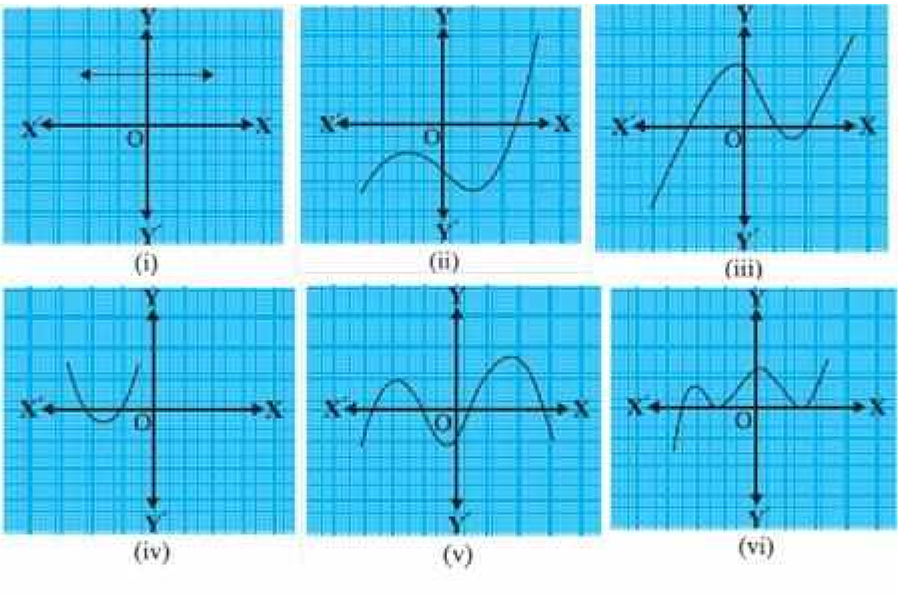
Chapter - 2
Polynomial

Exercise – 2. 1

Q. 1 The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials, p(x). Find the number of zeroes of p(x) in each case.



The number of zeroes for any graph is the number of values of x for which y is equal to zero. And y is equal to zero at the point where a graph cuts x axis.

So, to find the number of zeroes of a polynomial, watch the number of times it cuts the x axis.

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi)The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Exercise – 2. 2

Q. 1 Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) 6x2 - 3 - 7x

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x – 4$

Solution:

Zeroes of the polynomial are the values of the variable of the polynomial when the polynomial is put equal to zero.

Let p(x) be a polynomial with any number of terms any number of degree. Now, zeroes of the polynomial will be the values of x at which p(x) = 0. If $p(x) = ax^2 + bx + c$ is a quadratic polynomial (highest power is equal to 2) and its roots are α and β, then

Sum of the roots = $\alpha + \beta = - b/a$

Product of roots = $\alpha\beta = c/a$

(i) $p(x) = x^2 - 2x - 8$

So, the zeroes will be the values of x at which p(x) = 0.

Therefore,

$\Rightarrow x^2 - 4x + 2x - 8 = 0$

(We will factorize 2 such that the product of the factors is equal to 8 and difference is equal to 2)

$\Rightarrow x(x - 4) + 2(x - 4) = 0= (x - 4)(x + 2)$

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e, x = 4 or x = −2

Therefore, The zeroes of $x^2 - 2x - 8$ are 4 and −2.

Sum of zeroes = 4 + (-2) = 2

$$= \frac{-(-2)}{1} = \frac{-(-coefficient of\ x)}{(coefficient of\ x^2)}$$

Hence, it is verified that, sum of Zeros $= \frac{-(coefficient of\ x)}{coefficient of\ x^2}$

Product of zeroes = $4 + (-2) = -8 = \frac{(-8)}{1} = \frac{constant\ term}{coefficient of\ x^2}$

Hence, it is verified, Product of zeroes $= \frac{constant\ term}{coefficient of\ x^2}$

(ii) $4s^2 - 4s + 1$

$= (2s)^2 - 2(2s)1 + 1^2$

As, we know $(a - b)^2 = a^2 - 2ab + b^2$, the above equation can be written as $= (2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, when, s = 1/2 , 1/2.

