Chapter – 1

Real Numbers

Exercise- 1.1

Q. 1 Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Solutions: Concept used:

To obtain the HCF of two positive integers, say c and d, with $c > d$, we follow the steps below:

Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.

Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

(i) We know that,

$= 225 > 135$

Applying Euclid's division algorithm:

(Dividend = Divisor × Quotient + Remainder)

$225 = 135 \times 1 + 90$

Here remainder $= 90$,

So, Again Applying Euclid's division algorithm

$135 = 90 \times 1 + 45$

Here remainder $= 45$,

So, Again Applying Euclid's division algorithm

$90 = 45 \times 2 + 0$

Remainder $= 0$,

Hence, HCF of $(135, 225) = 45$

(ii) We know that,

$38220 > 196$

So, Applying Euclid's division algorithm

$38220 = 196 \times 195 + 0$ (Dividend $=$ Divisor $\times$ Quotient $+$ Remainder)

Remainder $= 0$

Hence, HCF of $(196, 38220) = 196$

(iii)We know that,

$867 > 255$

So, Applying Euclid's division algorithm

$867 = 255 \times 3 + 102$ (Dividend $=$ Divisor $\times$ Quotient $+$ Remainder)

Remainder $= 102$

So, Again Applying Euclid's division algorithm

$255 = 102 \times 2 + 51$

Remainder $= 51$

So, Again Applying Euclid's division algorithm

$102 = 51 \times 2 + 0$

Remainder $= 0$

Hence,

(HCF 0f 867 and 255) $= 51$

Q. 2 Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

To Prove: Any Positive odd integer is of the form 6q + 1, 6q + 3, 6q + 5

Proof: To prove the statement by Euclid's lemma we have to consider divisor as 6 and then find out the possible remainders when divided by 6 By taking,' a' as any positive integer and b = 6.

Applying Euclid's algorithm

$a = 6q + r$

As divisor is 6 the remainder can take only 6 values from 0 to 5

Here, r = remainder = 0, 1, 2, 3, 4, 5 and $q \geq 0$

So, total possible forms are 6q + 0, 6q + 1, 6q + 2, 6q + 3, 6q + 4 and 6q + 5

6q + 0, (6 is divisible by 2, its an even number)

6q + 1, (6 is divisible by 2 but 1 is not divisible by 2, its an odd number)

6q + 2, (6 and 2 both are divisible by 2, its an even number)

6q + 3, (6 is divisible by 2 but 3 is not divisible by 2, its an odd number)

6q + 4, (6 and 4 both are divisible by 2, its an even number)

6q + 5, (6 is divisible by 2 but 5 is not divisible by 2, its an odd number)

Therefore, odd numbers will be in the form 6q + 1, or 6q + 3, or 6q+5

Hence, Proved.

Q. 3 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution: Suppose, both groups are arranged in 'n' columns, for completely filling each column,

The maximum no of columns in which they can march is the highest common factor of their number of members. i.e. n = HCF(616, 32)

By using, Euclid's division algorithm

$616 = 32 \times 19 + 8$

Remainder $\neq 0$

So, again Applying Euclid's division algorithm

$32 = 8 \times 4 + 0$

HCF of (616, 32) is 8.

So, They can march in 8 columns each.

Q. 4 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solution:

To Prove: Square of any number is of the form 3 m or 3 m +1

Proof: to prove this statement from Euclid's division lemma, take any number as a divisor, in question we have 3m and 3m + 1 as the form

So, By taking, 'a' as any positive integer and b = 3.

Applying Euclid's algorithm $a = bq + r$.

$a = 3q + r$

Here, r = remainder = 0, 1, 2 and $q \geq 0$ as the divisor is 3 there can be only 3 remainders, 0, 1 and 2.

So, putting all the possible values of the remainder in, $a = 3q + r$

$a = 3q$ or $3q+1$ or $3q+2$

And now squaring all the values,

When a= 3qSquaring both sides we get,

$a^2 = (3q)^2$

$a^2 = 9q^2$

$a^2 = 3\ (3q^2)$

$a^2 = 3\ k_1$

Where $k_1 = 3q^2$

When $a = 3q+1$ Squaring both sides we get, $a^2 = (3q + 1)^2$

$a^2 = 9q^2 + 6q + 1$

$a^2 = 3(\ 3q^2 + 2q\ ) + 1$

$a^2 = 3k^2 + 1$

Where $k_2 = 3q^2 + 2q$

When $a = 3q+2$

Squaring both sides we get, $a^2 = (3q + 2)^2$

$a^2 = 9q^2 + 12q + 4$

$a^2 = 9q^2 + 12q + 3+1$

$a^2 = 3(3q^2 + 4q + 1) +1$

$a^2 = 3k_3 + 1$

Where $k_3 = 3q^2 + 4q + 1$

Where k_1, k_2 and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m+1.

