times 0.4 + 0.5\ y = 2.3$$

$$6.5 \times 0.4 - 1.5\ y \times 0.4 + 0.5\ y = 2.3$$
$$2.6 - 0.6\ y + 0.5\ y = 2.3$$

$$-0.1\ y = -0.3$$
$$y = \frac{-0.3}{-0.1} = 3$$

Putting value of y in equation (iii) we get,

x = 6.5 - 1.5 × 3 = 6.5 - 4.5 = 2

Hence, x = 2 and y = 3

v) $\sqrt{2}x + \sqrt{3}y = 0$ …. (i)

$\sqrt{3}x - \sqrt{8}y = 0$ …(ii)

From equation (i), we get,

$\sqrt{2}x = -\sqrt{3}y$

$x = \frac{-\sqrt{3}y}{\sqrt{2}}$ …. (iii)

Putting value of x in equation (ii). we get,

$\Rightarrow \quad \sqrt{3}\left(\frac{-\sqrt{3}}{\sqrt{2}}\right) - \sqrt{8}y = 0$

$\Rightarrow \quad -\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$ $\qquad \sqrt{8} = 2\sqrt{2}$

$\Rightarrow \quad y\left(\frac{-3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

so, y = 0

Putting value of y in equation (iii) we get.

x = 0

Hence, x = 0 and y = 0

vi) $\frac{3x}{2} - \frac{5y}{3} = -2$ ....(i)

and

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ .... (ii)

From equation (i) we get,

By taking L.C.M and solving we get,

$\frac{3\times 3x - 2\times 5y}{6} = -2$

9 x - 10 y = -12

$x = \frac{-12+10y}{9}$

Putting this value of x in equation (ii), we get,

$\Rightarrow \frac{\frac{-12+10y}{9}}{3} + \frac{y}{2} = \frac{13}{6}$

$\Rightarrow \frac{-12+10y}{27} + \frac{y}{2} = \frac{13}{6}$

⇒ 47y = 117 + 24

⇒ 47y = 141

⇒ y = $\frac{141}{47} = 3$

Putting value of y in (iii)

⇒ x = $\frac{-12+10y}{9} = \frac{-12+10\times 3}{9} = \frac{18}{9} = 2$

Hence, x = 2 and y = 3

Q. 2 Solve 2x + 3y = 11 and 2x - 4y = -24and hence find the value of 'm' for which y = mx + x.

Answer:

To Solve: 2x + 3y = 11 .... (i)

2x - 4y = - 24 .... (ii)

Solving by substitution method,

From equation (i)

2x = 11 - 3y .... (iii)

putting value of 2x from equation (iii) to equation (ii)

(11 - 3y) - 4y = - 24

⇒ 11 - 7y = - 24

⇒ - 7y = -35

$y = \frac{-35}{-7}$

$y = 5$

Putting value of y in equation (iii) we get,

2x = 11 - 3 × 5 = 11 - 15 = - 4

x = $-\frac{4}{2} = -2$

Now, putting values of x and y in equation

y = mx + 3

5 = -2 m + 3

= -2m = 2

$m = -\frac{2}{2} = -1$

Value of m is -1

Q. 3 Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Answer:

i) Let larger number = x

Let smaller number = y

According to the question,

$= x - y = 26$

$= x = 26 + y$ …….. (i)

And,

$= x = 3y$ …… (ii)

Comparing values of x from both equation, we get,

26 + y = 3y

-2y = -26

y = 13

So, x = 3y = 3 × 13 = 39

Hence, the numbers are 13 and 39.

ii) Let the first angle = x

Let second angle = y

According to the question,

= x + y = 180 (sum of supplementary angles is always 180)

x = 180 – y ….(i)

And,

= x - y = 18 ……(ii) Given

Putting value of x from equation (i) to (ii). we get,

= 180 – y – y = 18

= -2y = 18 - 180 = -162

$= y = \frac{-162}{-2} = 81$

so, x = 180 = y = 180 – 81 = 99

Hence the angles are 99^o and 81^o

iii) Let cost of each bat = Rs. X

Let cost of each ball = Rs. Y

According to the question,

= 7x + 6 y = 3800 Given

= 6y = 3800 – 7x

$= y = \frac{3800-7x}{6}$ … (i)

And

= 3x + 5 y = 1750 …. (ii) Given

Putting value of y from Equation (i) to equation (ii)

$= 3x + 5 \times \frac{3800-7x}{6} = 1750$

= 18x + 1900 – 35x = 1750 × 6

$= -17x = 10500 - 19000 = -8500$

$= x = \frac{-8500}{-17} = 500$

Putting value of x in equation (i), we get

$= y = \frac{3800-7\times500}{6} = \frac{300}{6} = 50$

Hence,

Cost of each bat = Rs.500 and Cost of each ball = Rs. 50

iv) Let the fixed charge for taxi = Rs. X

Let variable cost per km = Rs. y

We know that,

Total cost = Fixed charge + Variable Charge

According to the question,

= x + 10y = 105 given

= x = 105 – 10y …. (i)

And, For a journey of 15 km charge paid = Rs.155

so, x + 15y = 155 … (ii)

