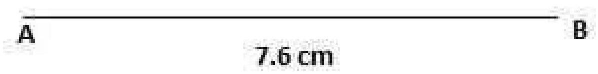
Chapter – 11
Constructions

Exercise 11.1

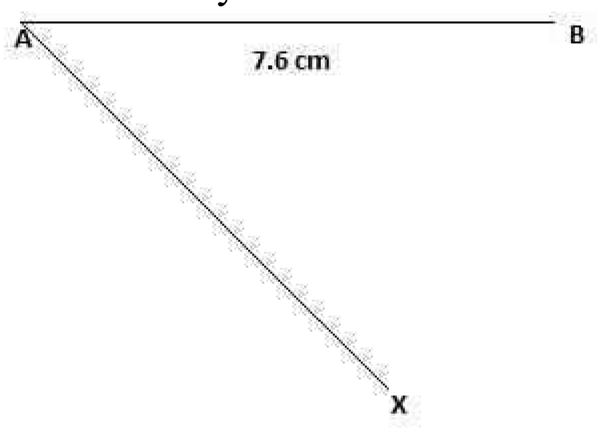
Q. 1 Draw a line segment of length 7.6 cm and divide it in the ratio 5: 8. Measure the two parts.

Steps of construction:

i. At first, we will draw a line segment AB = 7.6 cm.

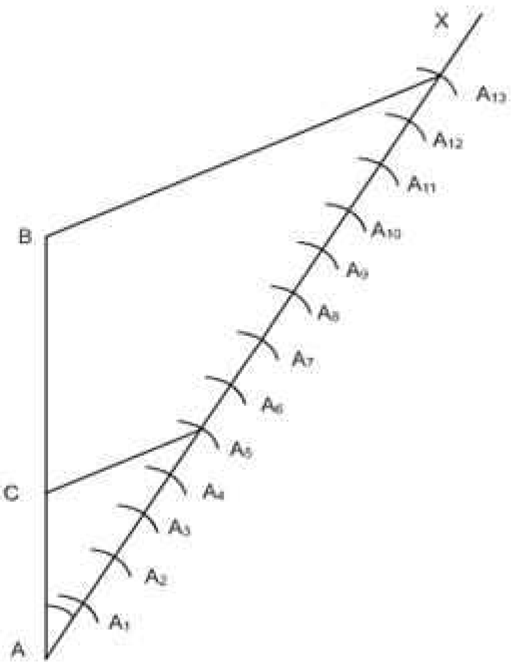


ii. Draw a ray AX such that it makes an acute angle with AB.



iii. Now, locate 13 points (5 + 8) A_1, A_2, A_3, …….A_{13} on AX so that;

$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7 = A_7A_8 = A_8A_9 = A_9A_{10} = A_{10}A_{11} = A_{11}A_{12} = A_{12}A_{13}$



iv. Join A_{13} to B

v. Draw a line $A_5C \parallel A_{13}B$; which intersects AB at point C and passes Through the point A_5.

Now we have, AC : CB = 5 : 8
On measuring with the scale.

AC = 2.92 cm

And

CB = 4.68 cm
Justification:
Since $\angle AA_{13}B = \angle AA_5C$
With AX is a transversal, corresponding angles are equal,
Hence $A_{13}B$ and A_5C are parallel.
So by basic proportionality theorem, $\frac{AA_5}{A_5A_{13}} = \frac{AC}{CB}$

By construction $\frac{AA_5}{A_5A_{13}} = \frac{5}{8}$

So, $\frac{AC}{CB} = \frac{5}{8}$

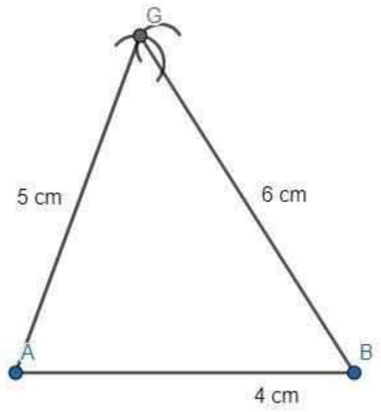
Hence C divides AB in the ratio 5:8.

Q. 2 Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Answer: Steps of construction:

i. Now in order to make a triangle, draw a line segment AB = 4 cm.

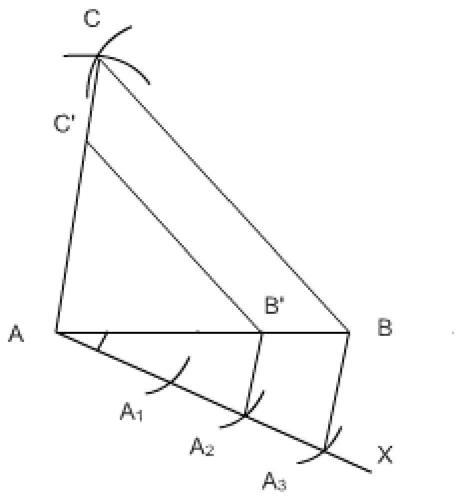
ii. Then, draw an arc at 5 cm from point A. From point B, draw an arc at 6 cm in such a way that it intersects the previous arc. Join the point of intersection from points A and B. This gives the required triangle ABC.



iii. Now, divide the base in the ratio 2 : 3. Draw a ray AX which is at an acute angle from AB. Then, plot three points on AX such that; $AA_1 = A_1A_2 = A_2A_3$. Join A_3 to B.

