Chapter – 12
Areas Related to Circles

Exercise 12.1

Q. 1 The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Answer:

Given: Radius (r_1) of 1st circle = 19 cm

Radius (r_2) or 2nd circle = 9 cm

Now,

Let the radius of 3rd circle be r

Circumference of 1st circle = $2\pi r_1 = 2\pi(19) = 38\pi$ cm

Circumference of 2nd circle = $2\pi r_2 = 2\pi(9) = 18\pi$ cm

Circumference of 3rd circle = $2\pi r$

Given:

Circumference of 3rd circle = Circumference of 1st circle + Circumference of 2nd circle

$2\pi r = 38\pi + 18\pi = 56\pi$ cm

$r = \frac{56\pi}{2\pi}$

= 28

Therefore, the radius of the circle which has circumference equal to the sum of the circumference of the given two circles is 28 cm.

Q. 2 The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer:

Radius (r_1) of 1st circle = 8 cm

Radius (r_2) of 2nd circle = 6 cm

Let the radius of 3rd circle be r

Area of 1^{st}circle = $\pi r_1^2 = \pi (8)^2 = 64\pi$

Area of 2^{nd} circle = $\pi r_2^2 = \pi (6)^2 = 36 \pi$

Given that,

Area of 3rd circle = Area of 1st circle + Area of 2nd circle

$\pi r^2 = \pi r_1^2 + \pi r_2^2$

$\pi r^2 = 64 \pi + 36 \pi$

$\pi r^2 = 100 \pi$

$r^2 = 100$

$r = \pm 10$

But we know that the radius cannot be negative. Hence, the radius of the circle having area equal to the sum of the areas of the two circles is 10 cm.

Q. 3 Fig. 12.3 depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Fig. 12.3

Answer: Radius (r_1) of gold region (i.e., 1st circle) $= \frac{21}{2} = 10.5$ cm

Given: Each circle is 10.5 cm wider than the previous circle.

Hence,

Radius (r_2) of 2nd circle = 10.5 + 10.5

21 cm

Radius (r_3) of 3rd circle = 21 + 10.5= 31.5 cm

Radius (r_4) of 4th circle = 31.5 + 10.5= 42 cm

Radius (r_5) of 5th circle = 42 + 10.5= 52.5 cm

Area of gold region = Area of 1stcircle

$= \pi r_1^2$

$= \pi (10.5)^2$

$= 346.5$ cm^2

Area of red region = Area of 2nd circle - Area of 1stcircle

$= \pi r_2^2 - \pi r_1^2$

$= \pi (21)^2 - \pi (10.5)^2$

$= 441\pi - 110.25\pi$

$= 330.75\pi$

$= 1039.5\ cm^2$

Area of blue region = Area of 3rd circle - Area of 2nd circle

$= \pi r_3^2 - \pi r_2^2$

$= \pi\ (31.5)^2 - \pi\ (21)^2$

$= 992.25\ \pi - 441\ \pi$

$= 551.25\ \pi$

$= 1732.5\ cm^2$

Area of black region = Area of 4th circle - Area of 3rd circle

$= \pi r_4^2 - \pi r_3^2$

$= \pi\ (42)^2 - \pi\ (31.5)^2$

$= 1764\ \pi - 992.25\ \pi$

$= 771.75\ \pi$

$= 2425.5\ cm^2$

Area of white region = Area of 5th circle - Area of 4th circle

$= \pi r_5^2 - \pi r_4^2$

$= \pi\ (52.5)^2 - \pi\ (42)^2$

$= 2756.25\ \pi - 1764\ \pi$

$= 992.25\ \pi$

$= 3118.5\ cm^2$

Therefore, areas of gold, red, blue, black, and white regions are 346.5 cm^2, 1039.5 cm^2, 1732.5 cm^2, 2425.5 cm^2, and 3118.5 cm^2 respectively.

Q. 4 The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

The diameter of the wheel of the car = 80 cm

Radius (r) of the wheel of the car = 40 cm

Circumference of wheel = $2\pi r$

$\Rightarrow 2\pi(40) = 80\pi$ cm

Speed of car = 66 km/hour

1 km = 1000 m and 1m = 100 cm

$\Rightarrow$ 1 km = 100000 cm $\Longrightarrow \frac{66\times100000}{60} = 110000\ cm/mi\ n$

= 110000 cm/min

As distance= speed × time

Distance traveled by car in 10 minutes

= 110000 × 10

= 1100000 cm

Let the number of revolutions of the wheel of the car be "n".

n × Distance travelled in 1 revolution (i.e., circumference)

(In one revolution of the wheel, a wheel covers the distance equal to its circumference)

= Distance travelled in 10 minutes

n × 80 π = 1100000

$n = \frac{1100000*7}{80*22}$

$= \frac{35000}{8}$

= 4375

Therefore, each wheel of the car will make 4375 revolutions.

