Chapter 13
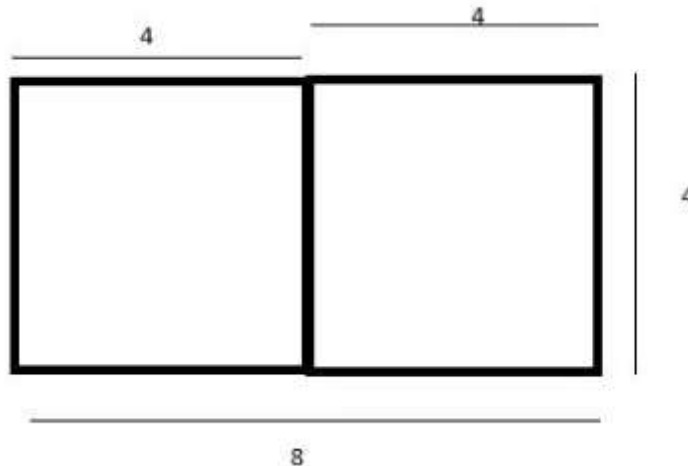
Surface Areas and Volumes

Exercise 13.1

Q.1 2 cubes each of volume 64 cm³ are joined end to end. Find the surface area of the resulting cuboid.

Answer:

In the figure above two cubes are joined end to end such that a cuboid is formed.

Volume of cube = 64 cm³

We know that,

Side of cube = $\sqrt[3]{volume}$

$= \sqrt[3]{64}$

$= 4$ cm

Side of cube = 4 cm

Now these two cubes when joined, we can see from the figure that, their lengths are added and breadth and height remains the same.

Length of new cuboid = 8 cm

Height of new cuboid = 4 cm

Width of new cuboid = 4 cm

We know that:

Surface Area = 2(lb + lh + bh)

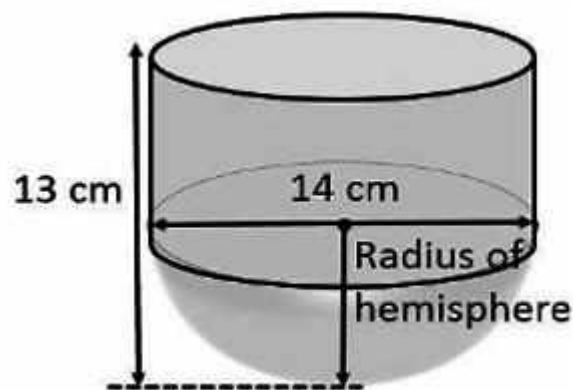Surface Area = 2(8 x 4 + 8 x 4 + 4 x 4)

= 2 x 80

= 160 cm²

Q. 2 A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer:

According to the question,

Radius (r) of the cylindrical part = 7cm

And

Radius(r) of the hemispherical part = 7 cm

Height of hemispherical part = Radius = 7 cm

Height of cylindrical part (h) = 13 – 7 = 6 cm

Now,

Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part = 2rh + 2 πr2

Inner surface area of vessel = $(2 \times \frac{22}{7} \times 7 \times 6) + (2 \times \frac{22}{7} \times 7 \times 7)$

= (44 × 6) + (44 × 7)

= 44 (6 + 7)
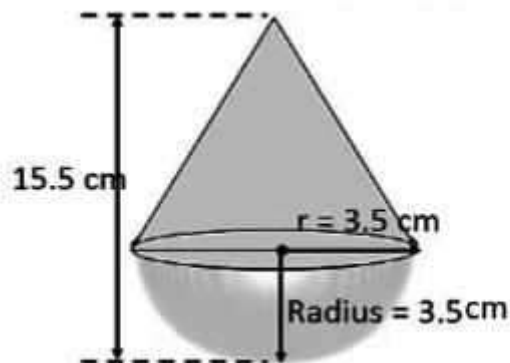
= 44 × 13

= 572 cm²

Q. 3 A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer:

As per the question:

The radius of the conical part = 3.5 cm

And,

Radius of the hemispherical part = 3.5 cm

Height of hemispherical part = Radius (r)

= 3.5 cm

$= \dfrac{7}{2}$ cm

Height of conical part (h) = 15.5 - 3.5 = 12 cm

for cone we know that,

$r^2 + h^2 = l^2$

where r is the radius of base, l is the slant height and h is the height of cone.

Slant height of the conical part

$l = \sqrt{r^2 + h^2}$

$l = \sqrt{\left(\dfrac{7}{2}\right)^2 + (12)^2}$

$l = \sqrt{\dfrac{49}{4} + 144}$

$l = \sqrt{\dfrac{49+576}{4}}$

$l = \sqrt{\dfrac{625}{4}}$

$l = \dfrac{25}{2}$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$= \pi r l + 2\pi r^2$

$= \left(\dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{25}{2}\right) + \left(2 \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}\right)$

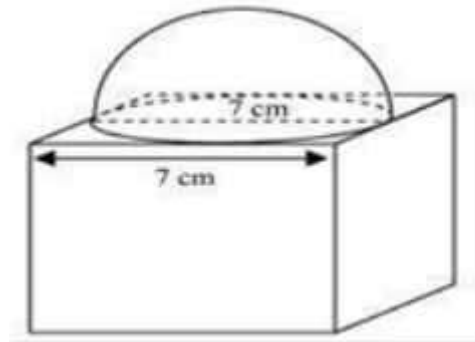$= \left(\frac{275}{2}\right) + (11 \times 7)$

$= 137.5 + 77$

$= 214.5$ cm²

Q. 4 A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer:

From the figure:

The greatest diameter possible for the hemisphere is equal to the cube's edge= 7cm

Radius (r) of hemispherical part = 7/2cm

The total surface area of solid = Surface area of cubical part + CSA of hemispherical part - Area of the base of hemispherical part
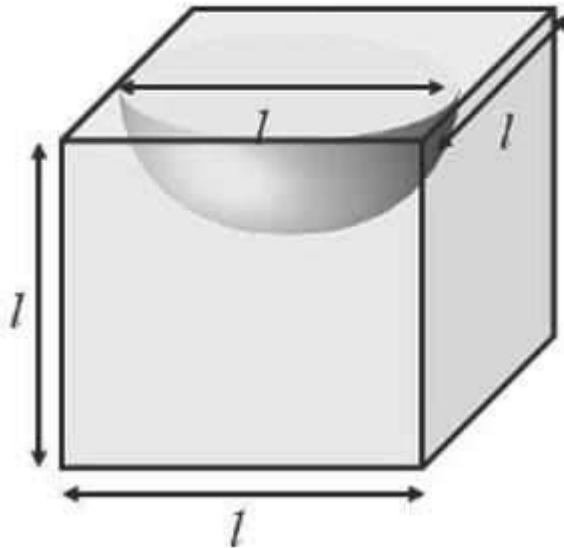
$= 6$ (Edge)² $+ 2\pi r^2 - \pi r^2$

$= 6$ (Edge)² $+ \pi r^2$

Total surface area of solid $= 6$ (7)² $+\left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$

$$= 294 + \frac{77}{2}$$

$$= 294 + 38.5$$

$$= 332.5 \text{ cm}^2$$

Q. 5 A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer:

According to the question,

Diameter of hemisphere = Edge of cube = l

Radius of hemisphere = $\frac{l}{2}$

Curved Surface Area of hemisphere = $2\pi r^2$

Surface Area of cube = $6(\text{Edge})^2$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part - Area of base of hemispherical part