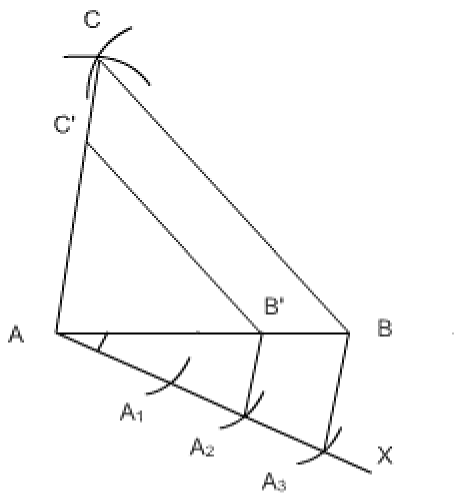
iv. Draw a line from point A_2 parallel to A_3B and intersects AB at point B'.

v. Draw a line from point B' that is parallel to BC intersecting AC at point C'.



Hence, triangle AB'C' is the required triangle.

justification:



Since the scalar factor is 2/3

We need to prove:

$$\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC} = \frac{2}{3}$$

By construction,

$$\frac{AB'}{AB} = \frac{AA_2}{AA_3} = \frac{2}{3} \qquad \ldots.(1)$$

Also, B'C' is parallel to BC.

So, both will make same angle with AB.

∴ ∠AB'C'=∠ABC (corresponding angles)

In ΔAB'C' and Δ ABC

∠A = ∠A (common)

∠AB'C'=∠ABC

So, by AA similarity

ΔAB'C' ~ Δ ABC

As we know if two triangles are similar the ratio of their corresponding sides are also equal.

So,

$$\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC}$$

From (1),

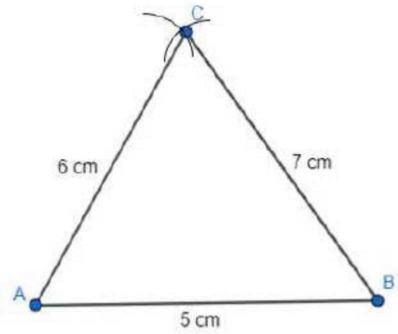
$$\frac{AB'}{AB} = \frac{AC'}{AC} = \frac{B'C'}{BC} = \frac{2}{3}$$

Hence proved.

Q. 3 Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

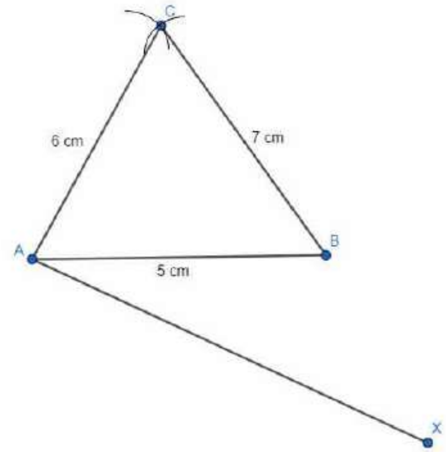
Answer: 1. Now in order to make a triangle, draw a line segment AB = 5 cm. From point A, draw an arc at 6 cm. Draw an arc at 7 cm from point B intersecting the previous arc.

2. Join the point of intersection from A and B.



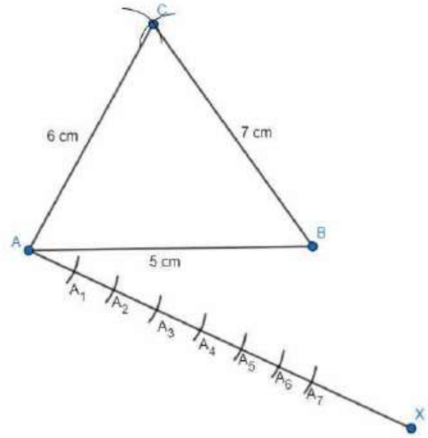
Hence, this gives the required triangle ABC

3. Dividing the base, draw a ray AX which is at an acute angle from AB



4. Plot seven points on AX such that:

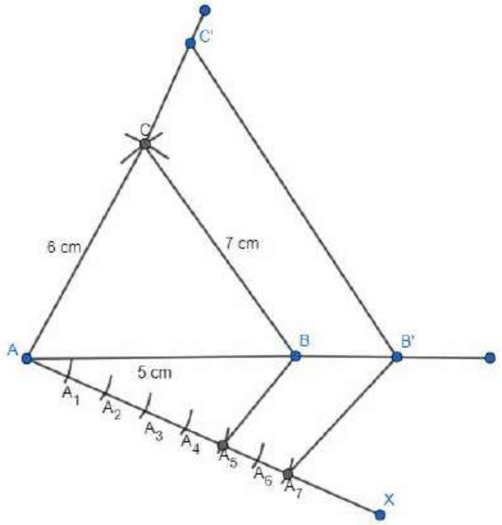
$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.



5. Join A_5 to B.

6. Draw a line from point A_7 that is parallel to A_5B and joins AB' (AB extended to AB').

7. Draw a line B'C' ∥ BC.



Hence, triangle AB'C' is the required triangle.

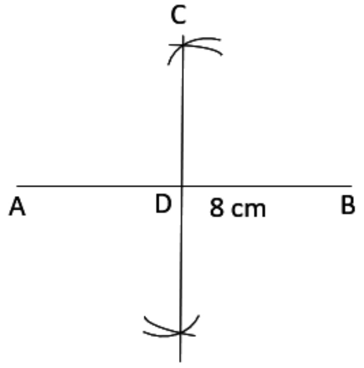
Q. 4 Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Answer: Steps of construction:

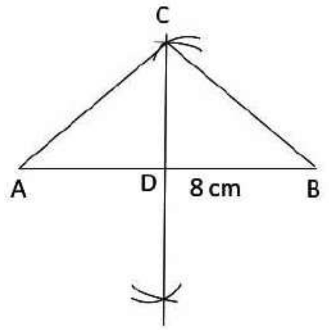
i. Now in order to make a triangle, draw a line segment AB = 8 cm.

ii. Draw two arcs intersecting at 4 cm distance from points A and B; on either side of AB.