Q. 5 Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

A. 2 units
B. π units
C. 4 units
D. 7 units

Answer: Let the radius of the circle be r

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given that, the circumference of the circle and the area of the circle are equal.

This implies $2\pi r = \pi r^2$

$2 = r$

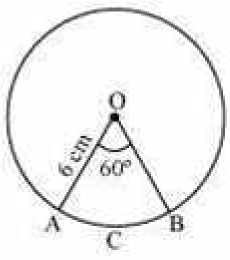
Therefore, the radius of the circle is 2 units

Hence, the correct answer is A

Exercise 12.2

Q. 1 Find the area of a sector of a circle with radius 6 cm if the angle of the sector is 60°.

Answer: Let OACB be a sector of the circle making 60° angle at centre O of the circle



Area of sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

Area of sector OACB $= \frac{60}{360} * \frac{22}{7} * 6 * 6$

$= \frac{132}{7}\ cm^2$

Hence,

The area of the sector of the circle making 60° at the centre of the circle is $\frac{132}{7}\ cm^2$.

Q. 2 Find the area of a quadrant of a circle whose circumference is 22 cm.

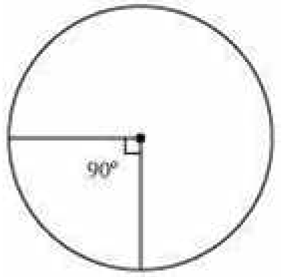
Answer: To find: Area of the quadrant of the circle
Given: Circumference of the circle
Let the radius of the circle be r

Circumference = 22 cm

Circumference = $2\pi r$
Therefore, $2\pi r = 22$

$r = \frac{22}{2\pi}$

$= \frac{11}{\pi}$



Quadrant of circle means 1/4 th of the circle.

Area of such quadrant of the circle $= \frac{1}{4}\ \pi r^2$

$= \frac{1}{4}\pi\left(\frac{11}{\pi}\right)^2$

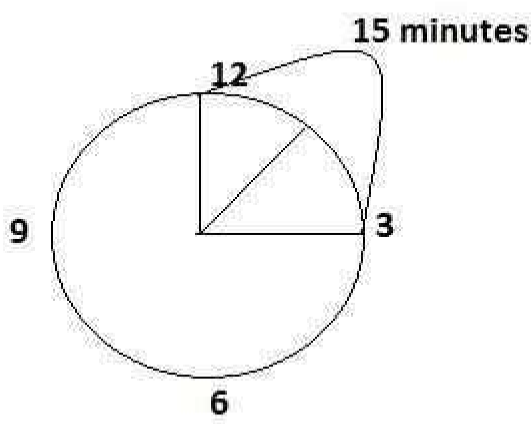
$\frac{121}{\pi}$

$= \frac{121\times7}{4\times22}$

$= \frac{77}{8}\ cm^2$

Hence, The area of the quadrant of the circle is $\frac{77}{8}\ cm^2$

Q. 3 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.



So 15 minutes $= 90°$ 1 min $= \frac{90°}{15}$

$= 6°$

5 min = 5 × 6°= 30°So 5 minutes subtend an angle of 30°.

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of 30° in a circle of 14 cm radius.

Area swept by min hand = Area of sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30°}{360°} \times \frac{22}{7} \times (14)^2$

$= \frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14$

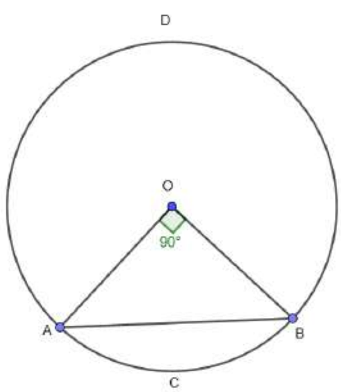
$= \frac{154}{3}\ cm^2$

Q. 4 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector (Use π = 3.14)

Answer:



Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

$Area\ of\ sector\ OACB\ = \frac{90°}{360°} \times 3.14 \times 10 \times 10$

$$= \frac{\cancel{90}^{1}}{\cancel{360}^{4}} \times 3.14 \times 10 \times 10$$

$= \frac{90°}{360°} \times 3.14 \times 10 \times 10$

$$= \frac{1}{\cancel{4}\ \cancel{2}1} \times 3.14 \times \cancel{10}^{5} \times \cancel{10}^{5}$$

$$= \frac{1}{4} \times 3.14 \times 10^5 \times 10^5$$

= 3.14 × 25= 78.5 cm^2

OAB is a right angles triangle, Area of triangle OAB = 1/2 (base x height)

Area of ΔOAB $=\frac{1}{2} \times$ OA × OB

$=\frac{1}{2} \times 10 \times 10$

= 50 cm^2

Area of minor segment ACB = Area of minor sector OACB -Area of ΔOAB

= 78.5 - 50

= 28.5 cm^2

ii) Let AB be the chord of the circle subtending 90° angle at centre O of the circle.

Area of major sector OACB =

$= \left(\frac{360°-90°}{360°}\right) \times \pi r^2$

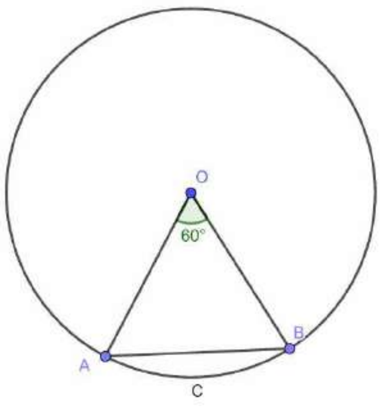
$= \left(\frac{270°}{360°}\right) \times \pi r^2$

=3/4 × 3.14 × 10 × 10

=235.5cm^2

Q. 5 In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc
(ii) Area of the sector formed by the arc
(iii) Area of the segment formed by the corresponding chord.



Radius (r) of circle = 21 cm

The angle subtended by the given arc = 60°

Length of an arc of a sector of angle θ $= \frac{\theta}{360°} \times 2\pi r$

(i) Length of arc ACB $= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= \frac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

(ii) Area of sector OACB $= \frac{\theta}{360°} \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

= 231 cm^2

(iii) In ΔOAB,

As OA = OB

⇒ ∠OAB = ∠OBA (Angles opposite to equal sides are equal)

⇒ ∠OAB + ∠AOB + ∠OBA = 180°

⇒ 2∠OAB + 60° = 180°

⇒ 2∠OAB = 180° - 60°

$\Rightarrow \angle OAB = 60°$

Hence,

ΔOAB is an equilateral triangle

Area of equilateral triangle $= \frac{\sqrt{3}}{4}(Si\ de)^2$

$\Rightarrow$ Area of $\Delta OAB = \frac{\sqrt{3}}{4}\ (Si\ de)^2$

$\frac{\sqrt{3}}{4} \times (21)^2$

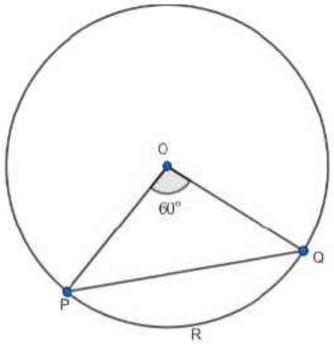
$= \frac{441\sqrt{3}}{4}\ cm^2$

Area of segment ACB = Area of sector OACB - Area of ΔOAB

$= \left(231 - \frac{441\sqrt{3}}{4}\right)\ cm^2$

Q. 6 A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

(Use π = 3.14 and $\sqrt{3}$= 1.73)



Radius (r) of circle = 15 cm

Area of sector OPRQ = $(60/360) \times \Pi r^2$

$= 1/6 \times 3.14 \times (15)^2$

$= 117.75\ cm^2$

In ΔOPQ,

∠OPQ = ∠OQP (As OP = OQ)

∠OPQ + ∠OQP + ∠POQ = 180°

2 ∠OPQ = 120°

∠OPQ = 60°

ΔOPQ is an equilateral triangle.