$= 6 \text{ (Edge)}^2 + 2\pi r^2 - \pi r^2$

$= 6 \text{ (Edge)}2 + \pi r2$

TSA of Solid $= 612 + \pi \times \left(\dfrac{l}{2}\right)^2$

$= 612 + \left(\pi \times \left(\dfrac{l}{2}\right)^2\right)$

$= 612 + \left(\dfrac{l^2}{4}\pi\right)$

$= 12 \left(\dfrac{24+\pi}{4}\right)$ sq. units

Q. 6 A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 13.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.
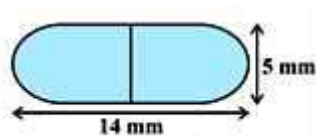

Fig. 13.10

Answer:

According to the question,

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$= \dfrac{diameter\ of\ the\ capsule}{2}$

$= \dfrac{5}{2}$

Length of cylindrical part (h) = Length of the entire capsule - 2 × r

$= 14 - 5 = 9$ cm

Surface area of capsule = 2 × C S A of hemispherical part + C S A of cylindrical part

(Curved Surface Area of cylinder = 2 πrh, Curved Surface Area of Hemisphere = 2 πr²)

= 2x 2πr² + 2πrh

$$= 4\pi \left(\frac{5}{2}\right)^2 + 2\pi \frac{5}{2} \times 9$$

= 25 π + 45 π

= 70 π

Q. 7b A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m². (Note that the base of the tent will not be covered with canvas)

Answer:

Given:

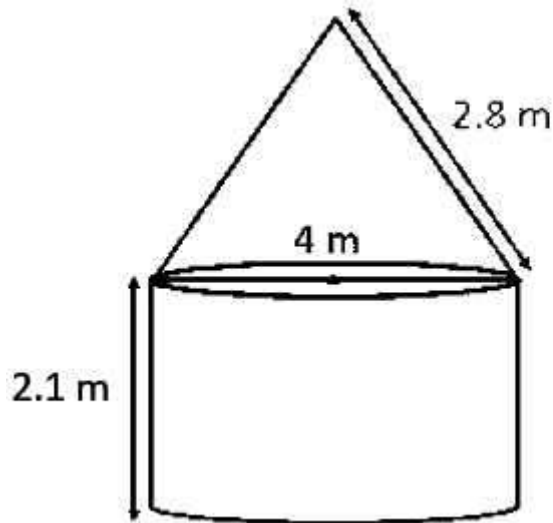Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) = 2.8 m

Solution:

We know for a cone,

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi r^l + 2\pi rh$

$= (\pi \times 2 \times 2.8) + (2\pi \times 2 \times 2.1)$

$= 2\pi [2.8 + (2 \times 2.1)]$

$= 2\pi [2.8 + 4.2]$
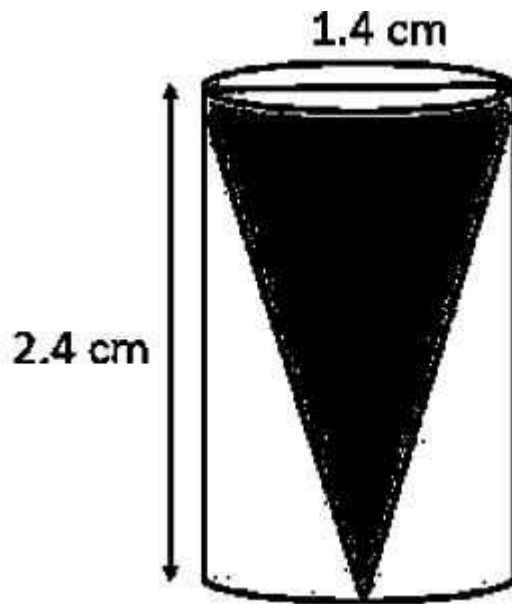
$= 2 \times \frac{22}{7} \times 7$

$= 44 \text{ m}^2$

Cost of 1 m² canvas = Rs 500

Cost of 44 m² canvas = 44 × 500 = Rs 22000

Q. 8 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Answer:

Given:

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

Radius (r) of the cylindrical part = 0.7 cm

we know, slant height of cone, $l = \sqrt{r^2 + h^2}$

Slant height of the cylindrical part $(l) = \sqrt{(0.7 \times 0.7) + (2.4 \times 2.4)}$

$= \sqrt{0.49 + 5.76}$

$= \sqrt{6.25}$

$= 2.5$

Total surface area of the remaining solid will be

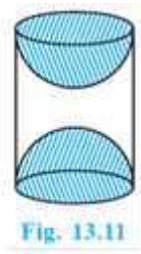= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$= 2\pi rh + \pi r^l + \pi r^2$

$= (2 \times \frac{22}{7} \times 0.7 \times 2.4) + (\frac{22}{7} \times 0.7 \times 2.5) + (\frac{22}{7} \times 0.7 \times 0.7)$

$= (4.4 \times 2.4) + (2.2 \times 2.5) + (2.2 \times 0.7)$

$= 10.56 + 5.50 + 1.54$

$= 12.10 + 5.50 \text{ cm}^2$

$= 17.60 \text{ cm}^2 \text{ (approx.)}$

Hence, the total surface area of remaining solid is 17.60 cm².

Q. 9 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

Fig. 13.11

Answer:

Given:

Radius (r) of cylindrical part = 3.5 cm

Radius (r) of hemispherical part = 3.5 cm

Height of cylindrical part (h) = 10 cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part = 2 πrh + 2 × 2 πr²

$= (2 \pi \times 3.5 \times 10) + (2 \times 2 \pi \times 3.5 \times 3.5)$

$= 70 \pi + 49 \pi$

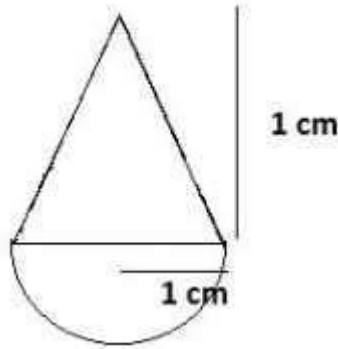$= 119 \pi$

$= 119 \times \dfrac{22}{7}$

$= 17 \times 22$

$= 374 \text{ cm}^2$

Hence, the total surface of the article $= 374 \text{ cm}^2$.