Join these arcs to get perpendicular bisector CD of AB. (Since, altitude is the perpendicular bisector of base of isosceles triangle).



iii. Join points A and B to C in order to get the triangle ABC.

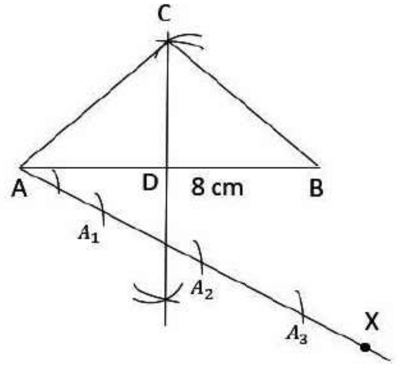


iv. Now, draw a ray AX which is at an acute angle from point A.

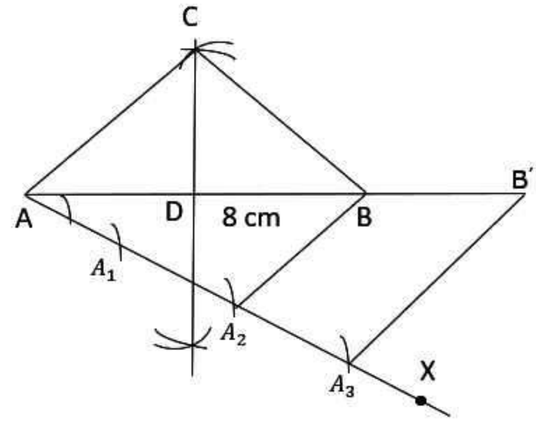
As $1\frac{1}{2} = \frac{3}{2}$

And 3 is greater between 3 and 2, So Plot 3 points on AX such that:

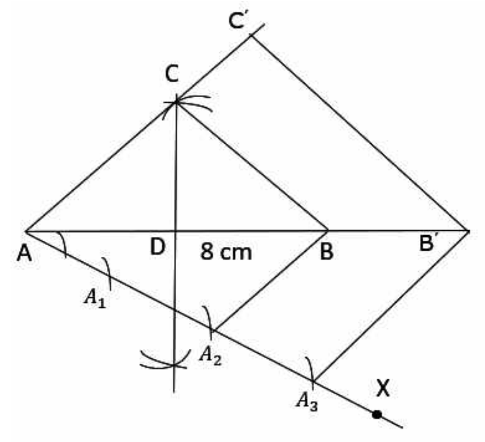
$AA_1 = A_1A_2 = A_2A_3$.



v. As 2 is smaller between 2 and 3. Join A_2 to point B.
Draw a line from A_3 which is parallel to A_2B meeting the extension of AB at B'.



vi. Draw B'C' ∥ BC. Then, draw A'C' ∥ AC.



Triangle AB'C' is the required triangle.

Justification:

We need to prove,

$$\frac{AC'}{AC} = \frac{AB'}{AB} = \frac{C'B'}{CB} = \frac{3}{2}$$

By construction $\frac{AB'}{AB} = \frac{AA_3}{AA_2} = \frac{3}{2}$ …. (1)

s C'B' ∥ CB

They will maker equal angles with line AB.∠ACB = ∠AC'B' ..... (corresponding angles)

In ΔACB and ΔAC'B'

∠A = ∠A (common) ∠ACB = ∠AC'B' (corresponding angles)

So ΔACB ~ ΔAC'B'

As corresponding sides of similar triangles are in ratio,Hence, $\frac{AC\prime}{AC} =$
$\frac{AC\prime}{AC} = \frac{AB\prime}{AB} = \frac{C\prime B\prime}{CB} = \frac{3}{2}$

Q. 5 Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$of the corresponding sides of the triangle ABC.

Answer:

Given in Δ ABC,

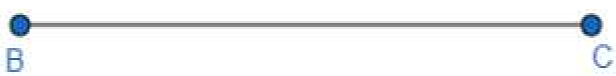
- Length of side BC = 6 cm.
- Length of side AB = 5 cm.
- ∠ ABC = 60°.

<u>Steps of Construction</u>:

1. Draw a line segment BC of length 6 cm.

2. With B as center, draw a line which makes an angle of 60° with BC.

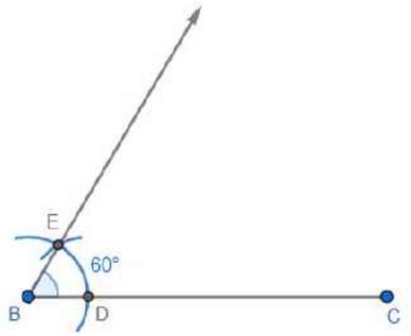
Construction of 60° angle at B:
a. With B as centre and with some convenient radius draw an arc which cuts the line BC at



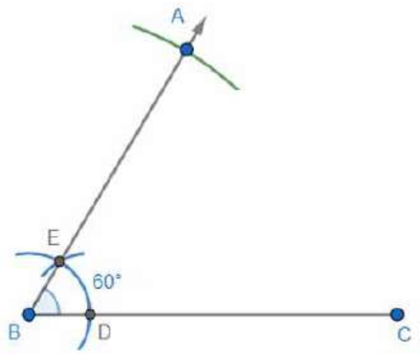
D.

b. With D as radius and with same radius (in step a), draw another arc which cuts the previous arc at E.

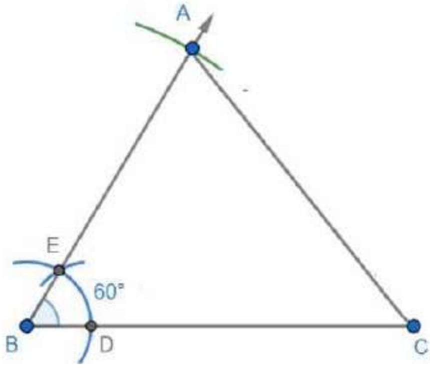
c. Join BE. The line BE makes an angle 60° with BC.