Putting value of x from equation (i) to equation (ii). we get,

= 105 – 10y + 15y = 155

= 5y = 155 – 105 = 50

$y = \frac{50}{5} = 10$

Putting value of y in equation (i). we get,

= x = 105 – 10 × 10 = 105 – 100 = 5

So, People have to pay for travelling a distance of 25 km

= x + 25y = 5 + 25 × 10 = Rs. 255

v) Let numerator = x

Let denominator = y

Fraction will be = $\frac{x}{y}$

According to the question,

Fraction become $\frac{9}{11}$, if 2 is added in both. Numerator and denominator

$= \frac{x+2}{y+2} = \frac{9}{11}$

= 11x +22 = 9y +18 (by cross multiplication)

= 11x = 9y – 4

$= x = \frac{9y-4}{11}$ ….(i)

And, if 3 is added to both numerator and denominator it become $\frac{5}{6}$

$= \frac{x+3}{y+3} = \frac{5}{6}$

= 6x + 18 = 5y + 15 …… (ii) By cross multiplication

Putting value of x from equation (i) to equation (ii)

$= 6 \times \frac{9y-4}{11} + 18 = 5y + 15$

$= 54y - 24 = 55y - 33$

$= -y = -9$

$= y = 9$

Putting value of y in equation (i)

$= x = \frac{9y-4}{11} = \frac{9\times9-4}{11} = \frac{77}{11} = 7$

Hence, the fraction is $\frac{7}{9}$.

vi) Let present age of Jacob = X years

Let present age of his son = Y years

Five year hence,

Age of Jacob = X + 5

Age of son = Y + 5

And, age of Jacob is 3 times of his son Given

= x + 5 = 3 (y +5)

= x + 5 = 3y + 15

= x = 3y + 10 …. (i)

Five years ago,

Age of Jacob = X - 5

Age of son = Y - 5

And, Jacob's age was 7 times of his son Given

= x – 5 = 7 (y- 5)

= x – 5 = 7y – 35

= x – 7y = - 30 …… (ii)

Putting value of X from equation (i) to equation (ii)

= 3y + 10 – 7y = - 30

= - 4y = -40

$= y = \frac{-40}{-4} = 10$

Putting value of Y in (i)

$= x = 3 \times 10 + 10 = 30 + 10 = 40$

Hence, present age of Jacob = 40 years and his son's age = 10 years.

Exercise 3.4

Q. 1 Solve the following pair of linear equations by the elimination method and the substitution method:

(i) x + y = 5 and 2x - 3y = 4

(ii) 3x + 4y = 10 and 2x - 2y = 2

(iii) 3x - 5y - 4 = 0 and 9x = 2y + 7

(iv) $\frac{x}{2} + \frac{2y}{3} = -1 \; and \; x - \frac{y}{3} = 3$

Answer:

i) By elimination method

x + y = 5 . ... (i)

2x - 3y = 4 ....(ii)

Multiplying equation (i) by 2 we get,

2x + 2y = 10 ....(iii)

Subtracting equation (ii) from equation (iii) we get,

5y = 6

$y = \frac{6}{5}$

Putting value of y in equation (i). we get,

$x = 5 - \frac{6}{5} = \frac{19}{5}$

By substitution method

x + y = 5 .... (i)

2x - 3y = 4 .... (ii)

from equation (i)

x = 5 – y ....(iii)

Putting value of x from equation (iii) to equation (ii) we get,

2(5 - y) - 3y = 4

= 10 - 2y - 3y = 4

= -5y = -6

= y = $\frac{6}{5}$

Putting value of y in equation (iii) we get,

$x = 5 - \frac{6}{5} = \frac{19}{5}$

ii) By elimination method

3x + 4y = 10 .... (i)

2x - 2y = 2 .... (ii)

Multiplying equation (ii) by 2 we get,

4x - 4y = 4 .... (iii)

Adding equations (i) and (iii) we get,

7x = 14

$= x = \frac{14}{7} = 2$

Putting value of x in equation (i) we get,

3 (2) + 4y = 10

4y = 10 - 6 = 4

y = $\frac{4}{4} = 1$

By substitution method

3x + 4y = 10 .... (i)

2x - 2y = 2 .... (ii)

From equation (ii)

2x = 2 + 2y

Dividing both side by 2, we getx = 1 + y

putting value of x in equation (i) we get,

3(1 + y) + 4y = 10

3 + 3y + 4y = 107y = 7y = 1

and

x = 1 + y = 1 + 1 = 2(iii) By elimination method

3x - 5y = 4. ....(i)

9x - 2y = 7..........(ii)

Multiplying equation (i) by 3 we get,

9x - 15y = 12.................(iii)

Subtracting equation (ii) from (iii) we get,

- 13y = 5

$= y = -\frac{5}{13}$

Putting value of y in equation (i) we get,

$3x\ - 5 \times \left(-\frac{5}{3}\right) = 4$

$3x + \frac{25}{13} = 4$

Taking LCM,

$3x = \frac{4}{1} - \frac{25}{13} = \frac{52-25}{13} = \frac{27}{13}$

or $x = \frac{27}{13\times 3} = \frac{9}{13}$

By substitution method

3x - 5y = 4 .... (i)

9x - 2y = 7 .... (ii)

From equation (i)

$x = \frac{4+5y}{3}$

Putting value of x in equation (ii) we get,

$= 9 \times \frac{4+5y}{3} - 2y = 7$

= 12 + 15y – 2y = 7

= 13y = -5

y = $-\frac{5}{13}$

Putting value of y in equation (i) we get

$3x - 5 \times -\frac{5}{13} = 4$

$3x = 4 - \frac{25}{13} = \frac{27}{13}$

$= x = \frac{27}{39} = \frac{9}{13}$

iv) By elimination method

$= \frac{x}{2} + \frac{2y}{3} = -1$ . .... (i)