Exercise 13.2

Q. 1 A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer:

Given:

Height (h) of conical part = Radius(r) of conical part = 1 cm

Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm

Volume of solid = Volume of conical part + Volume of hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi (1)^2 (1) + \frac{2\pi}{3}(1)^3$$

$$= \frac{\pi}{3} + \frac{2\pi}{3}$$

$$= \frac{3\pi}{3}$$

$$= \pi \ cm^3$$

Hence, Volume of the solid $= \pi \ cm^3$

Q. 2 Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same)

Answer:
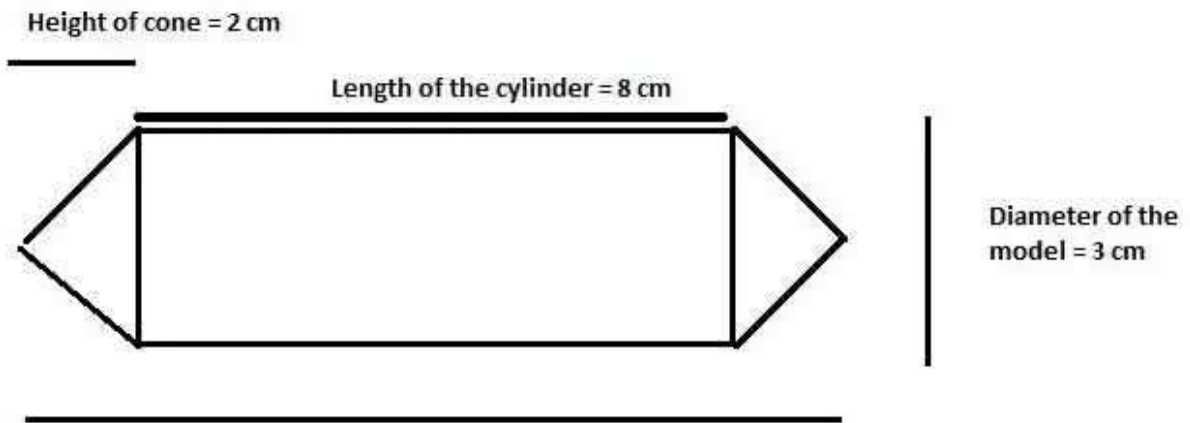
Height (h^1) of each conical part = 2 cm

Height (h^2) of cylindrical part = 1^2 - 2 × Height of conical part = 1^2 - 2 × 2 = 8 cm

Radius = Diameter/ 2

Radius (r) of cylindrical part = $\frac{3}{2}$ cm

Radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder + 2 × Volume of cones

volume of cylinder $= \frac{1}{3} \pi r^2 h$

Volume of air present in the model $= \pi r^2 h^2 + 2 \times \frac{1}{3} \pi r^2 h^1$

Volume of air present in the model $= \pi \left(\frac{3}{2}\right)^2 (8)\, 2 \times \frac{1}{3} \pi \left(\frac{3}{2}\right)^2 (2)$

$= 18\pi + 3\pi$

$= 21\pi$
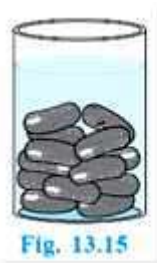
$= 21 \times 22/7$

$= 3 \times 22$

$= 66 cm^3$

Hence, volume of air contained in the model that Rachel made $= 66 cm3$

Q. 3 A gulabjamun contains sugar syrup up to about30% of its volume. Find approximately how much syrup would be found in 45 gulabjamun each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 13.15)

Fig. 13.15

Answer:

2.8 cm

5 cm

Radius (r) of cylindrical part $= \dfrac{2.8}{2} = 1.4$ cm

Radius (R) of hemispherical part $= \dfrac{2.8}{2} = 1.4$ cm

Length of each hemispherical part = 1.4 cm

The radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 - 2 × Length of hemispherical part

$= 5 - (2 \times 1.4)$

$= 5 - 2.8$

$= 2.2$ cm

The volume of one gulabjamun = Vol. of cylindrical part + 2 × Vol. of hemispherical part

$= (\pi r^2 h) + (2 \times \dfrac{2}{3}\pi R^3)$

$= (\pi r^2 h) + (\dfrac{4}{3}\pi R^3)$

$= [\pi \times (1.4)^2 \times 2.2] + [\dfrac{4}{3}\pi(1.4)^3]$

$= \left(\dfrac{22}{7} \times 1.4 \times 1.4 \times 2.2\right) + \left(\dfrac{4}{3} \times \dfrac{22}{7} \times 1.4 \times 1.4 \times 1.4\right)$

$= \left(\dfrac{94.864}{7}\right) + \left(\dfrac{241.472}{21}\right)$

$= 13.552 + 11.498$

$= 25.05$ cm³

Volume of 45 gulabjamun = 45 × 25.05

= 1,127.25 cm³
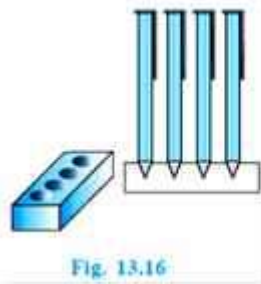
The volume of sugar syrup = 30% of the volume

$= \frac{30}{100} \times 1127.25$

= 338.17 cm³

= 338 cm³(approx.)

Q. 4 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 13.16)

Fig. 13.16

Answer:

Given:

Depth (h) of each conical depression = 1.4 cm

Radius (r) of each conical depression = 0.5 cm

Volume of wood = Volume of cuboid - 4 × Volume of cones

$= \text{lbh} - 4 \times \frac{1}{3} \pi \, r^2 h$

$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$

= 525 − 1.47

= 523.53 cm³

Hence,

The volume of wood in the entire stand= 523.53 cm³

Q. 5 A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer:
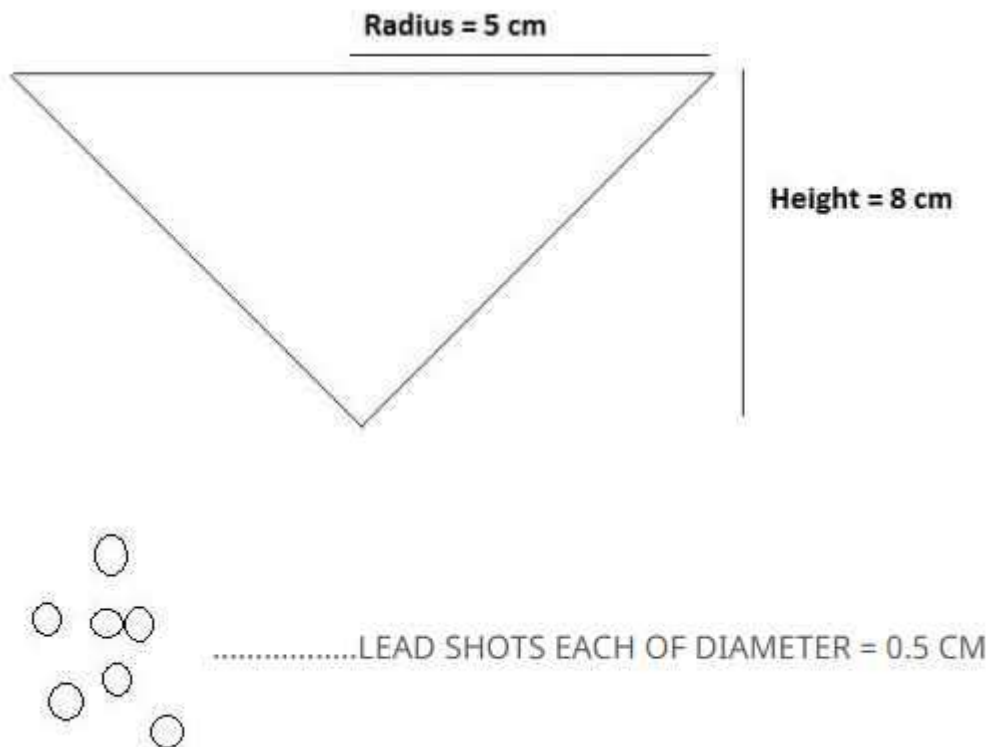
When lead shots are dropped in the Vessel, the same amount of water will come out of the vessel as the Total Volume of Lead Shots.