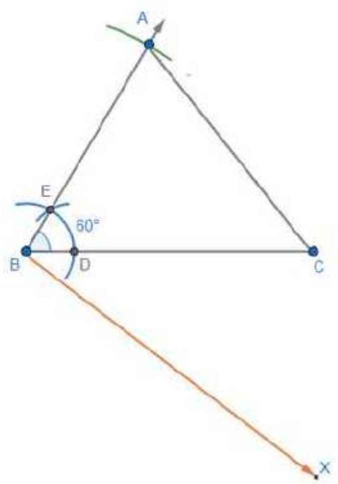
3. Again with B as centre and with radius of 5 cm, draw an arc which intersects the line BE at point A.



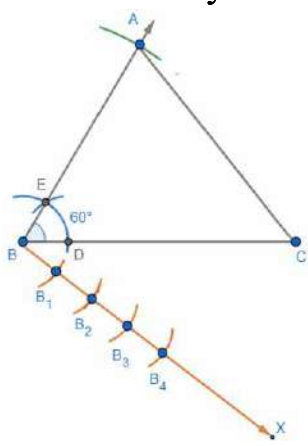
4. Join AC. This is the required triangle.



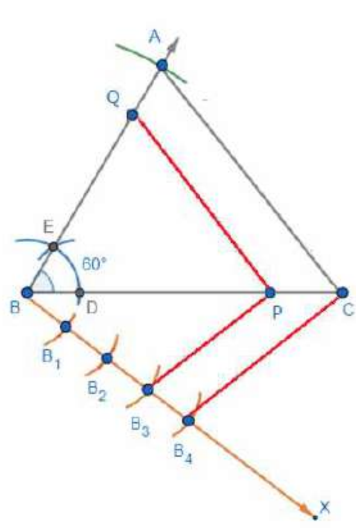
5. Now, from B, draw a ray BX which makes an acute angle on the opposite side of the vertex A.



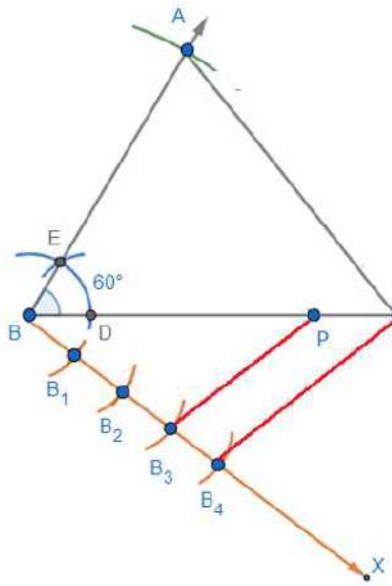
6. With B as center, mark four points B_1, B_2, B_3 and B_4 on BX such that they are equidistant. i.e. $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.



7. Join B_4C and then draw a line from B_3 parallel to B_4C which meets the line BC at P.



8. From P, draw a line parallel to AC and meets the line AB at Q. Thus ΔBPQ is the required triangle.



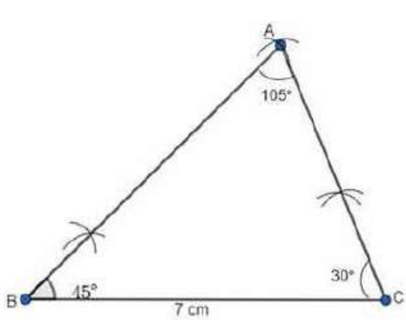
Q. 6 Draw a triangle ABC with side BC = 7 cm, ∠ B = 45°, ∠ A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of Δ ABC.

Answer:

Steps of construction:

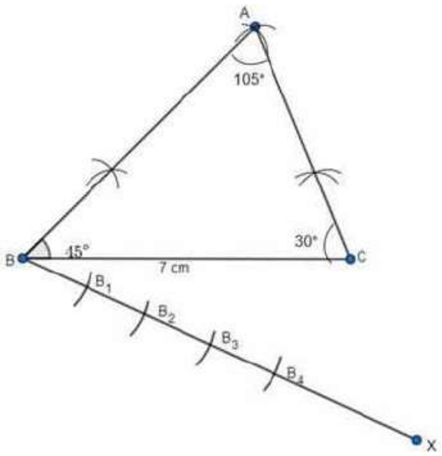
1. Draw a line segment BC = 7 cm.

2. Draw ∠ABC = 45° and ∠ACB = 30° i.e. ∠BAC = 105°.



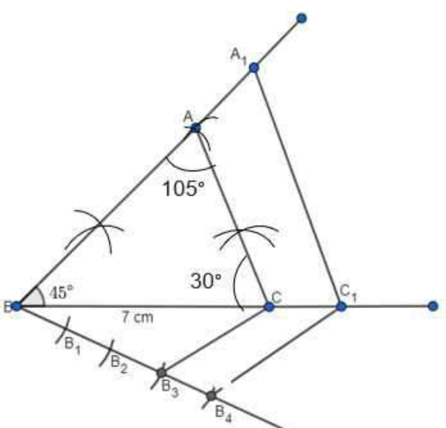
We obtain ΔABC.

3. Draw a ray BX making an acute angle with BC. Mark four points B_1, B_2, B_3, B_4 at equal distances.



4. Through B_3 draw B_3C and through B_4 draw B_4C_1 parallel to B_3C.

Then draw A_1C_1 parallel to AC.