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(coefficient of\ s)}{coefficient of\ s^2}$

Product of zeroes =

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{coefficient of\ s^2}$$

Hence Verified.

(iii) $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

(We will factorize 7 such that the product of the factors is equal to 18 and the difference is equal to - 7)= $6x^2 + 2x - 9x - 3$ $= 2x(3x + 1) - 3(3x + 1)$= $(3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$,

i.e. $x = \frac{-1}{3}\ or\ \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}\ or\ \frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(coefficient of\ x)}{coefficient of\ x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{constant\ term}{coefficient of\ x^2}$

Hence, verified.

(iv) $4u^2 + 8u$

$= 4u^2 + 8u + 0$

$= 4u\ (u + 2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$,

i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(coefficient of\ u)}{coefficient of\ u^2}$

Product of zeroes = $0 + (-2) = 0 = \frac{0}{4} = \frac{-constant\ term}{coefficient of\ u^2}$

(v) $t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$ [As, $x^2 - y^2 = (x - y)(x + y)$]

The value of $t^2 - 15$ is zero when $(t - \sqrt{15}) = 0$ or $(t + \sqrt{15}) = 0$,

i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2}$

$$\sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2}$$

Product of zeroes =

$$(\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{-\text{constant term}}{\text{coefficient of } x^2}$$

Hence verified.

(vi) $3x^2 - x - 4$

(We will factorize 1 in such a way that the product of factors is equal to 12 and the difference is equal to 1) $= 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$,

when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1

Sum of zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-coefficient of\ x}{coefficient of\ x^2}$

Product of zeroes = $\frac{4}{3}(-1) = \frac{-4}{3} = \frac{constant\ term}{coefficient of\ x^2}$

Hence, verified.

Q. 2 Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$
(ii) $\sqrt{2}, \frac{1}{3}$
(iii) $0, \sqrt{5}$
(iv) $1, 1$
(v) $-\frac{1}{4}, \frac{1}{4}$
(vi) $4, 1$

Solution: If α, β are roots of an equation, then the quadratic form of this equation can be given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

(i) $\frac{1}{4}$ -1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zeros are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be, $ax^2 + bx + c$, then

$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$

$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

Let a = 4, then b = -1, c = -4

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be, $ax^2 + bx + c$, then

$$\alpha + \beta = \sqrt{2} = \frac{-b}{a}$$

and

If a = 3, then b = $-3\sqrt{2}$, and c = 1

Therefore, the quadratic polynomial is $3x2 - 3\sqrt{2}x + 1$

(iii) 0, $\sqrt{5}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial b $ax^2 + bx + c$, then

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If a = 1, then b = 0, c = $\sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$, then

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial is $x^2 - x + 1$

(v) $-\frac{1}{4}, \frac{1}{4}$

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$, then

$$\alpha + \beta = 1 = \frac{-1}{4} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{4} = \frac{c}{a}$$

If a = 4, then b = 1, c = 1

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

we know that for a quadratic equation in the form $ax^2 + bx + c = 0$, and its zerors are α and β, then

sum of zeroes is $\alpha + \beta = \frac{-b}{a}$

and product of zeroes is $\alpha\beta = \frac{c}{a}$

Let the polynomial be $ax^2 + bx + c$, then

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then b = -4, c = 1

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

Exercise 2.3

Divide the polynomial by the polynomial and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

(i)By long division method we have,

$$
\begin{array}{r|l}
 & x-3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\qquad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \quad + 6 \\
\hline
 & 7x - 9
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) By long division method we have,

$$x^2 + x - 3$$

$$x^2 - x + 1 \sqrt{x^4 + ox^3 - 3x^2 + 4x + 5}$$

$$x^4 - \quad x^3 \quad + x^2$$

$$- \qquad + \qquad -$$

$$x^3 - 4x^2 + 4x + 5$$

$$x^3 - \quad x^2 + x$$

$$- \qquad + \qquad -$$

$$-3x^2 + 3x + 5$$

$$-3x^2 + 3x \; -3$$

$$+ \qquad - \qquad +$$

$$8$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) By long division method we have,

$$-x^2 - 2$$

$$-x^2 + 2\sqrt{x^4 + 0x^2 - 5x + 6}$$

$$x^4 - 2x^2$$

$$- \qquad +$$

$$2x^2 - 5x + 6$$

$$2x^2 \qquad - 4$$

$$- \qquad +$$

$$-5x \; +10$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q. 2 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^3 - 9t - 12$