$= x - \frac{y}{3} = 3$ ….. (ii)

Multiplying equation (i) by 2, we get,

$x + \frac{4y}{3} = -2$ …. (iii)

Subtracting equation (ii) from equation (iii) we get,

$= \frac{5y}{3} = -5$

$= y = -\frac{15}{3} = -3$

Putting value of y in equation (ii) we get,

$x + \frac{4\times-3}{3} = -2$

= x - 4 = - 2

= x = 2

By substitution method

$= \frac{x}{2} + \frac{2y}{3} = -1$ . .... (i)

$= x - \frac{y}{3} = 3$ ... (ii)

From equation (ii) we get,

x = 3 + $\frac{y}{3}$ (iii)

Putting value of x in equation (i) we get,

$= \frac{9+y}{6} + \frac{2y}{3} = -1$

$= \frac{9+y+4y}{6} = -1$

= 5y = - 6 - 9 = - 15

= y = $-\frac{15}{5}$

Putting this value in (iii), we get x = 3 - 1 = 2.

Q. 2 Form the pair of linear equations in the following problems, and find their solutions(if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice a sold as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer: i) Let the fraction is =

According condition (i)

$= \frac{x+1}{y-1} = 1$

$= x + 1 = y - 1$

$= x - y = -2$ .... (i)

According condition (ii)

$= \frac{x}{y+1} = \frac{1}{2}$

= 2x = y + 1

= 2x - y = 1 .... (ii)

Subtracting equation (i) from equation (ii) we get,

x = 3

Putting value of x in equation (i) we get,

3 - y = -2

= - y = - 5 and y = 5

Hence, the required fraction is $\frac{3}{5}$.

ii) Let present age of Nuri = x

Let present age of Sonu = y

According to condition (i)

five years before Nuri's age is 3 times that of Sonu's age

$(x- 5) = 3 (y - 5)$

$x - 3y = -10$ .... (i)

According to condition (ii)

$x + 10 = 2(y + 10)$

$= x - 2y = 10$ ... (ii)

Subtracting equation (i) from equation (ii) we get,

$y = 20$

Putting value of y in equation (i) we get,

$x - 60 = -10$

$x = 50$

Hence,

Present age of Nuri = 50 years

Present age of Sonu = 20 years

iii) Let the unit digit of number = x

Let tens digit = y

so number = 10y + x

Number formed after reversing the digits = 10x + y

According to the question,

$x + y = 9$ ....(i)

$9(10y + x) = 2(10x + y)$

$= 90y + 9x = 20x + 2y$

$= 88y -11x = 0$

$= - x + 8y = 0$..................(ii)

Adding equations (i) and (ii). we get,

$9y = 9$

$= y = \frac{9}{9} = 1$

Putting value of y in equation (i), we get,

$x = 8$

Hence,

The required number is $= 10y + x = 10\times1+8 = 18$

iv) Let the number of Rs, 50 notes = x

Let the number of Rs, 100 notes = y

According to the question,

$x + y = 25$ .... (i)

$50x + 100y = 2000$ .... (ii)

Multiplying equation (i) by 50 we get,

$50x + 50y = 1250$ .... (iii)

Subtracting equation (iii) from equation (ii). we get,

$50\ y = 750$

$= y = \frac{750}{50} = 15$

Putting value of y in equation (i) we get,

$x = 10$

Hence,

Meena has 10 notes of Rs, 50 and 15 notes of Rs, 100.

v) Let the fixed charge for first three days = Rs. x

Let each day charge there after = Rs. y

According to the question,

$x + 4y = 27$ .... (i)

$x + 2y = 21$ .... (ii)

Subtracting equation (ii) from equation (i) we get,

$2y = 6$

$$y = \frac{6}{2} = 3$$

Putting value of y in equation (i). we get,

$x + 12 = 27$

$= x = 27 - 12 = 15$

Hence,

Fixed charge = Rs. 15

Charge per day = Rs. 3

Exercise 3.5

Q. 1 Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(ii) $2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

For two linear equations: $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations have exactly one solution

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations has infinitely many solutions

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations has no solution

(i) Linear equations:

$x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}$,

$\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) Linear Equations:

2x + y = 5

3x + 2y = 8

$\frac{a_1}{a_2} = \frac{2}{3}$,

$\frac{b_1}{b_2} = \frac{1}{2}$,

$\frac{c_1}{c_2} = \frac{-5}{-8}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8-(-10)} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$$\frac{x}{2} = 1$$

$$\frac{y}{1} = 1$$

∴ x = 2, y = 1

(iii) Linear Equations:

3x – 5y = 20

6x – 10y = 40

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$,

$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$,

$\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) Linear Equations:

x – 3y – 7 = 0

3x – 3y – 15 = 0

$\frac{a_1}{a_2} = \frac{1}{3}$,

$\frac{b_1}{b_2} = \frac{-3}{-3} = 1$

$\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)} = \frac{y}{-21-(-15)} = \frac{1}{-3-(-9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\frac{x}{24} = \frac{1}{6} \; and \; \frac{y}{-6} = \frac{1}{6}$$

x = 4 and y = -1

∴ x = 4, y = -1

Q. 2 (A) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

2x + 3y = 7

(a – b)x + (a + b)y = 3a + b – 2

Answer: (i) 2x + 3y - 7 =0

(a - b)x + (a + b)y - (3a + b - 2) = 0

we know, a pair of linear equations (say $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$) have infinite solution,

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore from the given equations,

$\frac{a_1}{a_2} = \frac{2}{a-b}$

$\frac{b_1}{b_2} = \frac{3}{a+b}$

$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$

For infinitely many solutions,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{a-b} = \frac{7}{3a+b-2}$

6a + 2b – 4 = 7a – b

a – 9b = -4.............. (i)

$\frac{2}{a-b} = \frac{3}{a+b}$

2a + 2b = 3a – 3b

a – 5b = 0 ....(ii)

Subtracting (i) from (ii), we obtain

4b = 4

b = 1

Substituting this in equation (ii), we obtain

a - 5 × 1 = 0

a = 5

Hence, a = 5 and b = 1 are the values for which the given equations give infinitely many solutions.