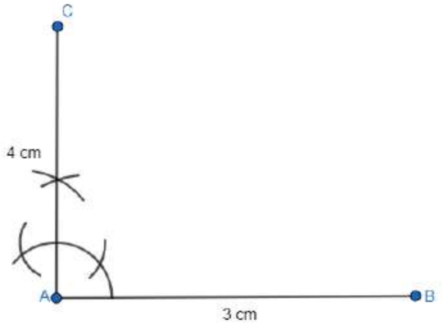
∴ A_1BC_1 is the required triangle.

Q. 7 Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

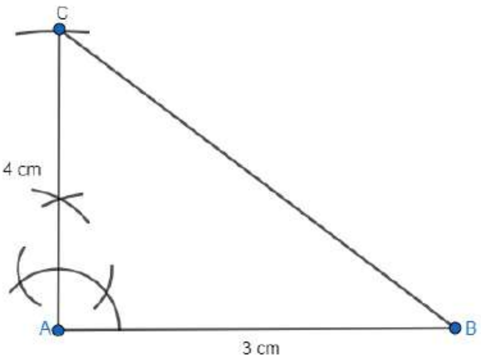
Answer: Steps of construction:

1. Now in order to make a triangle, draw a line segment AB = 3 cm.

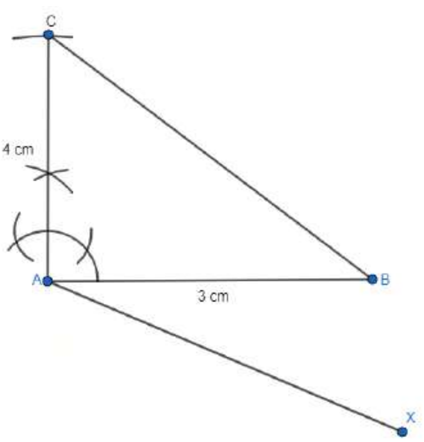
2. Make a right angle at point A and draw AC = 4 cm from this point.



3. Join points A and B to get the right triangle ABC.

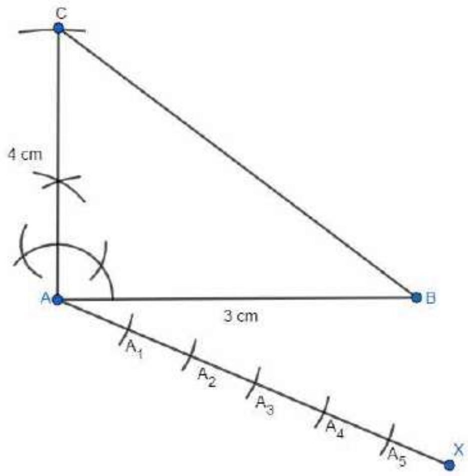


4. Now, Dividing the base, draw a ray AX such at it forms an acute angle from AB.



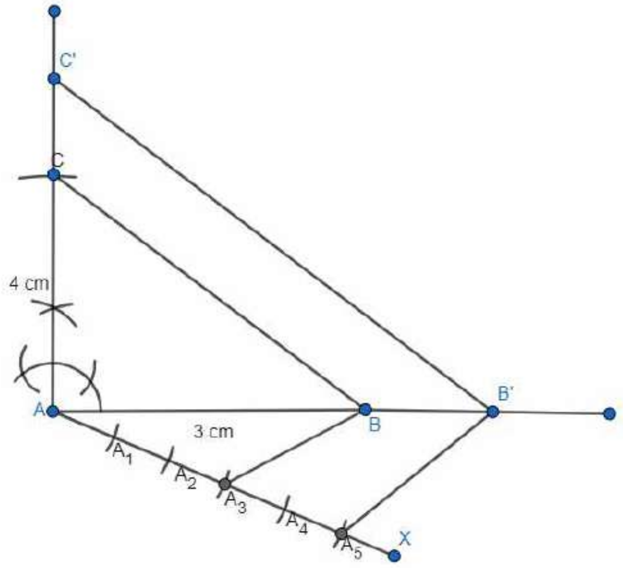
5. Then, plot 5 points on AX such that:

AG = GH = HI = IJ = JK.



6. Join I to point line AB and Draw a line from K which is parallel to IB such that it meets AB at point M.

7. Draw MN ∥ CB.



This is the required construction, thus forming AMN which have all the sides 5/3 times the sides of ABC

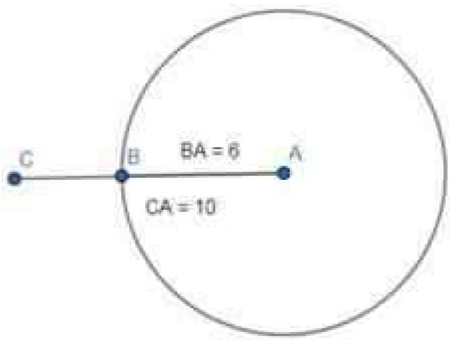
Triangle AMN is the required triangle.

Exercise 11.2

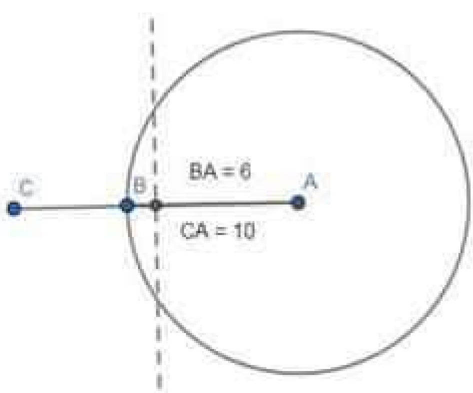
Q. 1 Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Answer:

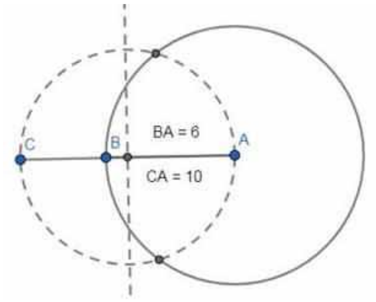
Step1: Draw circle of radius 6cm with center A, mark point C at 10 cm from center.