Height (h) of conical vessel = 8 cm

Radius (r_1) of conical vessel = 5 cm

Radius (r_2) of lead shots = 0.5 cm

Volume of water = Volume of Vessel Let n number of lead shots were dropped in the vessel

Volume of water spilled = Volume of dropped lead shots

Volume of vessel $= \frac{1}{3}\pi r_1^2 \, h$

Volume of vessel $= \frac{1}{3}\pi \, (5)^2 \, 8$

Volume of water spilled $= \frac{1}{4} \times$ volume of vessel

Volume of water spilled $= \frac{1}{12}\pi \times 25 \times 8$

Volume of lead shots $= n \times \frac{4}{3}\pi r_2^3$

Volume of lead shots $= n \times \frac{4}{3}\pi \, (0.5)^3$

Volume of lead shots = Volume of water spilled

$n \times \frac{4}{3}\pi \, (0.5)^3 = \frac{1}{12}\pi \times 25 \times 8$

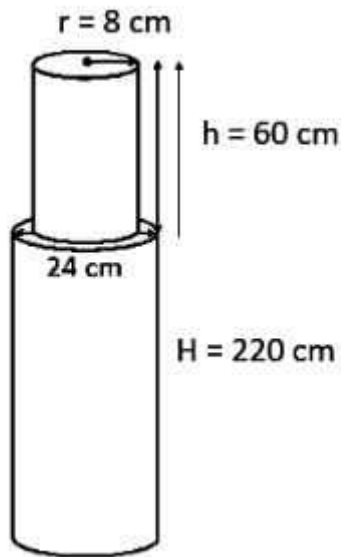$n = \frac{25 \times 8 \times 3}{12 \times 4 \times 0.5 \times 0.5 \times 0.5}$

$n = 100$

Hence, the number of lead shots dropped in the vessel is 100.

Q. 6 A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8gm mass. (Use $\pi = 3.14$)

Answer:

r = 8 cm

h = 60 cm

24 cm

H = 220 cm

Given:

Height (H) of larger cylinder = 220 cm

Radius (R) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h) of smaller cylinder = 60 cm

Radius (r) of smaller cylinder = 8 cm

Total volume = Volume (Larger cylinder) + Volume (Smaller cylinder)

$= \pi(R)^2H + \pi(r)^2h$

$= \pi (12)^2 \times 220 + \pi (8)^2 \times 60$

$= \pi [(144 \times 220) + (64 \times 60)]$

$= \pi [31680 + 3840]$

$= 35520 \times 3.14$

$= 111532.8$ cm³

Mass of 1 cm³ iron = 8 g

Mass of 111532.8 cm³ iron = 111532.8 × 8

= 892262.4 g

= 892.262 kg

Q. 7 A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer:
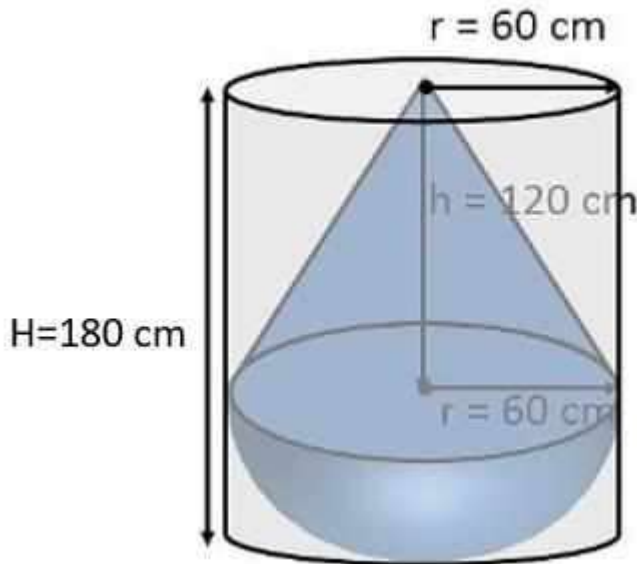
Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm

Height (h) of conical part of solid = 120 cm

Height (H) of cylinder = 180 cm

Radius (r) of cylinder = 60 cm

Volume of water left = Volume of cylinder - Volume of solid

= Volume of cylinder − (Volume of cone + Volume of hemisphere)

$$= \pi r^2 H - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

$$= [\pi\ (60)^2\ (180)\ ] - [\ \frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi\ (60)^3]$$

$= 648000\ \pi$ - $(144000\ \pi + 144000\ \pi) = 648000\ \pi$ - $288000\ \pi = 360000\pi$

$= 1131428.57\ cm^3$

$= 1.131\ m^3(approx.)$

Q. 8 A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$

Answer:
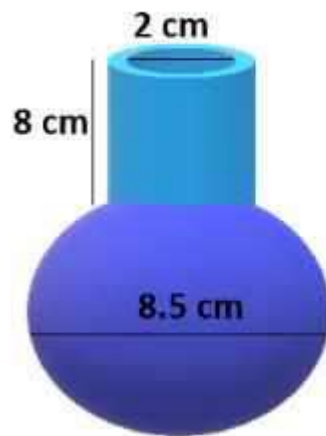
Given:

Height (h) of cylindrical part = 8 cm

Radius (r_2) of cylindrical part $= \frac{2}{2} = 1$ cm

Radius (r_1) spherical part $= \frac{8.5}{2} = 4.25$ cm

Volume of vessel = Volume of sphere + Volume of cylinder

$= \frac{4}{3}\ \pi\ (r_1)^3 + \pi(r_2)^2h$

$= \frac{4}{3} \pi (4.25)^3 + \pi (1)^2 (8)$

$= (\frac{4}{3} \times 3.14 \times 76.765625) + (8 \times 3.14)$

$= 321.392 + 25.12$

$= 346.512$

$= 346.51 \text{ cm}^3$

Hence,

From the result we can conclude that she is wrong.

Exercise 13.3

Q. 1 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder

Answer:

To find: Height of the cylinder (h)

Given:

Radius (r_1) of sphere = 4.2 cm

Radius (r_2) of cylinder = 6 cm

Let the height of the cylinder be h.

The object formed by recasting the sphere will be the same in volume.

Volume of sphere = Volume of cylinder

$$= \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2 = \frac{22}{7} \times 6 \times 6 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$
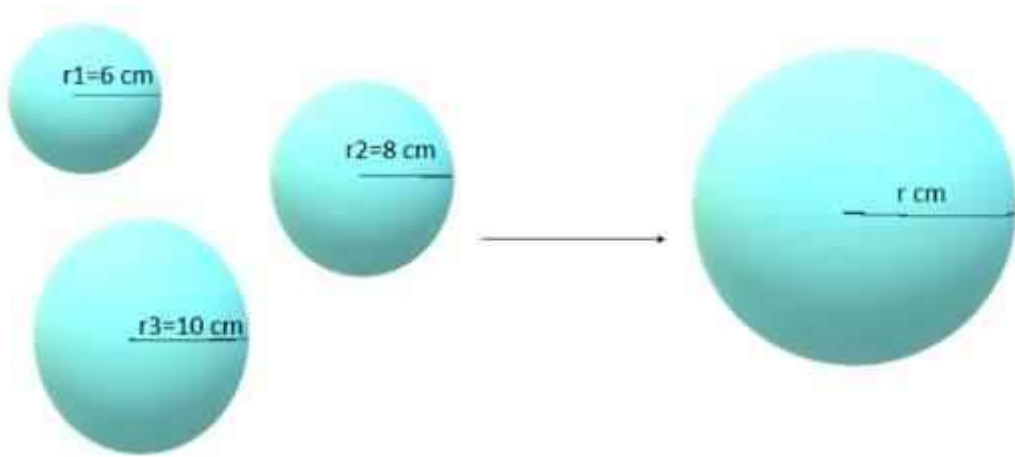
h = 2.74 cm

Hence, the height of the cylinder = 2.74 cm

Q. 2 Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer:

Radius (r_1) of 1st sphere = 6 cm

Radius (r_2) of 2nd sphere = 8 cm

Radius (r_3) of 3rd sphere = 10 cm

Let the radius of the resulting sphere be r.