Q. 2 (B) For which value of k will the following pair of linear equations have no solution?

3x + y = 1

(2k – 1)x + (k – 1)y = 2k + 1

Answer: 3x + y - 1 = 0

(2k - 1)x + (k - 1)y - 2k - 1 = 0

$$\frac{a_1}{a_2} = \frac{3}{2k-1}$$

$$\frac{b_1}{b_2} = \frac{1}{k-1}$$

$$\frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(This means that the coefficients of variables bear the same ratio, due to which they will eliminate together leaving no value for a variable)

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$\frac{3}{2k-1} = \frac{1}{k-1}$

⇒ 3k - 3 = 2k – 1

⇒ k = 2

Therefore, for k = 2, the given equation has no solution.

Also, k - 1 ≠ 2k + 12k - k ≠ -1 - 1k ≠ -2Hence, for k = 2 and k ≠ -2 the equation has no solution.

Q. 3 Solve the following pair of linear equations by the substitution and cross-multiplication methods:

8x + 5y = 9

3x + 2y = 4

Answer: 8x + 5y = 9 .... (i)

3x + 2y = 4 .... (ii)

From equation (ii), we get

$x = \frac{4-2y}{3}$ .... (iii)

Substituting this value in equation (i), we obtain

$8\left(\frac{4-2y}{3}\right) + 5y = 9$

32 - 16y + 15y = 27

-y = -5

y = 5............. (iv)

Substituting this value in equation (ii), we obtain

3x + 10 = 4

x = -2

Hence, x = -2, y =5

Again, by cross-multiplication method, we obtain

8 x + 5 y - 9 =0

3 x + 2 y - 4 = 0

$$\frac{x}{-20-(-18)} = \frac{y}{-27-(-32)} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\frac{x}{-2} = 1 \; and \; \frac{y}{5} = 1$$

x = -2 and y = 5

Q. 4 Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer: (i) Let x be the fixed charge of the food and y be the charge for food per day. According to the given information,

x + 20y = 1000 .... (1)

x + 26y = 1180 .... (2)

Subtracting equation (1) from equation (2), we obtain

6y = 180

y = 30

Substituting this value in equation (1), we obtain

X + 20 × 30 = 1000

x = 1000 - 600 = 400

x = 400

Hence, fixed charge = Rs 400 And charge per day = Rs 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$\frac{x-1}{y} = \frac{1}{3} \rightarrow 3x - y = 3$ .... (i)

$\frac{x}{y+8} = \frac{1}{4} \rightarrow 4x - y = 8$ ... (ii)

Subtracting equation (i) from equation (ii), we obtain

x = 5 .... (iii)

Putting this value in equation (i), we obtain

15 – y = 3

y = 12

Hence, the fraction is $\frac{5}{12}$.

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the given information,

Case I

$3x - y = 40$......(i)

Case II

$4x - 2y = 50$

$2x - y = 25$ .... (ii)

Subtracting equation (ii) from equation (i),

we obtain $x = 15$ (iii)

Substituting this in equation (ii), we obtain

$30 - y = 25$

$y = 5$

Therefore,

number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of car from A be 'a' and of car from B be 'b'

Speed = distance/time

Relative speed of cars when moving in same direction = a + b

Relative speed of cars when moving in opposite direction = a – b

Given, places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour

$\Rightarrow$ a – b = 100/5 = 20   (1)

Also, a + b = 100/1 = 100   (2)

Adding (1) and (2)

a - b + a + b = 20 + 100

$\Rightarrow$ 2a = 120

$\Rightarrow$ a = 60 km/hr

Putting value of a in (1) we get,

Thus, b = 60 – 20 = 40 km/hr

Speed of two cars are 60 km/h and 40 km/h

(v) Let length and breadth of rectangle be x unit and y unit respectively.

Area = xy

According to the question,

The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units.

(x - 5) (y + 3) = xy - 9

3x – 5y – 6 = 0   .... (i)

and if we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units

(x + 3) (y + 2) = xy + 67

2x + 3y – 61 = 0   .... (ii)

By cross-multiplication method, we obtain

$$\frac{x}{305-18} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

x = 17, y = 9

Exercise 3.6

Q. 1 Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2, \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2, \frac{4}{\sqrt{x}}+\frac{9}{\sqrt{y}}=-9$

(iii) $\frac{4}{x}+3y=14, \frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2, \frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5, \frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy, 2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4, \frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer: Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations becomes.