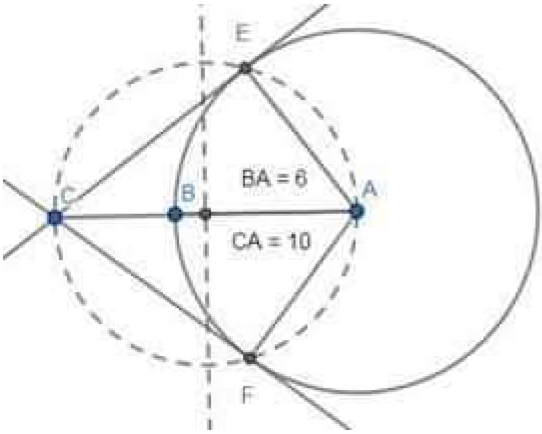
Step 2: find perpendicular bisector of AC



Step3: Take this point as center and draw a circle through A and C



Step 4: Mark the point where this circle intersects our circle and draw tangents through C



Length of tangents = 8cm

AE is perpendicular to CE (tangent and radius relation)

In ΔACE

AC becomes hypotenuse

$AC^2 = CE^2 + AE^2$

$10^2 = CE^2 + 6^2$

$CE^2 = 100-36$

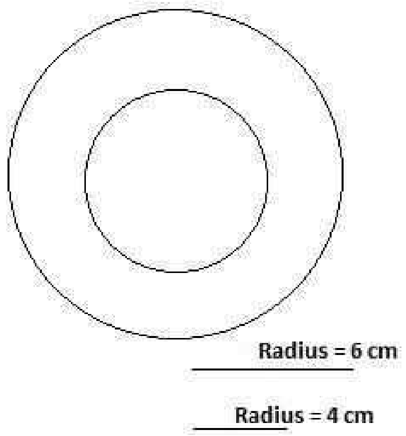
$CE^2 = 64$

CE = 8cm

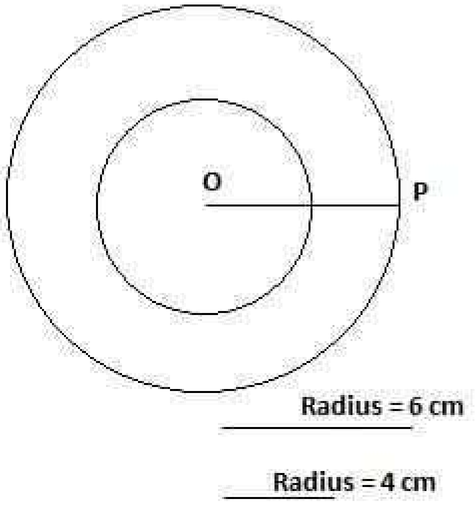
Q. 2 Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Answer: Steps of construction:

i. Draw two concentric circles with radii 4 cm and 6 cm respectively.



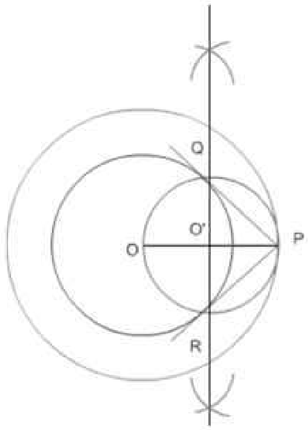
ii. Now, draw the radius OP of the larger circle.



iii. Construct a perpendicular bisector of OP intersecting OP at point O'.

iv. Considering O'P as radius, draw another circle.

v. From point P, draw tangents PQ and PR(can see in the figure)



Justification: By applying Pythagoras theorem, we have;

$PQ^2 = OP^2 - OQ^2$

$= 6^2 - 4^2$

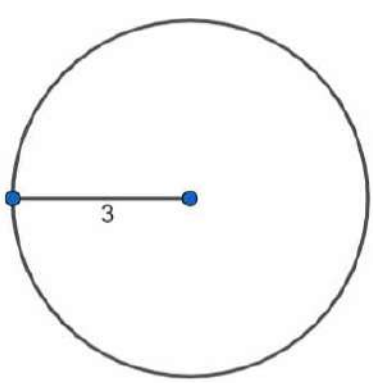
$= 36 - 16 = 20$

Or, $PQ = 2\sqrt{5}$ cm

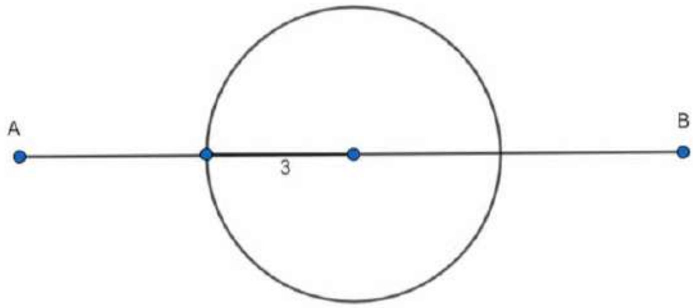
Q. 3 Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Answer: Steps of construction:

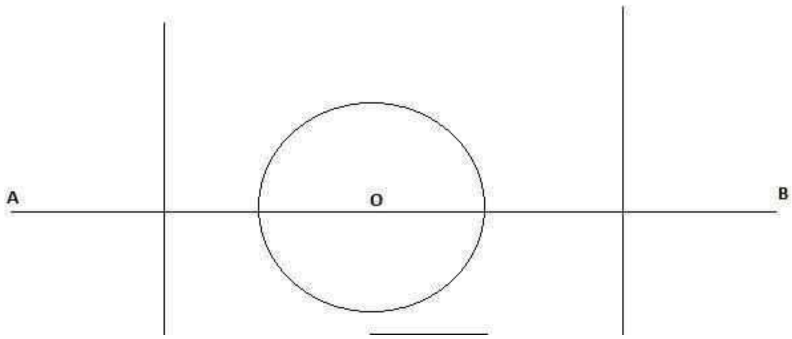
i. At first draw a circle with radius 3 cm.



ii. Now, extend the diameter to A and B on both the sides.



iii. Then, draw the perpendicular bisectors of OA and OB.