Area of ΔOPQ $= \frac{\sqrt{3}}{4} \times (Si\ de)^2$

$= \frac{\sqrt{3}}{4} \times (15)^2$

$= \frac{225\sqrt{3}}{4}$

$= 56.25 \times \sqrt{3}$

= 97.3125 cm^2

Area of segment PRQ = Area of sector OPRQ - Area of ΔOPQ

= 117.75 - 97.3125

= 20.4375 cm^2

Area of major segment PSQ = Area of circle - Area of segment PRQ

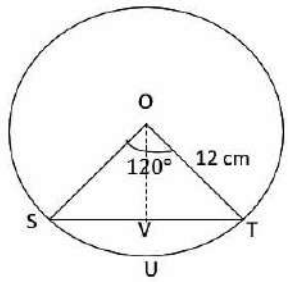
= π(15)^2 – 20.4375

= 3.14 ×225 – 20.4375

= 706.5 – 20.4375

= 686.0625 cm^2

Q. 7 A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)



Let us draw a perpendicular OV on chord ST. It will bisect the chord ST and the angle O.

SV = VT

In ΔOVS,

As, $\cos\theta = \frac{Base}{Hypotenuse}$

$\frac{OV}{OS} = \cos 60°$

$\frac{OV}{12} = 1$

OV= 6 cm

$\frac{sv}{so} = \sin 60°$

$\frac{sv}{12} = \frac{\sqrt{3}}{2}$

SV = 6√3 cm

ST = 2 × SV

= 2 × 6√3

= 12√3 cm

Area of ΔOST $= \frac{1}{2} \times 12\sqrt{3} \times 6$

= 36√3

= 36 × 1.73

= 62.28 cm^2

Area of sector OSUT $= \frac{120}{360} \times \pi \times (12)^2$

$= \frac{1}{3} \times 3.14 \times 144$

$= 150.72 \text{ cm}^2$

Area of segment SUT = Area of sector OSUT - Area of ΔOST

$= 150.72 - 62.28$

$= 88.44 \text{ cm}^2$

Q. 8 A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find

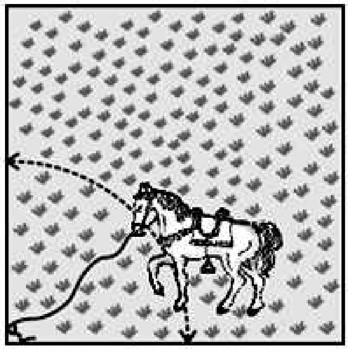


Fig. 12.11

(i) The area of that part of the field in which the horse can graze
(ii) The increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Answer: From the figure, it can be observed that the horse can graze a sector of 90° in a circle of 5 m radius

(i) Area that can be grazed by horse = Area of sector

$= \frac{90}{360} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625 \text{ m}^2$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625 \text{ m}^2$

The area that can be grazed by the horse when the length of rope is 10 m long = $\frac{90}{360} \times \pi\ (10)^2$

$= \frac{1}{4} \times 3.14 \times 100$

= 78.5 m^2

(ii) Increase in grazing area = Area grazed by a horse now - Area grazed previously
(78.5 - 19.625)

= 58.875 m^2

Q. 9 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12. Find:
(i) The total length of the silver wire required
(ii) The area of each sector of the brooch

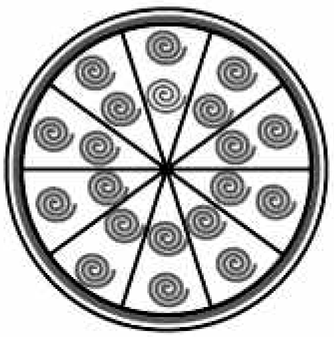


Fig. 12.12

(i) To Calculate total length of wire required to make brooch, we need to find Circumference of Brooch and the length of 5 diameters of Brooch.

Diameter of Circle = 35 mm $Radius = \frac{Diameter}{2}$

Radius of Circle = 35/2 mm Circumference of Circle = 2 π r

$Circumference of circle = 2 \times \frac{22}{7} \times \frac{35}{2}$
$Circumference of circle = 110\ mm$

Total Length of wire required = Circumference of Circle + 5 x Diameter of Circle Total Length of wire required = 110 + 5 x 35Total Length of wire required = 285 mm

(ii) A complete Circle subtends an angle of 360°10 sectors = 360°1 sector will subtend = 36°.