Q. 5 Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Solution: Let a be any positive integer. Then, it is of the form 3q or, 3q + 1 or, 3q + 2.

We know that according to Euclid's division lemma:

$a = bq + r$ So, we have the following cases:

Case I When $a = 3q$

In this case, we have

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 3q3$

Case II When $a = 3q + 1$

In this case, we have

$a^3 = (3q + 1)^3$

$\Rightarrow 27q^3 + 27q^2 + 9q + 1$

$\Rightarrow 9q(3q^2 + 3q + 1) + 1$

$\Rightarrow a^3 = 9m + 1$, where $m = q(3q^2 + 3q + 1)$

Case III When $a = 3q + 2$

In this case, we have

$a^3 = (3q + 1)^3$

$\Rightarrow 27q^3 + 54q^2 + 36q + 8$

$\Rightarrow 9q(3q^2 + 6q + 4) + 8$

$\Rightarrow a^3 = 9m + 8$, where $m = q(3q^2 + 6q + 4)$

Hence, a^3 is the form of $9m$ or, $9m + 1$ or, $9m + 8$

Exercise 1.2

Q. 1 Express each number as a product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Solution: (i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

2	140
2	70
5	35
7	7
	1

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

2	156
2	78
3	39
13	13
	1

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

3	3825
3	1275
5	425
5	85
17	17
	1

(iv) $5005 = 5\times7\times11\times13$

5	5005
7	1001
11	143

13	13
	1

(v) $7429 = 17\times19\times23$

17	7429
19	437
23	23
	1

Q. 2 Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Solution: (i) $26 = 2\times13$

$91 = 7\times13$

HCF $=13$

LCM $= 2\times7\times13 = 182$

Product of the two numbers $= 26\times91 = 2366$

HCF×LCM $= 13\times182 = 2366$

Hence, product of two numbers = HCF×LCM

(ii) $510 = 2\times3\times5\times17$

$92 = 2\times2\times23$

HCF $= 2$

LCM $= 2\times2\times3\times5\times17\times23 = 23460$

Product of the two numbers $= 510\times92 = 46920$

$HCF \times LCM = 2 \times 23460$

$HCF \times LCM = 46920$

Hence, product of two numbers = HCF×LCM

(iii) $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 24 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$HCF = 2 \times 3 = 6$

$LCM = 24 \times 3^3 \times 7 = 3024$

Product of the two numbers = $336 \times 54 = 18144$

$HCF \times LCM = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF×LCM

Q. 3 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

Solution:

Prime factors of any number is the representation of a number as a product of prime numbers it is composed of

for example: Prime factors of 20 = 2 × 2 × 5HCF = Highest common factor = The product of the factors that are common to the numbers LCF = Least Common Factor = Product of all the factors of numbers without duplicating the factor

(i)let us write prime factors of the given numbers12 = $2 \times 2 \times 3 = 22 \times 3$

$15 = 3\times5$

$21 = 3\times7$

Only 3 is common in all the three numbers, therefore

HCF = 3

As 3 is common in all three numbers, it will be taken as 1 time in the product of calculating LCM

LCM = $2^2\times3\times5\times7 = 420$

(ii) Let us write the prime factors of the given numbers

$17 = 1\times17$

$23 = 1\times23$

$29 = 1\times29$

As only 1 is common from all the three factors

HCF = 1

As nothing except 1 is common from all three numbers, simply multiplying them will give LCM of numbers.

LCM = $17\times23\times29 = 11339$

(iii) Let us write the prime factors of the given numbers

$8=2\times2\times2$

$9 = 3\times3$

$25 = 5\times5$

As nothing is common in the numbers

HCF = 1

As nothing is common in factors of numbers, numbers are simply multiplied to obtain LCM

LCM = $2\times2\times2\times3\times3\times5\times5 = 1800$

Q. 4 Given that HCF (306, 657) = 9, find LCM (306, 657).

Solutions: Given: HCF of (306, 657) = 9

We know that,

LCM × HCF = product of two numbers

LCM×HCF = 306×657

$$\text{LCM} = \frac{306\times657}{HCF} = \frac{306\times657}{9}$$

LCM = 22338

Q. 5 Check whether 6^n can end with the digit 0 for any natural number n.

Solution: We need to find can 6^n end with zero

If any number has last digit 0,

Then, it should be divisible by 10

Factors of 10 = 2×5

So, Value 6^n should be divisible by 2 and 5

Prime factorisation of $6^n = (2\times3)^n$

Hence,

6^n is divisible by 2 but not by 5.