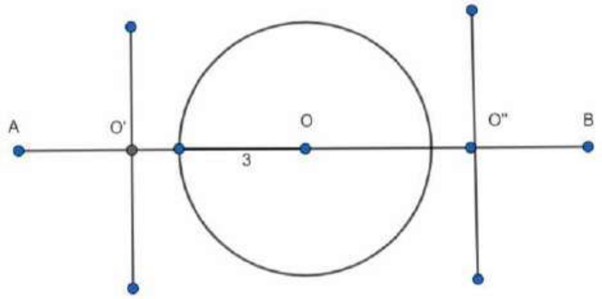


Such that:

Perpendicular bisector of OA intersects it at point O'.

And,

Perpendicular bisector of OB intersects it at point O".

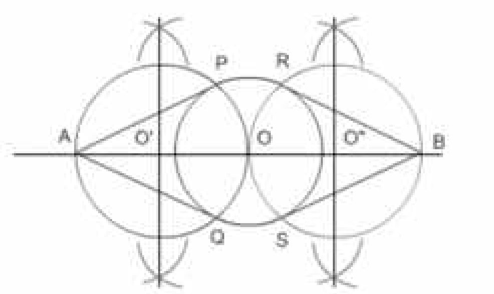


iv. Considering O'A as radius, construct another circle.

v. Considering O"B as radius, construct the third circle.

vi. From point A, draw tangents AP and AQ.

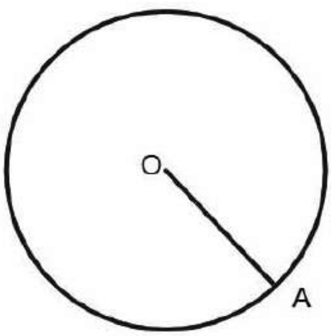
vii. From point B, draw tangents BR and BS.



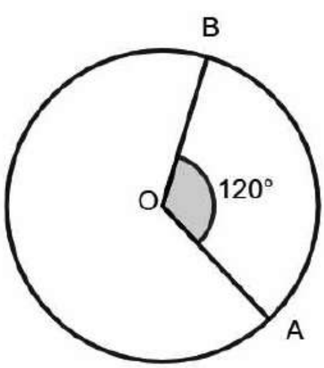
Q. 4 Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Answer: Steps of construction:

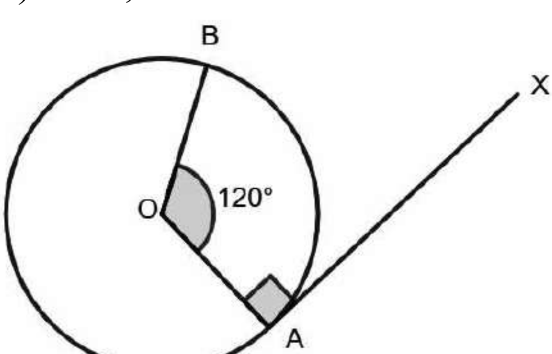
1) Draw a circle of radius 5 cm, and draw a radius OA anywhere in the circle.



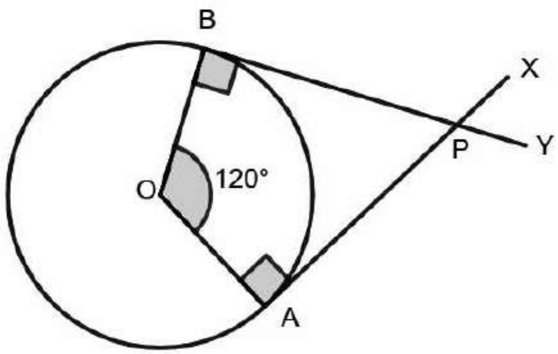
2) Taking OA as base, draw an angle AOB such that ∠AOB = 120°.



3) At A, Draw a line AX such that AX ⊥ OA.



4) At B, Draw a line BY such that BY ⊥ OB.



5) AX and BY intersect at P; AP and BP are required tangents.

Justification:

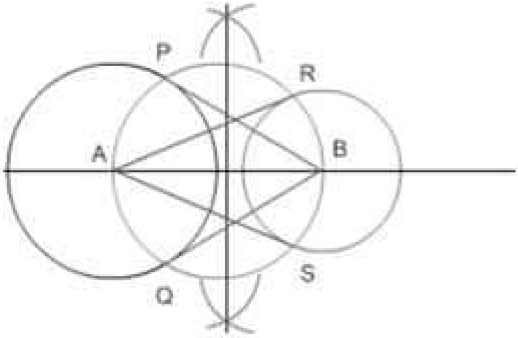
1) Clearly, AP and BP are tangents since tangent at a point on the circle is perpendicular to the radius through point of contact.

2) In Quadrilateral OAPB, we have ∠OAP + ∠APB + ∠OBP + ∠AOB = 360° [By Angle Sum Property] ⇒ ∠OAP + 90° + 90° + 120° = 360°⇒ ∠OAP = 60°.

Q. 5 Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

Answer: Steps of construction:

i. At first, draw a line segment, AB = 8 cm.



ii. Considering A as centre, construct a circle of radius 4 cm.

iii. Considering B as centre, draw another circle with radius 3 cm.

iv. Draw perpendicular bisector of AB.

v. Now, considering midpoint of AB as centre and AB as diameter, draw the third circle.

vi. From point A, draw tangents AR and AS.