Area of sector of Circle $= \frac{\theta}{360°}\pi r^2$

Area of Each Sector of Brooch $= \frac{36°}{360°} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$

Area of Each sector $= \frac{385}{4}\ mm^2$

Q. 10 An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

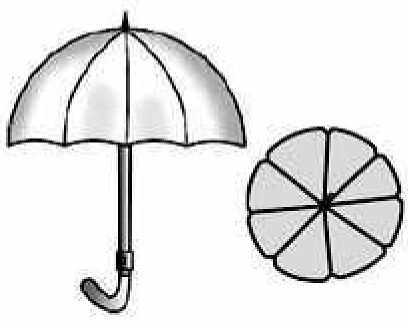


Fig. 12.13

Answer:

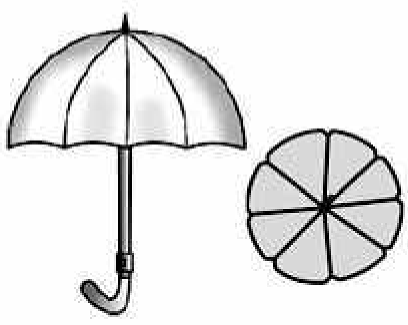


Fig. 12.13

There are 8 ribs in the given umbrella. The area between two consecutive ribs is subtending $\frac{360°}{8} = 45°$ at the centre of the assumed flat circle

Now we know that area of a sector of a circle subtending an angle x is given by

Area $= \frac{x}{360°}\pi r^2$

Now as the value of x for given umbrella is 45°

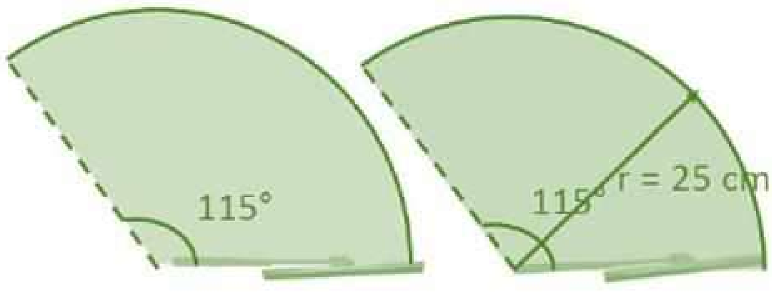
Area between two consecutive ribs $= \frac{45°}{360°} \times \frac{22}{7} \times 45 \times 45$

Area between two consecutive ribs $= \frac{11 \times 2025}{28}$

Area between two consecutive ribs $= \frac{22275}{28}\ cm^2$

Q. 11 A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Answer:



It can be seen from the figure that each blade of wiper would sweep an area of a sector of 115° in a circle with 25 cm radius

Area of sector $= \frac{\theta}{360°}\pi r^2$

$= \frac{115°}{360°}\pi(25)^2$

$= \frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25$

$= \frac{1581250}{2520}\ cm^2$

Area swept by 2 blades $= 2 \times \frac{1581250}{2520}\ cm^2$

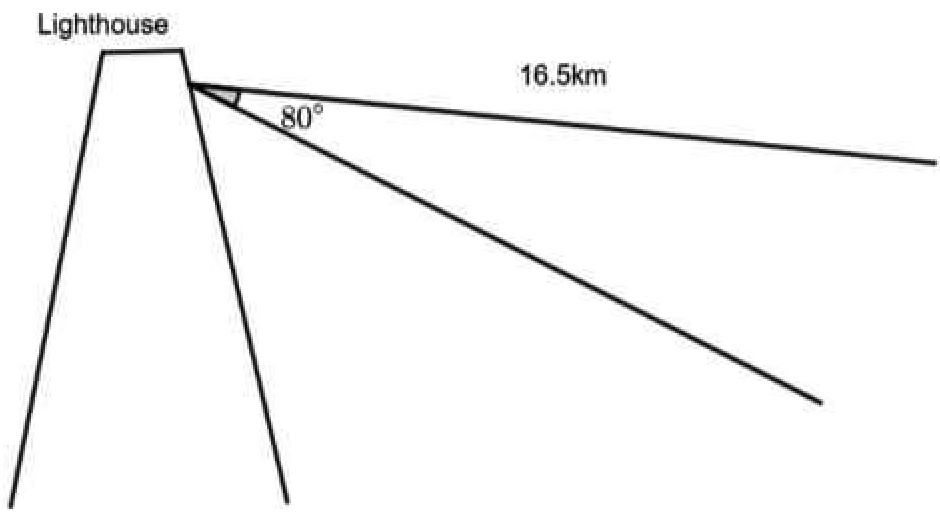
$= \frac{1581250}{2520}\ cm^2$

$= 1254.96\ cm^2$

$=1255\ cm^2$ (approx)