The object formed by recasting these spheres will be the same in volume as the sum of the volumes of these spheres.

The volume of 3 spheres = Volume of resulting sphere

$$\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi \left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi \left[6^3 + 8^3 + 10^3\right] = \frac{4}{3}\pi r^3$$
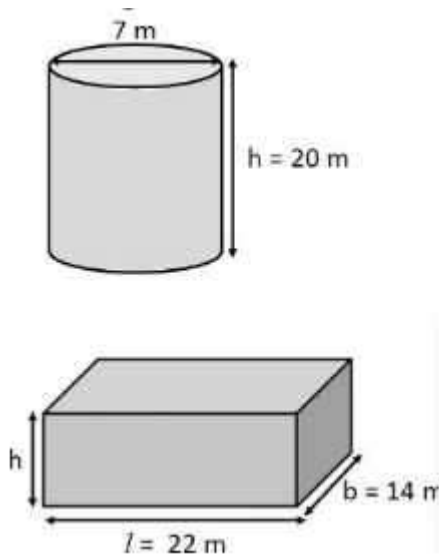
$$r^3 = 216 + 512 + 1000$$

$$r^3 = 1728$$

$$r = 12 \text{ cm}$$

Hence, the radius of the resulting sphere, r = 12 cm.

Q. 3 A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer:

The diagram for the situation is as follows:

As per the question,

The shape of the well will be cylindrical

Depth (h) of well = 20 m

Radius (r) of circular end of well = $\dfrac{7}{2}$

Area of platform = Length × Breadth = 22 × 14 m²

Let height of the platform = H

Volume of soil dug from the well would be equal to the volume of soil that is scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

$\pi r^2 h$ = Area × Height

$\pi \times \left(\dfrac{7}{2}\right)^2 \times 20 = 22 \times 14 \times H$
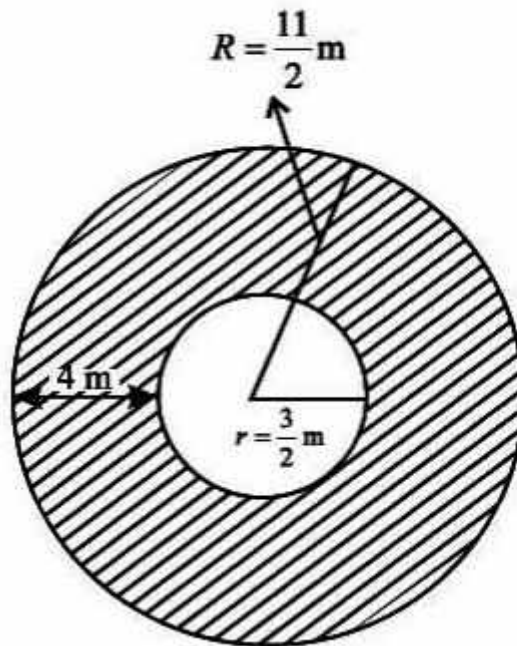
$H = \dfrac{22}{7} \times \dfrac{49}{4} \times \dfrac{20}{22*14}$

= 2.5 m

Hence, the height of the platform = 2.5 m

Q. 4 A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer:

The shape of the well will be cylindrical.

Depth (h_1) of well = 14 m

Radius (r) of the circular end of well = 3/2 m

Width of embankment = 4 m

As per the question,

Our embankment will be in a cylindrical shape having outer radius (R) as 11/2 m and inner radius (r) = 3/2 m

Let the height of embankment be h_2

Volume of soil dug from well = Volume of earth used to form embankment

$\pi r^2 \times h_1 = \pi \times (R^2 - r^2) \times h_2$

$= \pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times h$

$= \pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times \left[\frac{121}{4} - \frac{9}{4}\right] \times h$

$= \pi \times \frac{9}{4} \times 14 = \pi \times \left[\frac{112}{4}\right] \times$

$\Rightarrow 31.5 = 28\,h$

$= \frac{31.5}{28} = h$

$\Rightarrow h = 1.125\ m$

Q. 5 A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer:
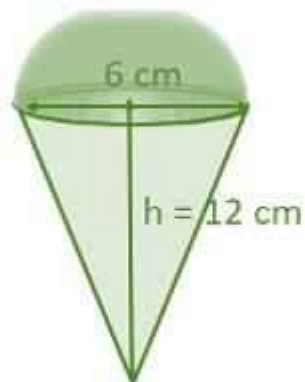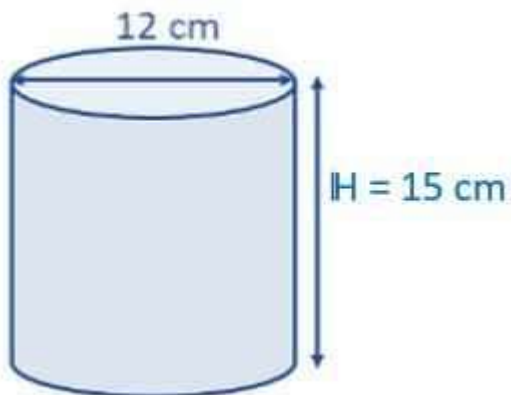
Height (H) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2}$ = 6 cm

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2}$ = 3 cm

Height (h) of conical part of ice-cream cone = 12 cm

Let n cones be filled with ice-cream of the container

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi \times (r_1)^2 \times H = n \left[\left(\frac{1}{3}\pi \times (r_2)^2 \times h \right) + \left(\frac{2}{3}\pi \times (r_2)^2 \right) \right]$$

$$n = \frac{\pi \times (r_1)^2 \times H}{\pi \left(\frac{1}{3} \times (r_2)^2 \times h \right) + \left(\frac{2}{3} \times (r_2)^2 \right)}$$

$$n = \frac{(r_1)^2 \times H}{\left(\frac{1}{3} \times (r_2)^2 \times h \right) + \left(\frac{2}{3} \times (r_2)^2 \right)}$$

$$= n = \frac{6 \times 6 \times 15}{\left(\frac{1}{3} \times 9 \times 12 \right) + \left(\frac{2}{3} \times 3 \times 3 \times 3 \right)}$$

$$= n = \frac{540}{36 + 18} = \frac{540}{54}$$

$$= n = 10$$

Q. 6 How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Answer:

Coins are cylindrical in shape.