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

(i) $t^2 - 3 = t^2 + 0t - 3$

$$2t^2 + 3t + 4$$

$$t^2 + 0t - 3 \,\overline{\big)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12}$$

$$2t^4 + 0t^3 - 6t^2$$

$$- \quad - \quad +$$

$$3t^3 + 4t^2 - 9t - 12$$

$$3t^3 + 0t^2 - 9t$$

$$- \quad - \quad +$$

$$4t^2 + 0t - 12$$

$$4t^2 + 0t - 12$$

$$- \quad - \quad +$$

$$0$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii)

$$3x^2 - 4x + 2$$

$$x^2+3x+1 \;\; \sqrt{3x^4 + 5x^3 - 7x^2 + 2x + 2}$$

$$3x^4 + 9x^3 + 3x^2$$

$$- \quad - \quad -$$

$$-4x^3 - 10x^2 + 2x + 2$$

$$-4x^3 - 12x^2 - 4x$$

$$+ \quad + \quad +$$

$$2x^2 + 6x + 2$$

$$2x^2 + 6x + 2$$

$$0$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii)

$$x^2 - 1$$

$$x^2 - 3x + 1 \;\; \sqrt{x^5 - 4x^3 + x^2 + 3x + 1}$$

$$x^5 - 3x^2 + x^2$$

$$- \quad + \quad -$$

$$-x^3 \qquad + 3x + 1$$

$$-x^3 \qquad + 3x - 1$$

$$+ \qquad - \quad +$$

$$2$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Q. 3 Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
x^2 + 0x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 + 0x^3 - 5x^2 \\
 & - \quad - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 + 0x^2 - 10x \\
 & - \quad - \quad + \\
\hline
 & 3x^2 + 0x - 5 \\
 & 3x^2 + 0x - 5 \\
 & - \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

We know, Dividend = (Divisor × quotient) + remainder

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

As $(a+b)^2 = a^2 + b^2 + 2ab$

So, $x^2 + 2x + 1 = (x+1)^2$

$3x^4 + 6x^3 - 2x^2 - 10x - 5 = 3\left(x^2 - \frac{5}{3}\right)(x+1)^2$Therefore, its zero is given by $x + 1 = 0$.

$\Rightarrow x = -1, -1$

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. and - 1 , - 1.

Q. 4 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x) the quotient and remainder were (x - 2) and (-2x + 4), respectively. Find g(x).

Solution: Given,

Polynomial, $p(x) = x^3 - 3x^2 + x + 2$ (dividend)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

To find: divisor = g(x)we know, Dividend = Divisor × Quotient + Remainder

$\Rightarrow x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$

$\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$

g(x) is the quotient when we divide (x^3 - $3x^2$ + 3x - 2) by (x - 2)

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 \quad + 3x - 2 \\
 & -x^2 \quad + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
\end{array}
$$

∴ g(x) = ($x^2 - x + 1$)

Q. 5 Give examples of polynomials and which satisfy the division algorithm and

(i) deg p (x) = deg q (x)

(ii) deg q (x) = deg r (x)

(iii) deg r (x) = 0

Degree of a polynomial is the highest power of the variable in the polynomial. For example if f(x) = x^3 - $2x^2$ + 1, then the degree of this polynomial will be 3.