Q. 12 To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14).

Answer:



It can be seen from the figure that the lighthouse spreads light across a sector of 80° in a circle of 16.5 km radius

Area of sector OACB $= \frac{80}{360} \times \pi r^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

$= 189.97\ km^2$

Q. 13 A round table cover has six equal design as shown in Fig.12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2. (Use $= \sqrt{3} = 1.7$)

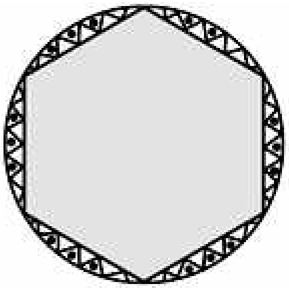
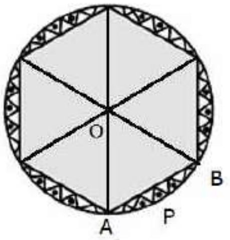


Fig. 12.14

Answer:



It can be concluded that these designs are segments of the circle.

Let us take segment APB.

Chord AB is a side of the regular hexagon.

And,

Each chord will substitute $\frac{360}{6}$ = 60° at the centre of the circle

In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠AOB = 60°

∠OAB + ∠OBA + ∠AOB = 180°

2∠OAB = 180° - 60°
= 120°

∠OAB = 60°

Hence,

ΔOAB is an equilateral triangle.

Area of ΔOAB $= \frac{\sqrt{3}}{4} \, (Si\ de)^2$

$= \frac{\sqrt{3}}{4} \times (28)^2$

$= 196\sqrt{3}$

= 333.2 cm^2

Area of sector OAPB = $\frac{60}{360} \times \pi r^2$

= $\frac{1}{6} \times \frac{22}{7} \times 28 \times 28$

$= \frac{1232}{3} cm^2$

Now,

Area of segment APB = Area of sector OAPB - Area of ΔOAB

$= \left(\frac{1232}{3} - 333.2\right) cm^2$

Area of design $= 6 \times \left(\frac{1232}{3} - 333.2\right)$

= 2464 – 1999.2

= 464.8 cm^2

Cost of making 1 cm^2 designs = Rs 0.35

Cost of making 464.76 cm^2 designs = 464.8 × 0.35

= Rs 162.68

Hence, the cost of making such designs would be Rs 162.68.

Q. 14 Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

A. $\frac{P}{180} \times 2\pi R$

B. $\frac{P}{180} \times \pi R^2$

C. $\frac{P}{380} \times 2\pi R$

D. $\frac{P}{720} \times 2\pi R^2$

Answer: Area of sector of angle θ $= \frac{\theta}{360} \times \pi R^2$

Where θ = angle, r = radius of circle

Here θ = p and radius = R

Area of sector of angle p$= \frac{p}{360} \times (\pi R^2)$

Multiply and divide by 2,

Area of sector of angle p $= (\frac{p}{360})(2\pi R^2)$

Hence, (D) is the correct answer

Exercise 12.3

Q. 1 Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

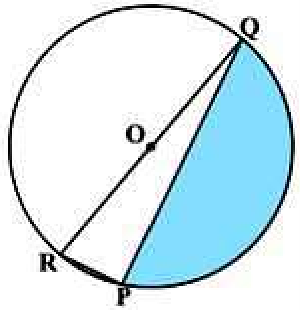
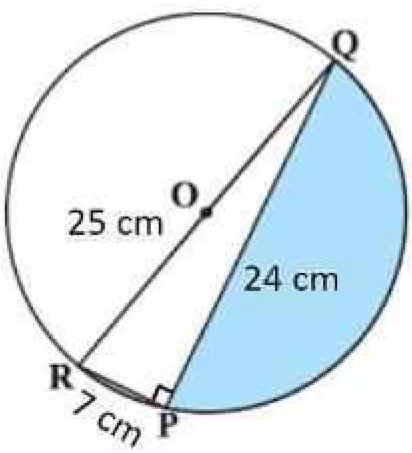


Fig. 12.19

Answer:



As, ΔPQR is in the semicircle and we know angle in the semicircle is a right angle, therefore PQR is a right-angled triangle.[If an **angle** is inscribed in a **semi-circle**, that **angle** measures 90 degrees.]
Here, QR is the hypotenuse of ΔPQR as lines from two ends of diameter always make a right angle when they meet at the circumference of that circle.