Height (h_1) of cylindrical coins = 2 mm = 0.2 cm

Radius (r) of circular end of coins = $\frac{1.75}{2}$ = 0.875 cm

Let n coins be melted to form the required cuboids

Volume of n coins = Volume of cuboids

$n \times \pi \times r^2 \times h_1 = l \times b \times h$

$n \times \pi \times (0.875)^2 \times 0.2 = 5.5 \times 10 \times 3.5$

$n = \dfrac{5.5*1.*3.5*7}{0.875*0.875*0.2*22}$

$= 400$

Hence,

The number of coins melted to form the given cuboid is 400.

Q. 7 A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap

Answer:

Given:

Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$\pi r_1^2 \times h_1 = \dfrac{1}{3} \pi \times r_2^2 \times h_2$

$\pi \times (18)^2 \times 32 = \dfrac{1}{3} \times \dfrac{22}{7} \times r_2^2 \times 24$

$$r_2^2 = \frac{18 \times 18 \times 32 \times 3}{24}$$

$r_2 = \sqrt{18 \times 18 \times 4}$

$= 36$ cm
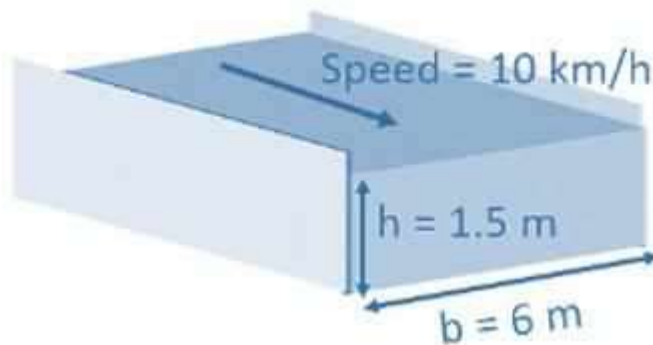
If h is the height and r is the radius of cone, then slant height is given by

$l = \sqrt{h^2 + r^2}$

$l = \sqrt{36^2 + 24^2}$

$l = 12\sqrt{13}$ cm

Q. 8 Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Area of cross-section $= 6 \times 1.5 = 9$ m²

Speed of water $= 10$ km/h

Water flows through canal in 60 min $= 10$ km

Water flows through canal in 1 min $= \frac{1}{60} \times 10$ km

Water flows through canal in 30 min $= \frac{30}{60} \times 10$

$= 5$ km $= 5000$ m

Hence length of the canal is 5000 m.

The volume of canal = L × B × H= (5000 × 6 × 1.5) = 45000 m³

Let the irrigated area be A.

Now, we know that,

The volume of water irrigating the required area will be equal to the volume of water that flowed in 30 minutes from the canal.

Vol. of water flowing in 30 minutes from canal = Vol. of water irrigating the reqd. area ⇒ 45000 = Area × height

Given height = 8 cm = 0.08 m⇒ 45000 = Area × 0.08

= area = $\frac{45000}{0.08}$

Area = 562500 m²

Q. 9 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:
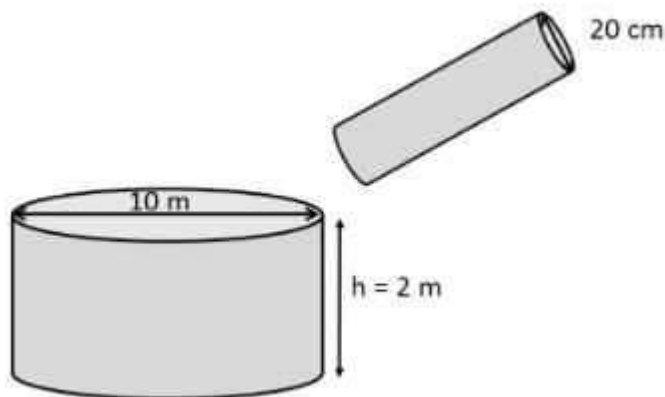
As we know that $1m = 100cm$ Therefore, $1cm = \dfrac{1}{100}$ m

$$20 \text{ cm} = \dfrac{20}{200} \text{ m}$$

Radius $(r1)$ of circular end of pipe $= \dfrac{20}{200} = 0.1$ m

Area of cross-section $= \pi \times r_1^2$

$= \pi \times (0.1)^2$

$= 0.01 \; \pi \; m^2$

Speed of water $= 3$ km/h

$= \dfrac{3000}{60}$

$= \dfrac{300}{6}$

$= 50$ meter/min

The volume of water that flows in 1 minute from pipe $= 50 \times 0.01 \; \pi$

$= 0.5\pi \; m^3$

The volume of water that flows in t minutes from pipe $= t \times 0.5\pi \; m^3$

Radius (r_2) of circular end of cylindrical tank $= \dfrac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank $= 2$ m

Let the tank be filled completely in t minutes

The volume of water filled in the tank in t minutes is equal to the volume of water flowed in t minutes from the pipe

The volume of water that flows in t minutes from pipe = Volume of water in the tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = (5)^2 \times 2$

$t = 100$

Hence, the cylindrical tank will be filled in 100 minutes.

Exercise 13.4

Q. 1 A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass

Answer:

Given:

Radius (r_1) [Upper base] $= \frac{4}{2} = 2$ cm

Radius (r_2) [Lower base] $= \frac{2}{2} = 1$ cm

Capacity of glass = Volume of frustum of cone

$= \frac{1}{3}\pi\ h\ (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3}\pi\ h\ [(2)^2 + (1)^2 + (2)\ (1)]$

$= \frac{1}{3} * \frac{22}{7} * 14\ [4 + 1 + 2]$

$= \frac{308}{3}$

$= 102\frac{2}{3}$ cm³

Hence, the capacity of the glass $= 102\frac{2}{3}$ cm³

Q. 2 The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum

Answer:

According to the question,

Perimeter of upper circular end of frustum = 18 cm

$[2\pi r_1 = 18]$

$r_1 = \dfrac{9}{\pi}$

Perimeter of lower end of frustum = 6 cm

$2\pi r^2 = 6$

$r_2 = \dfrac{3}{\pi}$

Slant height (l) of frustum = 4 cm

CSA of frustum = $\pi (r_1 + r_2) \, l$

$= \pi \left(\dfrac{9}{\pi} + \dfrac{3}{\pi} \right) * 4$

$= 12 * 4$

$= 48 \text{ cm}^2$

Q. 3 A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig. 13.24). If its radius on the open side is 10 cm, the radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Fig. 13.24

Answer:

Given:

Radius (r_2) at upper circular end = 4 cm

Radius (r_1) at lower circular end = 10 cm

Slant height (l) of frustum = 15 cm

Area of material used for making the fez = CSA of frustum + Area of upper circular end

$= \pi (r_1 + r_2)\, l + \pi r_2^2$

$= \pi (10 + 4) \times 15 + \pi (4)^2$

$= \pi (14) \times 15 + 16\pi$

$= 210\pi + 16\pi$

$= \dfrac{226 * 22}{7}$

$= 710\,\dfrac{2}{7}\ \text{cm}^2$

Hence, the area of material used $= 710\,\dfrac{2}{7}\ \text{cm}^2$.