(i) By division Algorithm : p(x) = g(x) x q(x) + r(x)

It means when P(x) is divided by g(x) then quotient is q(x) and remainder is r(x)We need to start with p(x) = q(x)This means that the degree of polynomial p(x) and quotient q(x) is same. This can only happen if the degree of g(x) = 0 i.e p(x) is divided by a constant. Let p(x) = x^2 + 1 and g(x) = 2

$$\frac{p(x)}{g(x)} = \frac{x^2+1}{2}$$

The,

$p(x) = g(x) \times \left(\frac{x^2+1}{2}\right)$

Clearly, Degree of p(x) = Degree of q(x)

2. Checking for division algorithm,

p(x) = g(x) × q(x) + r(x)

$= 6x^2 + 2x + 2 = 3\ (3x^2 + x\ + 1)$

$= 6x^2 + 2x + 2$

Thus, the division algorithm is satisfied.

(ii) Let us assume the division of $x^3 + x$ by x^2,

Here,

$p(x) = x^3 + x$

$g(x) = x^2$

q(x) = x and r(x) = x

Clearly, the degree of q(x) and r(x) is the same i.e.,

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)\ x^3 + x$

$= (x^2) \times x + x\ x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii) Degree of the remainder will be 0 when the remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here,

$p(x) = x^3 + 1$ $g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x) x^3 + 1$

$= (x^2) \times x + 1$ $x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Q. 1 Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Answer:

(i) $P(x) = 2x^3 + x^2 - 5x + 2$

Now for zeroes, putting the given values in x.

$P(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$

$= (1/4) + (1/4) - (5/2) + 2 = (1 + 1 - 10 + 8)/2 = 0/2 = 0$

$P(1) = 2 \times 1 + 1 - 5 \times 1 + 2 = 2 + 1 - 5 + 2 = 0$

$P(-2) = 2\times (-2)^3 + (-2)^2 - 5(-2) + 2 = (2\times -8) + 4 + 10 + 2 = -16 + 16 = 0$

Thus, 1/2, 1 and -2 are zeroes of given polynomial.

Comparing given polynomial with $ax^3 + bx^2 + cx + d$ and Taking zeroes as α, β, and γ, we have

$a = 2$, $b = 1$, $c = -5$, $d = 2$ and $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

Now, We know the relation between zeroes and the coefficient of a standard cubic polynomial as

$\alpha + \beta + \gamma = -\frac{b}{a}$

Substituting value, we have

$\frac{1}{2} + 1 - 2 = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

Since, LHS = RHS (Relation Verified)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\left(\frac{1}{2} \times 1\right) + (1 \times -2) + \left(-2 \times \frac{1}{2}\right) = -\frac{5}{2}$

$\frac{1}{2} - 2 - 1 = -\frac{5}{2}$

$-\frac{5}{2} = -\frac{5}{2}$

Since LHS = RHS, Relation verified.

$\alpha\beta\gamma = -\frac{d}{a}$

$\left(\frac{1}{2} \times 1 \times -2\right) = -\frac{2}{2}$

$-\frac{2}{2} = -\frac{2}{2}$

Since LHS = RHS, Relation verified.

Thus, all three relationships between zeroes and the coefficient is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Now for zeroes , put the given value in x.

$P(2) = 2^3 - 4(2)^2 + 5 \times 2 - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$

$P(1) = 1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

$P(1) = 1^3 - 4(1)^2 + 5 \times 1 - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

Thus, 2, 1, 1 are the zeroes of the given polynomial.

Now,

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 1$, $b =- 4$. $c = 5$, $d = -2$ and $\alpha = 2, \beta = 1, \gamma = 1$

Now,

$$2 + 1 + 1 = \frac{4}{1}$$

$4 = 4$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$(2 \times 1) + (1 \times 1) + (1 \times 2) = \frac{5}{1}$$

$2 + 1 + 2 = 5$

$5 = 5$

$$\alpha\beta\gamma = -\frac{d}{a}$$

$2 \times 1 \times 1 = 2$

$2 = 2$

Thus, all three relationships between zeroes and the coefficient is verified.

Q. 2 Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Answer:

For a cubic polynomial equation, ax3 + bx^2 + cx + d, and zeroes α, β and γ

we know that

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Let the polynomial be $ax^3 + bx^2 + cx + d$, and zeroes α, β and γ.