It can not end with 0.

Q. 6 Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Solutions: Composite numbers are those numbers, which can be written in the form of the product of two or more integers, and at least one of them should not be 1

(i) 7 × 11 × 13 + 13

$= (7 \times 11 \times 13) + (13 \times 1)$

Taking 13 as common, we get,

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1) = 13 \times 78$

$= 13 \times 13 \times 6$

As the given no is a multiple of two or more integers, one of them being other than 1.

Hence, it is a composite number.

Therefore, it is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= (7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + (5 \times 1)$

Taking 5 as common, we get,

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

As the given no is a multiple of two integers, one of them being other than 1.

Hence, it is a composite number.

Q. 7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution: Both of them start from the same point and start moving in same direction

Sonia takes 18 minutes and Ravi takes 12 minutes to complete the circle. After 12 minutes Ravi will be back to the starting point and Sonia must have covered (12/18) = 2/3 of the rounds. After 12 more minutes Ravi has completed 2 rounds and Sonia must have covered $(24/18) = 4/3$ of the rounds. After 12 more minutes Ravi has completed 3 rounds and Sonia must have completed $(36/18) = 2$ rounds. Hence after 36 minutes both will again meet at the starting point.

Alternate Method:

They will meet again after LCM of both values at starting point.

$18 = 2\times3\times3$

And

$12 = 2\times2\times3$

LCM of 12 and $18 = 2\times2\times3\times3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Exercise 1.3

Q. 1 Prove that is irrational.

Solution: Let's assume that $\sqrt{5}$ is a rational number.

Hence, $\sqrt{5}$ can be written in the form a/b [where a and b ($b \neq 0$) are co-prime (i.e. no common factor other than 1)]

$\therefore \sqrt{5} = a/b$

$\Rightarrow \sqrt{5}\, b = a$

Squaring both sides,

$\Rightarrow (\sqrt{5}\, b)^2 = a^2$

$\Rightarrow 5b^2 = a^2$

$\Rightarrow a^2/5 = b^2$

Hence, 5 divides a^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides a too

Hence, we can say $a/5 = c$ where, c is some integer

So, $a = 5c$

Now we know that,

$5b^2 = a^2$

Putting $a = 5c$,

$\Rightarrow 5b^2 = (5c)^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\therefore b^2/5 = c^2$

Hence, 5 divides b^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides b too

By earlier deductions, 5 divides both a and b

Hence, 5 is a factor of a and b

∴ a and b are not co-prime.

Hence, the assumption is wrong.

∴ By contradiction,

∴ √5 is irrational

Q. 2 Prove that is irrational.

To Prove: $3 + 2\sqrt{5}$ is irrational

Proof: Let $3 + 2\sqrt{5}$ is rational

A number is said to be rational if it can be expressed in the form p/q where $q \neq 0$

Therefore,

We can find two integers p & q where, $(q \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{p}{q}$

$2\sqrt{5} = \frac{p}{q} - 3$

Since p and q are integers, $\frac{1}{2}\left(\frac{p}{q} - 3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

We know that √5 is irrational but according to above statement it has to be rational

So, both the comments are contradictory, Hence, the number should have been irrational to make the statement correct.

Therefore, $3 + 2\sqrt{5}$ is irrational.

Q. 3 Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

(ii) $7\sqrt{5}$

(iii) $6 + \sqrt{2}$

Solution: **(i)** Let $\frac{1}{\sqrt{2}}$ is rational

Therefore, we can find two integers p & q where, q ≠0 such that

$$\frac{1}{\sqrt{2}} = \frac{p}{q}$$

$$\sqrt{2} = \frac{q}{p}$$

$\frac{q}{p}$ is rational as p and q are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let is rational.

Therefore, we can find two integers p & q where, q≠0 such that

$7\sqrt{5} = \frac{p}{q}$ for some integers p and q

$$\therefore \sqrt{5} = \frac{p}{7q}$$

$\frac{p}{7q}$ is rational as p and q are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Therefore our assumption that is rational is false.

Hence, $7\sqrt{5}$ is irrational.

(iii) Let be $6 + \sqrt{2}$ rational.