Q. 4 A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per liter. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm² (Take $\pi = 3.14$)

Answer:
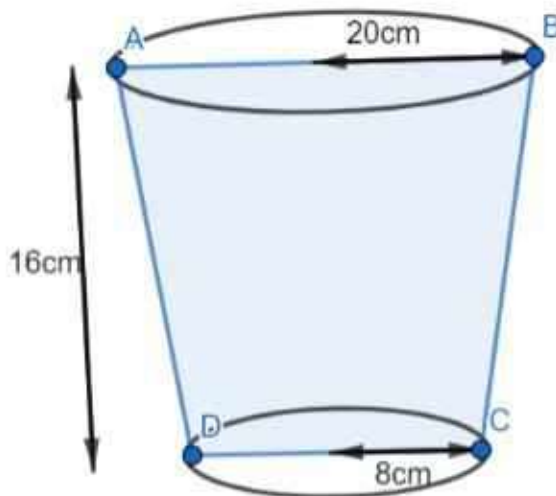
Radius (r_1) of upper end of container = 20 cm

Radius (r_2) of lower end of container = 8 cm

Height (h) of container = 16 cm

Slant height (l) of frustum = $\sqrt{((r_1 - r_2)_2 + h_2)}$

where r_1 and r_2 ($r_1 > r_2$) are radii of frustum and h is height of frustum

$= \sqrt{12 * 12 + 16 * 16}$

$= \sqrt{144 + 256}$

= 20 cm

The capacity of container = Volume of a frustum

$= \frac{1}{3} \pi$ h ($r_1^2 + r_2^2 + r_1r_2$)

$= \frac{1}{3} \times 3.14 \times 16 \times [(20)^2 + (8)^2 + (20)(8)]$

$= \frac{1}{3} \times 3.14 \times 16 (400 + 64 + 160)$

$= \frac{1}{3} \times 3.14 \times 16 \times 624$

= 10449.92 cm³

= 10.45 liters

Cost of 1 liter milk = Rs 20

Cost of 10.45 liters' milk = 10.45 × 20

= Rs 209

Area of metal sheet used to make the container = Curved surface area of frustum + area of lower base

$= \pi (r_1 + r_2) l + \pi r_2^2$

$= \pi (20 + 8) 20 + \pi (8)^2$

$= 560\,\pi + 64\,\pi$
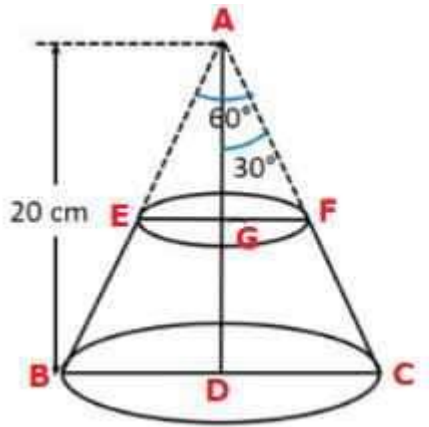
$= 624\,\pi\ cm^2$

Cost of 100 cm² metal sheet = Rs 8

Cost of 624 π cm² metal sheet $= \dfrac{624*3.14*8}{100}$

= 156.75 Rs.

Q. 5 A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\dfrac{1}{16}$ cm find the length of the wire.

Answer:

The figure is given below:

In ΔAEG,

$\tan \theta = \dfrac{perpendicular}{base}$

$\dfrac{EG}{AG} = \tan30°$

$\Rightarrow EG = \tan 30° \times AG$

$\Rightarrow EG = \left(\dfrac{1}{\sqrt{3}}\right) \times 10\ cm$ [tan 30° = 1/ √3]

$$EG = \frac{10}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow EG = \frac{10\sqrt{3}}{3}$$

In $\triangle ABD$,

$$\frac{BD}{AD} = \tan 30°$$

$$\Rightarrow BD = \tan 30° \times AD$$

$$\Rightarrow BD = \left(\frac{1}{\sqrt{3}}\right) \times 20 \text{ cm}$$

$$\Rightarrow BD = \frac{20}{\sqrt{3}}$$

$$\Rightarrow BD = \frac{20\sqrt{3}}{3}$$

Radius $(r_1) = \frac{10\sqrt{3}}{3}$ cm

Radius $(r_2) = \frac{20\sqrt{3}}{3}$ cm

Height $(h) = 10$ cm

Volume of frustum $= (1/3) \pi h (r_1{}^2 + r_2{}^2 + r_1 r_2)$

$$= \frac{1}{3} \times \pi \times 10 \left[\left(\frac{10\sqrt{3}}{3}\right)^2 + \left(\frac{20\sqrt{3}}{3}\right)^2 + \frac{10\sqrt{3}*20\sqrt{3}}{3*3}\right]$$

$$= \frac{10}{3} \pi \left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right]$$

$$= \frac{10}{3} \times \frac{22}{7} \times \frac{700}{3}$$

$$= \frac{22000}{9} \text{ cm}^3$$

Radius (r) of wire $= \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$ cm

Let the length of wire be l.

Volume of wire = Area of cross-section × Length

$= (\pi r^2) (l)$

$= \pi \text{ x } (\frac{1}{32})^2 * l$

Volume of frustum = Volume of wire

$\frac{22000}{9} = \frac{22}{7} \times \left(\frac{1}{32}\right)^2 * l$

$\frac{7000}{9} \times 1024 = l$

$l = 796444.44 \text{ cm}$

$= 7964.44 \text{ meters}$

Q.1 A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3

Answer:

We know that,

No. of rounds $= \dfrac{Height}{Diameter}$

$= \dfrac{12}{0.3} = 40$ rounds

Length of wire required in 1 round $=$ Circumference of base of cylinder

$= 2\pi r$

$= 2\pi \times 5$

$= 10\pi$

Length of wire in 40 rounds $= 40 \times 10\pi$

$= \dfrac{400 * 22}{7}$

$= \dfrac{8800}{7} = 12.57$ cm

Radius of wire $= \dfrac{0.3}{2} = 0.15$ cm

Volume of wire = Area of cross-section of wire × Length of wire
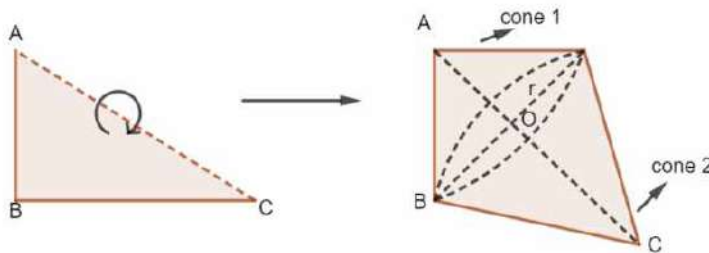
$= \pi(0.15)^2 \times 1257.14$

$= 88.898 \text{ cm}^3$

Mass = Volume × Density

$= 88.898 \times 8.88$

$= 789.41 \text{ gm}$

Q.2 A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose the value of π as found appropriate)

Answer:

Let ABC be the triangle, withAB = 3 cmBC = 4 cmand is revolved around hypotenuse.By Pythagoras theorem,Hypotenuse,