A cubic polynomial with respect to its zeroes is given by,x^3 - (sum of zeroes) x^2 + (Sum of the product of roots taken two at a time) x - Product of Roots = 0

$x^3 - (2)\, x^2 + (-\ 7)\, x - (-\ 14) = 0$

$x^3 - (2)\, x^2 + (-\ 7)\, x + 14 = 0$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q. 3 If the zeroes of the polynomial $x^3 – 3x^2 + x + 1$ are (a - b), a and (a + b). Find a and b.

Answer:

Given

$P(x) = x^3 - 3x^2 + x + 1$

Zeroes are = a – b , a+ b, a

Comparing the given polynomial with $mx^3 + nx^2 + px + q$, we get,

= m = 1, n = - 3, p = 1, q = 1

Sum of zeroes = a – b + a+ a + b = - $\frac{n}{m}$

$3a = -\frac{-3}{1} = 3$

$a = \frac{3}{3} = 3$

The zeroes are = (1 - b), 1 and (1 + b)

Product of zeroes = (1 - b)(1 + b)

(1 - b)(1 + b) = -q/m

$1 - b^2 = -\frac{1}{1} = -1$

$b^2 = 2$

b $= \pm\sqrt{2}$

So,

We get, a = 1 and b $= \pm\sqrt{2}$

Q. 4 If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ find other zeroes.

Answer:

Given:

2+√3 and 2-√3 are zeroes of given equation,

Therefore, (x - 2 + √3)(x - 2 - √3) should be a factor of given equation.
Also, (x - 2 + √3)(x - 2 - √3) = x^2 - 2x - √3x -2x + 4 + 2√3 + √3x - 2√3 – 3

= $x^2 - 4x + 1$

To find other zeroes, we divide given equation by $x^2 - 4x + 1$

$x^2 - 2x - 35$

$x^2 - 4x + 1 \sqrt{x^4 - 6x^3 - 26x^2 + 138x - 35}$

$x^4 - 4x^3 + x^2$

$- \quad + \quad -$

$-2x^3 - 27x^2 + 138x - 35$

$-2x^3 + 8x^2 - 2x$

$+ \quad - \quad +$

$-35x^2 + 140x - 35$

$-35x^2 + 140x - 35$

$+ \quad - \quad +$

0

We get ,

$x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now factorizing $x^2 - 2x - 35$ we get,

$x^2 - 2x – 35$ is also a factor of given polynomial and $x^2 - 2x – 35= (x-7) (x + 5)$

The value of polynomial is also zero when,

$x - 7 = 0$

or $x = 7$

And, $x + 5 = 0$

Or $x = -5$

Hence, 7 and -5 are also zeroes of this polynomial.

Q. 5 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2k + k$ the remainder comes out to be $x + a$, find k and a.

Answer:

To solve this question divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ by long division method

Let us divide, by $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$

$$
\begin{array}{rl}
 & x^2 - 4x + (8-k) \\
x^2 - 2x + k & \overline{\big) x^4 - 6x^3 + 16x^2 - 25x + 10} \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 \qquad -4kx \\
 & + \quad - \qquad + \\
\hline
 & (8-k)x^2 + (4k-25)x + 10 \\
 & (8-k)x^2 + (16-2k)x + (8-k)x \\
 & - \quad + \quad - \\
\hline
 & (2k-9)x + (10 - 8k + k^2)
\end{array}
$$

So, remainder $= (2k - 9)x + (10 - 8k + k^2)$

But given remainder $= x + a \Rightarrow (2k - 9)x + (10 - 8k + k^2) = x + a$

Comparing coefficient of x, we have$2k - 9 = 1 \Rightarrow 2k = 10 \Rightarrow k = 5$andComparing constant term,$10 - 8k + k^2 = a$

$\Rightarrow a = 10 - 8(5) + 5^2$

$\Rightarrow a = 10 - 40 + 25 \Rightarrow a = -5$. So, the value of k is 5 and a is -5.