Hence by Pythagoras theorem we get, **Pythagoras Theorem : It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.**

$QR^2 = PQ^2 + PR^2$

$QR^2 = 24^2 + 7^2$

$QR^2 = 576 + 49$

QR = 25 cm

Diameter = 25 cm
Or, radius = 12.5cm

Area of right angled triangle triangle $= \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 24 \times 7$

$= 12 \times 7 = 84\ cm^2$

Area of semicircle $= \frac{1}{2}\pi r^2$

Area of semicircle $= \frac{1}{2} \times 3.14 \times (12.5)^2$

= 245.3125 sq cm

Hence,

Area of shaded region = Area of semicircle - area of ΔPQR
= 245.3125 – 84

Area of shaded region = 161.3125 sq cm

Q. 2 Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠ AOC = 40°.

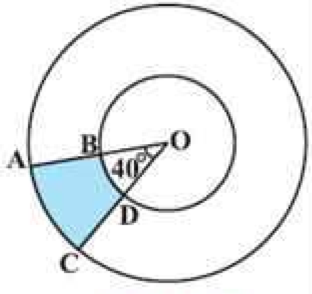


Fig. 12.20

Area of the shaded region = Area of Bigger sector – Area of Smaller Sector

$Area\ of\ a\ sector = \frac{\theta}{360°} \times \pi r^2$

Where $\theta = angl\ esubtended\ by\ sector$

r = radius of sector

If r_2 and r_1 are the two radii of Bigger and Smaller circles respectively, then:
Area of bigger sector = 40/360 πr_2^2
Area of smaller sector = 40/360 πr_1^2

Area of shaded region = $\frac{40°}{360°} \times \frac{22}{7} \times (r_2{}^2 - r_1{}^2)$

Area of shaded region $= \frac{1}{9} \times \frac{22}{7} \times (14^2 - 7^2)$

Area of shaded region = 51.33 cm^2

Q. 3 Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

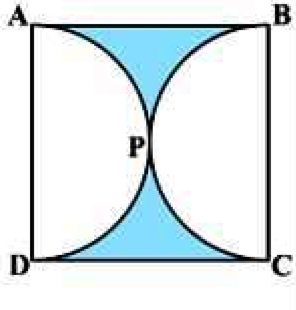


Fig. 12.21

Answer: It can be observed from the figure that the radius of each semi-circle is 7 cm

Area of each semi-circle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} * \frac{22}{7} * (7)^2$

= 77 cm^2

Area of square ABCD = (Side)^2

$= (14)^2$

$= 196\ cm^2$

Area of the shaded region= Area of square ABCD - Area of semi-circle APD - Area of semi-circle BPC

= 196 - 77 - 77

= 196 - 154

$= 42\ cm^2$

Q. 4 Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

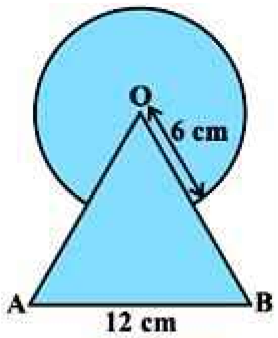


Fig. 12.22

Answer: We know that each interior angle of an equilateral triangle is of measure 60°

Area of sector $= \frac{60}{360}\pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}\ cm^2$

Area of triangle OAB $= \frac{\sqrt{3}}{4} \times (12)^2$

$= \frac{\sqrt{3} \times 12 \times 12}{4}$

$36\sqrt{3}\ cm^2$

Area of circle = πr^2

$= \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7} cm^2$

Area of shaded region = Area of ΔOAB + Area of circle - Area of sector

$= 36\sqrt{3} + \frac{792}{7} - \frac{132}{7}$

$= \left(36\sqrt{3} + \frac{660}{7}\right)\ cm^2$

Q. 5 From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

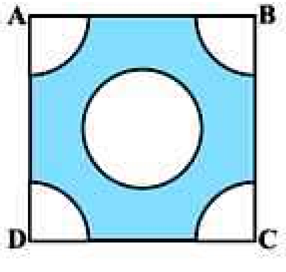


Fig. 12.23

Answer: Each quadrant is a sector of 90° in a circle of 1 cm radius

Area of each quadrant $= \frac{90}{360} * \pi r^2$

$= \frac{1}{4} * \frac{22}{7} * (1)^2$

$= \frac{22}{28}\ cm^2$

Area of square = $(\text{Side})^2$

$= (4)^2$

$= 16\ cm^2$

Area of circle = $\pi r^2 = \pi (1)^2$

$= \frac{22}{7} cm^2$

Area of the shaded region = Area of square - Area of circle - 4 × Area of quadrant