$AC = \sqrt{3*3+4*4}$

$= 5 \text{ cm}$

Area of $\triangle ABC = \frac{1}{2} \times AB \times BC$

$\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 3 \times 4$

$OB = \frac{12}{5} = 2.4$ cm

Volume of double cone = Volume of cone 1 + Volume of cone 2

$$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{2}\pi\ r^2 h_2$$

$$= \frac{1}{3}\pi r^2\ (h_1 + h_2)$$

$$= \frac{1}{3}\pi r^2\ (OA + OC)$$

$$= \frac{1}{3} \times 3.14 \times (2.4)^2\ (5)$$

$= 30.14$ cm³

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi r l_1 + \pi r l_2$

$l_1 = AB = 3$cm

$l_2 = BC = 4$ cm

$= \pi r\ [3 + 4]$

$= 3.14 \times 2.4 \times 7$

$= 52.75$ cm²

Q.3 A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm³ of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Answer:

According to the question:

Volume of cistern = 150 × 120 × 110

[Volume of cube = lbh where l, b and h are length, breadth and height respectively]

= 1980000 cm³

Volume to be filled in cistern = 1980000 - 129600

= 1850400 cm³

Let n numbers of porous bricks were placed in the cistern

Volume of n bricks = n × 22.5 × 7.5 × 6.5

= 1096.875n

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks $= \frac{n}{17}(1096.875)$

$1850400 + \frac{n}{17}(1096.875) = (1096.875)n$

$1850400 = \frac{16n}{17}(1096.875)$

n = 1792.41

Q.4 In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km², show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Answer:

Area of the valley = 7280 km²

If there was a rainfall of 10 cm in the valley then amount of rainfall in the valley = area of the valley × 10 cm

Amount of rainfall in the valley = 7280 km²× 10 cm

[As 1 km = 1000 m = 100000 cm]

$1 \text{ cm} = \dfrac{1}{100000} \text{km}$

Amount of rainfall in one day

$= 7280 \text{ km}^2 \left(\dfrac{10}{100000} km \right)$

= 0.728 km³ ------ [i]

Length of each river, l = 1072 km

The breadth of each river, b = 75 m

$= \dfrac{75}{1000} \text{km}$

Depth of each river, h = 3 m

$= \dfrac{3}{1000} \text{km}$

The volume of each river = $l \times b \times h$

$= 1072 \times \dfrac{75}{1000} \times \dfrac{3}{1000}$

$= \dfrac{241200}{1000000}$

= 0.2412 km³

Volume of three such rivers = 3 × Volume of each river

= 3 × 0.2412 km³

= 0.7236 km³   ------ [ii]

[i] and [ii] are approximately equal.

Q.5 An oil funnel made of tin sheet consists of a10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig. 13.25)

Fig. 13.25
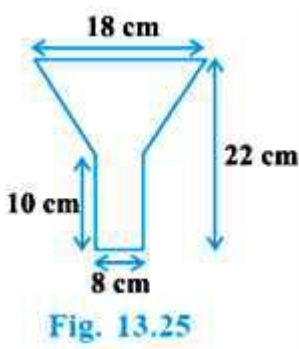
Answer:

Given:

Radius (r_1) of upper circular end of frustum part $= \dfrac{18}{2} = 9$ cm

Radius (r_2) of lower circular end of frustum part = Radius of circular end of cylindricalpart $= \dfrac{8}{2} = 4$ cm

Height (h_1) of frustum part = 22 - 10 = 12 cm

Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part

$= \sqrt{(9-4)*(9-4)+(12)*12}$

$= 13$ cm

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$$= \pi \, (r_1 + r_2) * 1 + 2\pi r_2 h_2$$

$$= \frac{22}{7} \, (9 + 4) * 13 + 2 * \frac{22}{7} * 4 * 10$$

$$= \frac{22}{7} \, [169 + 80]$$

$$= \frac{22*249}{7}$$

$$= 782\frac{4}{7} \, cm^2$$

Q.6 Derive the formula for the curved surface area and total surface area of the frustum of acone, given to you in Section 13.5, using the symbols as explained

Answer:

Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone
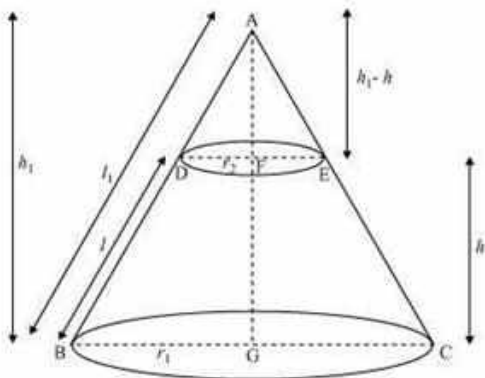
In $\triangle ABG$ and $\triangle ADF$, DF‖BG

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$

$$\frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\frac{l_1}{l} = \frac{r_1}{r_1 - r_2}$$

$$l_1 = \frac{r_1 l}{r_1 - r_2}$$

CSA of frustum DECB = CSA of cone ABC - CSA cone ADE

$$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$$

$$= \pi r_1 \left(\frac{l r_1}{r_1 - r_2} \right) - \pi r_2 \left[\frac{r_1 l}{r_1 - r_2} - l \right]$$

$$= \frac{\pi * r_1 * r_1 * l}{r_1 - r_2} - \frac{\pi * r_2 * r_2 * l}{r_1 - r_2}$$

$$= \pi l \left[\frac{r_1 * r_1 - r_2 * r_2}{r_1 - r_2} \right]$$

CSA = π (r_1 + r_2) * l

Total surface area of frustum = CSA + Area of upper circular end + Area of lower circular end

$$= \pi\,(r_1 + r_2) * 1 + \pi r2^2 + \pi r1^2$$

$$= \pi\,[(r_1 + r_2) * 1 + r1^2 + r2^2]$$

Q.7 Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5,using the symbols as explained
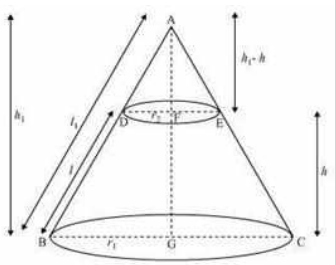
Answer:

Let ABC be a cone.

And,

A frustum DECB is cut by a plane parallel to its base

Now,

Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone

In $\triangle ABG$ and $\triangle ADF$, $DF \| BG$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$

$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\frac{h_1}{h} = \frac{r_1}{r_1 - r_2}$$

$$h_1 = \frac{r_1 h}{r_1 - r_2}$$

Volume of frustum of cone = Volume of cone ABC - Volume of cone ADE

$$= \frac{1}{3}\pi r1^2 h_1 - \frac{1}{3}\pi r2^2 (h_1 - h)$$

$$= \frac{\pi}{3}[r1^2 h_1 - r2^2 (h_1 - h)]$$

$$= \frac{\pi}{3}[r1^2 \left(\frac{h r_1}{r_1 - r_2}\right) - r2^2 \left(\frac{h r_1}{r_1 - r_2} - h\right)]$$

$$= \frac{\pi}{3}\left[\frac{h * r_1 * r_1 * r_1}{r_1 - r_2} - \frac{h * r_2 * r_2 * r_2}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3} *h \left[\frac{r_1 *r_1 *r_1 - r_2 *r_2 *r_2}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3}h \left[\frac{(r_1 - r_2)(r_1 *r_1 + r_2 *r_2 + r_1 *r_2)}{r_1 - r_2}\right]$$

$$= \frac{1}{3}h \left[r1^2 + r2^2 + r_1 r_2\right]$$