$\frac{p}{2}+\frac{q}{3}=2 \Rightarrow 3p+2q-12=0$ … (i)

$\frac{p}{2}+\frac{q}{2}=\frac{13}{6} \Rightarrow 2p+3q-13=0$ … (ii)

Using cross-multiplication method, we obtain,

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5}\ and\ \frac{q}{15}=\frac{1}{5}$$

p = 2 and q = 3

Note: These questions can also be solved by elimination method and the substitution method.

In the elimination method, the coefficient of one variable in both equations is made the same by multiplying the equation and the variable is eliminated. In the substitution method, the value of one variable is calculated in terms of another variable by equation one and then that value is put into another equation.

$\frac{1}{x} = 2 \; and \; \frac{1}{y} = 3$

$x = \frac{1}{2} \; and \; y = \frac{1}{3}$

(ii) Putting $\frac{1}{\sqrt{x}} = p \; and \frac{1}{\sqrt{y}} = q$ in the given equations, we obtain

2p +3q = 2........(i)

4p - 9q = -1 ....(ii)

Multiplying equation (1) by 3,

we obtain 6p + 9q = 6....... (iii)

Adding equation (ii) and (iii), we obtain

10p = 5

$p = \frac{1}{2}$ … (iv)

$2 \times \frac{1}{2} + 3q = 2$

= 3q = 1

$= q = \frac{1}{3}$

$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2 = x = 4$

$p = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3 = y = 9$

Hence, x = 4 and y = 9.

(iii) Putting $\frac{1}{x} = p$ in given equations, we get

= 4p + 3y = 14

= 4p + 3y - 14 = 0....................(i)

And, 3p - 4y = 23

= 3p - 4y -23 = 0 ....(ii)

By cross- multiplication, we get,

$= \frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$

$= \frac{p}{-125} = \frac{y}{50} = -\frac{1}{25}$

Now,

$= \frac{p}{-125} = -\frac{1}{25}$, so, p = 5

$= \frac{y}{50} = -\frac{1}{25}$, So y = -2

(iv) Putting $\frac{1}{x-1}$ = p and $\frac{1}{y-1}$ = 1, we get

= 5p + q = 2 ………….(i)

= 6p – 3q = 1 ………..(ii)

Now, multiplying equation (i) by 3 we get,

= 15p + 3q = 6 ………….(iii)

Adding equations (ii) and (iii)

21p = 7

$= p = \frac{7}{21} = \frac{1}{3}$

Putting value of p in equation (iii) we get,

$= 6\times \frac{1}{3} - 3q = 1$

$= -3q = -1$

$= q = \frac{1}{3}$

we know that,

$p = \frac{1}{x-1} = \frac{1}{3}$

$= 3 = x - 1$

$= x = 4$

and, $q = \frac{1}{y-2} = \frac{1}{3}$

$= 3 = y - 2$

$= y = 5$

Hence $x = 4, y = 5$

v). $\frac{7}{y} - \frac{2}{x} = 5$ ……..(i)

$= \frac{8x+7y}{xy} = 15$

$= \frac{8}{y} + \frac{7}{x} = 15$ …(ii)

Putting $\frac{1}{x} = p \; and \; \frac{1}{y} = q$ in (i) and (ii) to get,

$7q - 2p = 5$ ....(iii)

$8q + 7p = 15$ ....(iv)

multiplying equation (iii) by 7 and equation (iv) by 2 . we get,

$49q - 14p = 35$.................(v)

16q + 14p = 30..................(vi)

After adding equations (v) and (vi) . we get,

65q = 65

= q = 1

Putting value of q in equation (iv) , we get,

8 + 7p = 15

= 7p = 15 - 8 = 7

= p = 1

Now,

p = $\frac{1}{x} = \frac{1}{1}$ = 1

q = $\frac{1}{y} = \frac{1}{1}$ = 1

Hence , x = 1 and y = 1

(vi) 6x + 3y = 6xy

dividing equation by xy $\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy} = \frac{6}{y} + \frac{3}{x} = 6$ …….(i)

2x + 4y = 5xy

dividing equation by xy, $\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy} = \frac{2}{y} + \frac{4}{x} = 5$ …….(i)

Putting $\frac{1}{x}$ = p and $\frac{1}{y}$ = q, we get,

6q + 3p - 6 = 0

2q + 4p - 5 = 0

By cross multiplication method , we get,

$= \frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$

$= \frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$

After comparing we get,

p = 1 and q = $\frac{1}{2}$

Now ,

p = $\frac{1}{x}$ =1 and q = $\frac{1}{y} = \frac{1}{2}$

Hence, x = 1 and y = 2

(vii) Putting $\frac{1}{x+y}$ = p and $\frac{1}{x-y}$ = q, we get

10p + 2q - 4 = 0....................(i)

15p - 5q +2 = 0 ....,,(ii)

By applying cross multiplication method , we get,

$= \frac{p}{4-20} = \frac{q}{-60-(-20)} = \frac{1}{-50-30}$

$= \frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$

After comparing we get,

p = $\frac{1}{5}$ and q =1

Now,

p = $\frac{1}{x+y} = \frac{1}{5}$ So, x + y = 5 ……….(iii)

q = $\frac{1}{x-y}$ = 1 = So, x - y = 1 ……….(iv)

Adding equations (iii) and (iv) we get,

2x = 6

= x = $\frac{6}{2}$ = 3

Putting value of equation (iii) we get,

y = 2

Hence, x = 3 and y = 2

(viii) Putting $\frac{1}{3x+y}$ = p and $\frac{1}{3x-y}$ = q we get,

p + q = $\frac{3}{4}$ ………..(i)