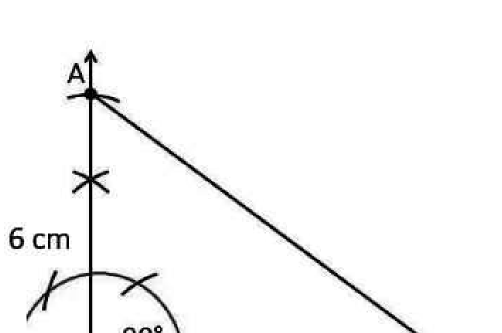
vii. Then, from point B, draw tangents BP and BQ.

Q. 6 Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and ∠ B = 90°. BD is theperpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

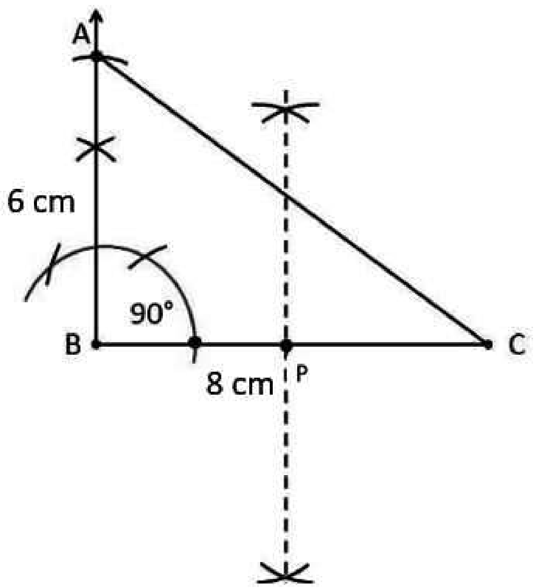
Answer: Steps of construction:

i. Draw a line segment AB = 6 cm.

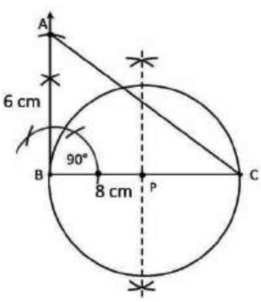
ii. Draw a right angle ∠ABC at point B, such that BC = 8 cm.



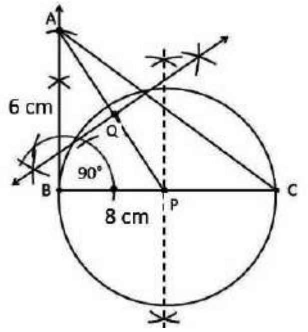
iii. Now, draw a perpendicular bisector of BC which will intersect it at P .



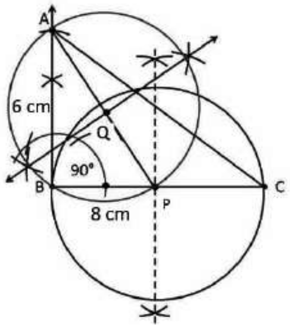
iv. Now P is a mid point of BC. Taking P as a centre and BP as radius draw a circle.



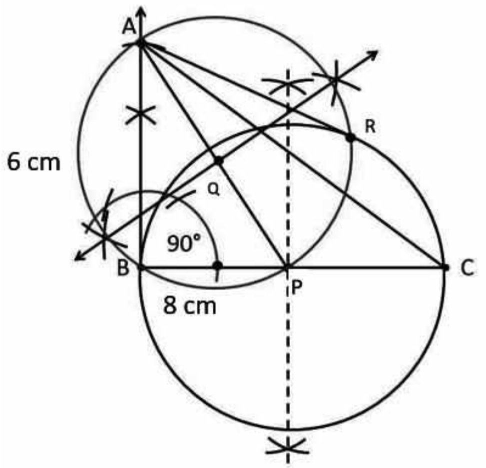
v. Join A to the centre of circle i.e. P.Make perpendicular bisector of AP. Let Q be the mid point of AP.



vi. Taking Q as centre and AQ as a radius draw a circle.



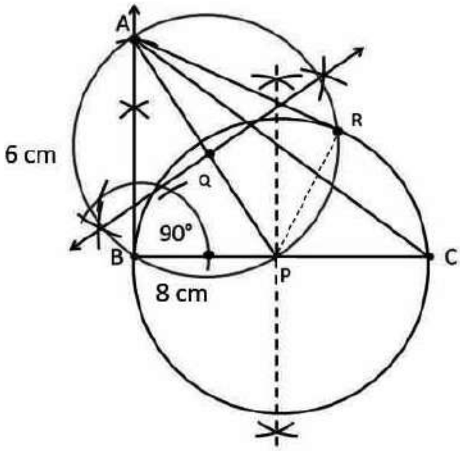
vii. Now Both circles intersect each other at B and R.Join AR.



Hence AB and AR are the required tangents.

Justification:

We need to prove AB and AR are tangents.
Construction: Join PR.



As ARP is an angle on the semicircle BPR. And angles in semicircles are of 90°.∴ ∠ARP = 90°⇒ AR ⊥ PR

And PR is the radius of circle, From the theorem which states that tangent is perpendicular to the radius. So AR has to be tangent. Similarly AB is a tangent. Hence proved.

Q. 7 Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

Answer: Steps of construction:
[We will take a bangle of some fixed radius, say it is 6cm]

i. So at first draw a circle with the help of a bangle having a certain radius (say 6 cm) and centre O.

ii. Take a point P outside the circle.

iii. Draw a line segment OP = 10 cm

iv. Make perpendicular bisector of OP which intersects OP at point O'.

v. Take O'P as radius and draw another circle.

vi. From point P, draw tangents to points of intersection between the two circles.