$= 16 - \frac{22}{7} - 4 * \frac{22}{28}$

$= 16 - \frac{22}{7} - \frac{22}{7}$

$= 16 - \frac{44}{7}$

$= \frac{112-44}{7}$

$= \frac{98}{7} \ cm^2$

Q. 6 In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design (shaded region).

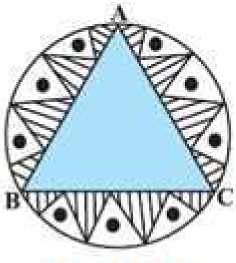


Fig. 12.24

Answer: Radius of the circle "R" = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.
⇒ BD = AB/2

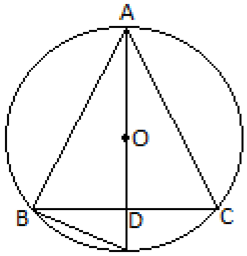
Since, AD is the median of the triangle

∴ AO = Radius of the circle = 2/3 AD [By the property of equilateral triangle inscribed in a circle]

⇒ 2/3 AD = 32 cm

$\Rightarrow$ AD = 48 cm

In ΔADB,



By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = 48^2 + (AB/2)^2$

$\Rightarrow AB^2 = 2304 + AB^2/4$

$\Rightarrow 3/4\ (AB^2) = 2304$

$\Rightarrow AB^2 = 3072$

$\Rightarrow AB = 32\sqrt{3}$ cm

Area of $\Delta ABC = \sqrt{3}/4 \times (32\sqrt{3})^2\ cm^2$

$= 768\sqrt{3}\ cm^2$

Area of circle = πR^2

$= 22/7 \times 32 \times 32 = 22528/7\ cm^2$

Area of the design = Area of circle - Area of ΔABC

$= (22528/7 - 768\sqrt{3})\ cm^2$

Q. 7 In Fig. 12.25, ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

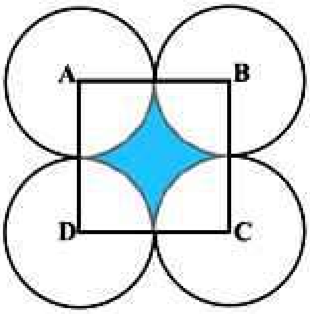


Fig. 12.25

Answer: To Find: Area of shaded region
Given: Side of square ABCD = 14 cm
Radius of circles with centers A, B, C and D = 14/2 = 7 cm

Area of shaded region = Area of square - Area of four sectors subtending right angle

Area of each of the 4 sectors is equal to each other and is a sector of 90° in a circle of 7 cm radius. So, Area of four sectors will be equal to Area of one complete circle

Area of 4 sectors = πr^2

Area of 4 sectors $= \frac{22}{7} \times 7 \times 7$

Area of 4 sectors = 154 cm^2

Area of square ABCD = $(Side)^2$

Area of square ABCD = $(14)^2$

Area of square ABCD = 196 cm^2

Area of shaded portion = Area of square ABCD - 4 × Area of each sector

= 196 – 154

= 42 cm^2

Therefore, the area of shaded portion is 42 cm^2

Q. 8 Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

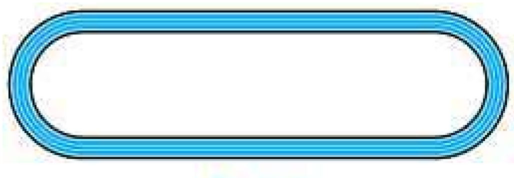


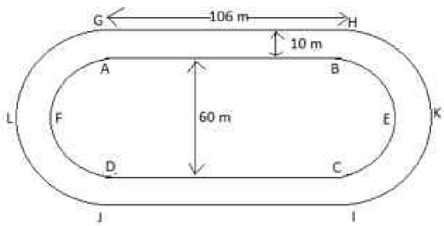
Fig. 12.26

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(i) The distance around the track along its inner edge

(ii) The area of the track.

Answer:



(i) Distance around the track along its inner edge = AB + semicircle BEC + CD + semicircle DFA

$= 106 + \left(\frac{1}{2} \times 2\pi r\right)$