$\frac{p}{2}$ - $\frac{q}{2}$ = -$\frac{1}{8}$

p – q = - $\frac{1}{4}$ …..(ii)

Addomh (i) and (ii) we get,

2p = $\frac{3}{4}$ - $\frac{1}{4}$ = $\frac{2}{4}$ = $\frac{1}{2}$

= p = $\frac{1}{4}$

Putting value of p in (ii) we get,

$= \frac{1}{4} - q = - \frac{1}{4}$

$= q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Now, $p = \frac{1}{3x+y}$ so, $3x + y = 4$ ……..(iii)

$q = \frac{1}{3x-y} =$ $3x - y = 2$ ……..(iv)

Adding equations (iii) and (iv) we get,

$6x = 6$

$= x = 1$

Putting value of x in equation (iii) we get,

$3(1) + y = 4$

$= y = 1$

Hence, $x = 1$ and $y = 1$

Q. 2 Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer: **(i)** Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

While rowing upstream, Ritu's speed slows down and the speed will be her speed minus speed of stream and while rowing downstream her speed will increase and will be equal to sum of her speed and speed of stream. Therefore,

The speed of Ritu while rowing

Upstream = $(x - y)$ km/h

Downstream = $(x + y)$ km/h

According to the question:

Ritu can row downstream 20 km in 2 hours, and

distance = speed × time

$\Rightarrow 2(x+y) = 20$

$\Rightarrow x+y = 10$............(1)

also, Ritu can row upstream 4 km in 2 hours

$\Rightarrow 2(x - y) = 4$

⇒ x-y = 2 ....(2)

Adding equation (1) and (2),

we obtain

⇒ x + y + x - y = 10 + 2

⇒ 2 x = 12

⇒ x = 6

Putting this in equation (1),

6 + y = 10

we obtain y = 4

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii)Let the number of days taken by a woman and a man be x and y respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

and work done by a man in 1 day = $\frac{1}{y}$

According to the question,

2 women and 5 men take 4 days to complete the work

i.e. they take 4 days to complete one work

$$\Rightarrow 4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\Rightarrow \frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

Also, 3 women and 6 men take 3 days to complete the work i.e. they take 3 days to complete one work

$$\Rightarrow 3\left(\frac{3}{x}+\frac{6}{y}\right)=1$$

$\Rightarrow \frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

Putting $\frac{1}{x} = p \; and \; \frac{1}{y} = q$ in these equations,

We obtain

$2p + 5q = \frac{1}{4}$

$\Rightarrow$ 8p + 20q = 1

And

$3p = 6q = \frac{1}{3}$

$\Rightarrow$9p + 18q = 1

By cross multiplication, we obtain

$$\frac{p}{-20-(-18)} = \frac{q}{-9-(-8)} = \frac{1}{144-180}$$

$$\frac{p}{-2} = \frac{q}{-1} = \frac{1}{-36}$$

$$\frac{p}{-2} = \frac{-1}{36} \; and \; \frac{q}{-1} = \frac{1}{-36}$$

$$p = \frac{1}{18} \; and \; q = \frac{1}{36}$$

$$p = \frac{1}{x} = \frac{1}{18} \; and \; q = \frac{1}{y} = \frac{1}{36}$$

x = 18, y = 36

Hence, number of days taken by a woman, x = 18

Number of days taken by a man, y = 36

(iii) Let the speed of train and bus be u km/h and v km/h respectively.

According to the given information,

It takes her 4 hours if she travels 60 km by bus and rest (i.e. 240 km) by train

As $time = \frac{distance}{speed}$

We have

$\frac{60}{u} + \frac{240}{v} = 4$ … (1)

and also, If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer i.e. 4 hours and 10 minutes. Also, 1 hour = 60 minutes.

$\Rightarrow 10\ minutes = \frac{1}{60} \times 10 = \frac{1}{6}\ hour$

And $\Rightarrow 4\ hours\ and\ 10\ minutes = \left(4 + \frac{1}{6}\right) = \frac{25}{6}$ hours

We have

$\frac{100}{u} + \frac{200}{v} = \frac{25}{6}$ ….. (2)

Putting $\frac{1}{u} = p\ and\ \frac{1}{v} = q$ is these equations, we obtain

60p+240q = 4 ......(3)

100p+200q =

600p+1200q = 25 ....(4)

Multiplying equation (3) by 10, we obtain

600p+2400q = 40 ....(5)

Subtracting equation (4) from (5), we obtain

1200q = 15

$q = \frac{15}{1200} = \frac{1}{80}$ … (6)

Substituting in equation (3), we obtain

60p+3 = 4

60p = 1

$p = \frac{1}{60}$

$p = \frac{1}{u} = \frac{1}{60}\ and\ q = \frac{1}{v} = \frac{1}{80}$

u = 60km/h and v = 80km/h

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h.

Exercise 3.7

Q. 1 The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer: The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = 2 × x = 2x years

And age of Biju's sister Cathy $=\frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years, x - y = 3 (i)

4x - y = 60 (ii)

Subtracting (i) from (ii), we obtain 3x = 60 - 3 = 57

x = 57/3 = 19

Therefore, age of Ani = 19 years

And age of Biju = 19 - 3 = 16 years

Case (II) When Biju is older than Ani, y - x = 3 (i)

$$2x - \frac{y}{2} = 30$$

4x - y = 60 (ii)

Adding (i) and (ii), we obtain 3x = 63

x = 21

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years

Q. 2 One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[Hint: $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer: Let those friends were having Rs x and y with them. Using the information given in the question,

we obtain

From the first condition, $x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$ (i)

And, From the second condition$6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$6x - y = 70$ (ii)

Multiplying equation (ii) by 2, we obtain

$12x - 2y = 140$ (iii)

Subtracting equation (i) from equation (iii),

we obtain

$11x = 140 + 300$

$11x = 440$

$x = 40$

Using this in equation (i), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

2y = 340

y = 170

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Q. 3 A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer: Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km.