Therefore, we can find two integers p & q where q≠0, such that

$6 + \sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 6$

Since p and q are integers, $\frac{p}{q} - 6$ is also rational

Hence, $\sqrt{2}$ should be rational, this contradicts the fact that is irrational.

So, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.

Since p and q are integers, $6 + \sqrt{2}$ is also rational.

Exercise – 1.4

Q. 1 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^3}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Solution: (i) $\frac{13}{3125}$

Factorize the denominator we get,

$3125 = 5\times5\times5\times5\times5 = 5^5$

The denominator is of the form 5m

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

Factorize the denominator we get,

$8 = 2 \times 2 \times 2 = 2^3$

The denominator is of the form 2^m

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

Factorize the denominator we get,

$455 = 5\times7\times13$

Since, the denominator is not in the form of $2^m \times 5^n$, and it also contains 7 and 13 as its factors,

Its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

Factorize the denominator we get,

$1600 = 2^6 \times 5^2$

The denominator is in the form $2^m \times 5^n$

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

Factorize the denominator we get,

$343 = 7^3$

Since the denominator is not in the form of $2^m \times 5^n$, it has 7 as its factors.

So, the decimal expansion of non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

The denominator is in the form $2^m \times 5^n$

Hence, the decimal expansion of $\frac{23}{2^3 5^2}$ is terminating.

(vii) $\frac{129}{2^2 5^7 7^5}$

Since, the denominator is not in the form of $2^m \times 5^n$, as it has 7 in denominator.

So, the decimal expansion of $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2\times3}{3\times5} = \frac{2}{5}$

The denominator is in the form 5^n

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50} = \frac{7\times5}{10\times5} = \frac{7}{10}$

Factorize the denominator we get,

$10 = 2\times5$

The denominator is in the form $2^m\times5^n$

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210} = \frac{11\times7}{30\times7} = \frac{11}{30}$

Factorize the denominator we get,

$30 = 2\times3\times5$

Since the denominator is not in the form of $2^m \times 5^n$, as it has 3 in denominator.

So, the decimal expansion of $\frac{77}{210}$ non-terminating repeating.

Q. 2 Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solution: (i) $\frac{13}{3125} = 0.00416$

$$
\begin{array}{r|l}
 & 0.00416 \\
\hline
3125 & 13.00000 \\
 & \underline{0} \\
 & 130 \\
 & \underline{0} \\
 & 1300 \\
 & \underline{0} \\
 & 13000 \\
 & \underline{12500} \\
 & 5000 \\
 & \underline{3125} \\
 & 18750 \\
 & \underline{18750} \\
 & \times
\end{array}
$$

(ii) $\frac{17}{8} = 2.125$

$$\begin{array}{r|l} & 2.125 \\ \hline 8 & 17 \\ & \underline{16} \\ & 10 \\ & \underline{8} \\ & 20 \\ & \underline{16} \\ & 40 \\ & \underline{40} \\ & \times \end{array}$$

(iv) $\frac{15}{1600} = 0.009375$

$$\begin{array}{r|l} & 0.009375 \\ \hline 1600 & 15.000000 \\ & \underline{0} \\ & 150 \\ & \underline{0} \\ & 1500 \\ & \underline{0} \\ & 15000 \\ & \underline{14400} \\ & 6000 \\ & \underline{4800} \\ & 12000 \\ & \underline{11200} \\ & 8000 \\ & \underline{8000} \\ & \times \end{array}$$

(vi) $\frac{23}{2^3 \times 5^2} = 0.115$

$$\begin{array}{r} 0.115 \\ 200\overline{)23.000} \\ \underline{0} \\ 230 \\ \underline{200} \\ 300 \\ \underline{200} \\ 1000 \\ \underline{1000} \\ \times \end{array}$$

(viii) $\frac{6}{15} = \frac{2\times3}{2\times5} = \frac{2}{5} = 0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ \underline{0} \\ 20 \\ \underline{20} \\ \times \end{array}$$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ \underline{0} \\ 350 \\ \underline{350} \\ \times \end{array}$$

Q. 3 The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$what can you say about the prime factors of q?

(i) 43.123456789
(ii) 0.120120012000120000…
(iii) $43.\overline{123456789}$

Solution: (i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number $\frac{p}{q}$ of the form and q is of the form 2m×5n

That is, the prime factor of q will be 2 or 5 or both.

(ii) 0.120120012000120000...

The decimal expansion is neither terminating nor recurring.

Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating but recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ that is, the prime factors of q will also have a factor other than 2 or 5.