We know that,

$$Speed = \frac{distance\ travelled}{time\ taken\ to\ travel\ that\ distance}$$

$$x = \frac{d}{t}$$

Or, d = xt (i)

If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time

⇒ $(x + 10) = \frac{d}{t-2}$

⇒ (x + 10)(t - 2) = d

⇒ xt + 10t - 2x - 20 = d

From (i), we have

⇒ d + 10t - 2x - 20 = d

⇒ - 2x + 10t = 20

⇒ x - 5t = -10 ⇒ x = 5t - 10 (ii)

Also,

if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time⇒ $(x - 10) = \frac{d}{t-3}$

⇒ (x - 10)(t + 3) = d

⇒ xt - 10t + 3x - 30 = d

By using equation (i),

⇒ d - 10t + 3x - 30 = d⇒ 3x - 10t = 30 (iii)

Substituting the value of x from eq (ii) into eq (iii), we get

⇒ 3(5t - 10) - 10t = 30

⇒ 15t - 30 - 10t = 30

⇒ 5t = 60⇒ t = 12 hours

Putting this in eq(ii)⇒ x = 5t - 10

= 5(12) - 10

= 50 km/h

From equation (i), we obtain

Distance to travel = d = xt

= 50 × 12

= 600 km

Hence, the distance covered by the train is 600 km.

Q. 4 The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer: Let the number of rows be x and number of students in a row be y.

Total students of the class = Number of rows x Number of students in a row = xy

Using the information given in the question,

Condition 1

Total number of students = (x - 1) (y + 3)

= (x - 1) (y + 3)

= xy - y + 3x - 3

3x - y - 3 = 0

3x - y = 3 ....(i)

Condition 2

Total number of students = (x + 2) (y - 3)

= xy + 2y - 3x - 6

⇒3x - 2y = -6 ... (ii)

Subtracting equation (ii) from (i),

(3x - y) - (3x - 2y) = 3 - (-6)

⇒- y + 2y = 9

⇒3 + 6 y = 9

By using equation (i), we obtain 3x - 9 = 3,

⇒3x = 9 + 3

⇒3x = 12

⇒ x = 4

From (i),

⇒ 3(4) - y = 3

⇒ 12 - y = 3⇒ 9 = y

Number of rows = $x = 4$

Number of students in a row = $y = 9$

Number of total students in a class = Number of students in 1 row × Number of rows

$= xy$

$= 4 \times 9 = 36.$

Q. 5 In a Δ ABC, <C=3<B=2 (<A+<B) Find the three angles.

Answer: Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

Let's take $3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ … (i)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180°$ … (ii)

Multiplying equation (i) by 4, we obtain

$8\angle A - 4\angle B = 0$ … (iii)

Adding equations (ii) and (iii), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (ii), we obtain

$20° + 4\angle B = 180°$

$4 \angle B = 160°$

$\angle B = 40°$

and

$\angle C = 3 \angle B$

$= 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20°, 40°, and 120° respectively.

Q. 6 Draw the graphs of the equations 5x - y = 5 and 3x - y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y axis.

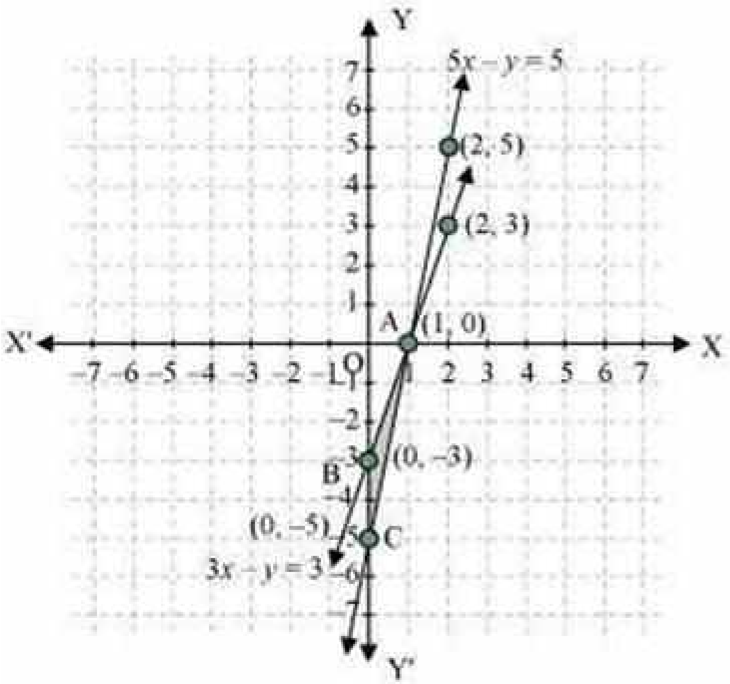
Answer: 5x - y = 5 Or,

y = 5x – 5

The solution table will be as follows.

x	0	1	2
y	-5	0	5
3x-y =3 or, y =3x - 3			
The solution table will be as follows.			
x	0	1	2
y	-3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and y-axis. The coordinates of vertices are A (1, 0), B (0, - 3), C (0, - 5).

Q. 7 Solve the following pair of linear equations.

(i) px + qy = p - q

qx - py = p + q

(ii) ax + by = c

bx + ay = 1 + c

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

ax + by = $a^2 + b^2$

(iv) (a - b) x + (a + b) y = a^2 - 2ab - b^2

(a + b) (x + y) = $a^2 + b^2$

(v) 152 x - 378 y = - 74

-378 x + 152 y = - 604

Answer:

(i) $px + qy = p - q$ … (1)

$qx - py = p + q$ … (2)

Multiplying equation (1) by p and equation (2) by q,

we obtain $p^2 x + pq\, y = p^2 - pq$ … (3)

$q^2 x - pq\, y = pq + q^2$ … (4)

Adding equations (3) and (4),

we obtain $p^2 x + q^2 x = p^2 + q^2$

$(p^2 + q^2)\, x = p^2 + q^2$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation putting the value of x (1),

we obtain $p(1) + qy = p - q$

$qy = -q$

$y = -1$

(ii) $ax + by = c$ … (1)

$bx + ay = 1 + c$ … (2)

Multiplying equation (1) by a and equation (2) by b,

we obtain $a^2 x + ab\, y = ac$ … (3)

$b^2 x + ab\, y = b + bc$ … (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2)\, x = ac - bc - b$

$$x = \frac{c(a-b) - b}{a^2 - b^2}$$

From equation (1), we obtain ax + by = c, now putting the value of x in the equation

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\} + by = c$$

$$\frac{ac\,(a-b)-ab}{a^2-b^2} + by = c$$

$$by = c - \frac{ac\,(a-b)-ab}{a^2-b^2}$$

$$by = \frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by = \frac{abc-b^2c+ab}{a^2-b^2}$$

$$by = \frac{bc(a-b)+ab}{a^2-b^2}$$

$$y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

or, bx - ay = 0 … (1)

ax + by = $a^2 + b^2$ … (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain b2 x - ab y = 0 … (3)

a^2 x + ab y = $a^3 + ab^2$ … (4)

Adding equations (3) and (4), we obtain $b^2x + a^2x = a^3 + ab^2$

x ($b^2 + a^2$) = a ($a^2 + b^2$) x

Thus, x = a

By using (1), and putting the value of x in the equation we obtain b (a) - ay = 0

ab - ay = 0

$ay = ab$

$y = b$

(iv) $(a - b) x + (a + b) y = a^2 - 2ab - b^2$ … (1)

$(a + b) (x + y) = a^2 + b^2$

$(a + b) x + (a + b) y = a^2 + b^2$… (2)

Subtracting equation (2) from (1),

we obtain

$(a - b) x - (a + b) x = (a^2 - 2ab - b^2) - (a^2 + b^2)$ $(a - b - a - b) x = - 2ab - 2b^2$

$- 2bx = - 2b (a + b)$

$x = a + b$

Using equation (1), and putting the value of x in the equation we obtain

$(a - b) (a + b) + (a + b) y = a^2 - 2ab - b^2 a^2 - b^2 + (a + b) y = a^2 - 2ab - b^2$

$(a + b) y = - 2ab$

$$y = \frac{-2ab}{a+b}$$

(v) $152 x - 378 y = - 74$ ------------ (1)

$-378 x + 152 y = - 604$ -------- (2)

Multiply eq (2) by 152 and equation (1) by 378

$378 \times 152x – 3782y = -74 \times 378$

$-378 \times 152x + 1522y = -604 \times 152$

Adding both the questions we get

$(152^2 – 378^2)y = -119780$

$-119780 y = -119780$

$y = 1$

put the value in eq 1,

152 x - 378 x 1 = - 74 152 x = 378 - 74152 x = 304x = 2we get x = 2.

Q. 8 ABCD is a cyclic quadrilateral (see Fig. 3.7). Find the angles of the cyclic quadrilateral.

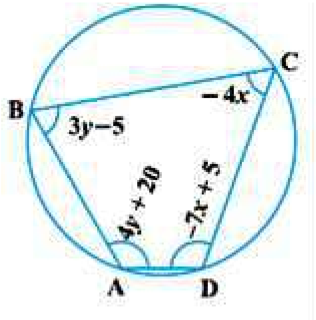


Fig. 3.7

Answer: We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°. Therefore, ∠ A + ∠ C = 180

⇒4y + 20 - 4x = 180

⇒- 4x + 4y = 160

⇒x - y = - 40 ....(i)

Also, ∠ B + ∠ D = 180

⇒3y - 5 - 7x + 5 = 180

⇒- 7x+ 3y = 180 .... (ii)

Multiplying equation (i) by 3, we obtain 3x - 3y = - 120.... (iii)

Adding equations (ii) and (iii), we obtain

- 7x + 3x = 180 - 120

- 4x = 60

x = -15

By using equation (i), we obtain x - y = - 40

-15 - y = - 40

y = -15 + 40

= 25

$\angle A = 4y + 20$

$= 4(25) + 20$ $\angle A = 120°$

$\angle B = 3y - 5$

$= 3(25) - 5$ $\angle B = 70°$

$\angle C = -4x$

$= -4(-15)$

$\angle C = 60°$

$\angle D = -7x + 5$ $= -7(-15) + 5$

$= 105 + 5$

$\angle D = 110°$.