Chapter – 10

Circles

Exercise 10.1

Q. 1 How many tangents can a circle have?

Answer: Tangent is a line that intersects the circle at one point.

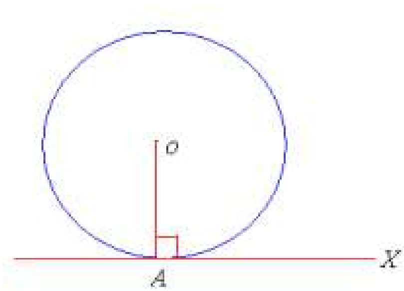
Since, there are infinite number of points on circle. At every point, there is one tangent.

Hence, there are infinite number of tangents in a circle.

Q. 2 Fill in the blanks :
(i) A tangent to a circle intersects it in point (s).
(ii) A line intersecting a circle in two points is called a .
(iii) A circle can have parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called.

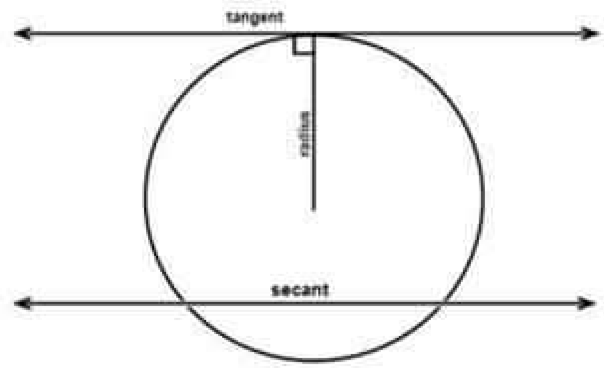
Answer:

(i) A tangent to a circle intersects it in One point.

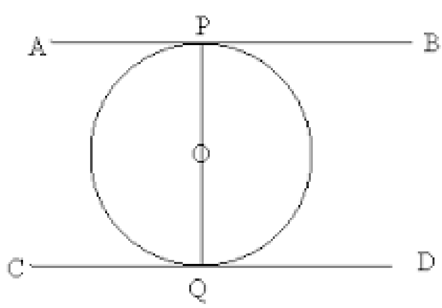


XY is a tangent which intersect the circle at A.

(ii) A line intersecting a circle in two points is called a Secant.

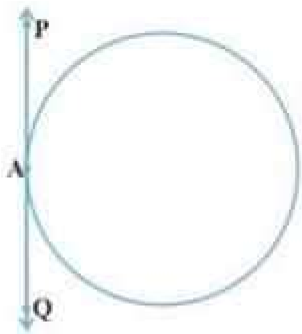


(iii) A circle can have Two parallel tangents at the most.



AB and CD are two parallel tangents

(iv) The common point of a tangent to a circle and the circle is called the Point of contact.



A is point of contact here.

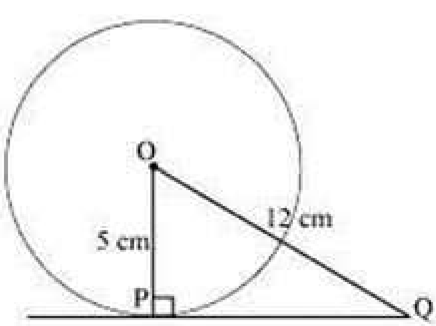
Q. 3 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

A. 12 cm B. 13 cm
C. 8.5 cm D. 119 cm.

Answer: We know that the line drawn from the centre of the circle to the tangent is perpendicular to the tangent.

$OP \perp PQ$

By applying Pythagoras theorem in ΔOPQ,



$OP^2 + PQ^2 = OQ^2$

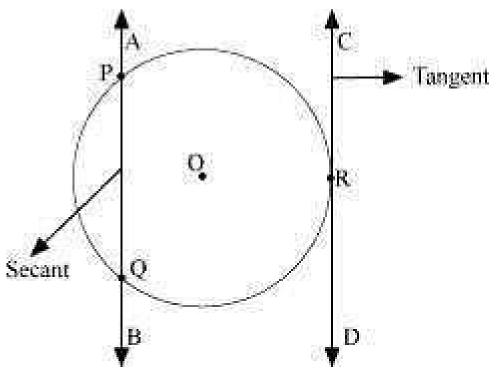
$5^2 + PQ^2 = 12^2$

$PQ^2 = 144 - 25$

$PQ = \sqrt{119}$cm.

Q. 4 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer:



Steps of Construction:

1) Draw a circle with any radius and center O.

2) Choose any point P on the circumference of circle, and draw a line passing through P, Let's name it CD.3) Draw a line AB parallel to CD, such that AB intersects the circle at two points P and A.Here, AB and CD are two parallel lines.AB intersects the circle at exactly two points, P and Q. Therefore, line AB is the secant of this circle.

CD intersects the circle at exactly one point, R, line CD is the tangent to the circle.

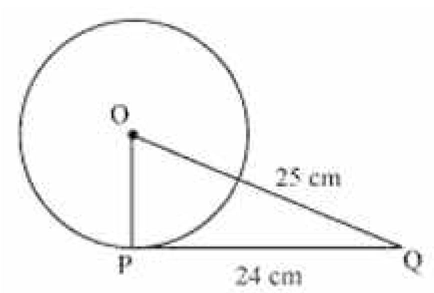
Exercise 10.2

Q. 1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

A. 7 cm B. 12 cm
C. 15 cm D. 24.5 cm

Answer:

(A)



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, $OP \perp PQ$

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

$OP = \sqrt{49} = 7$

Therefore, the radius of the circle is 7 cm.

Q. 2 In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that ∠ POQ = 110°, then ∠ PTQ is equal to

A. 60° B. 70°
C. 80° D. 90°

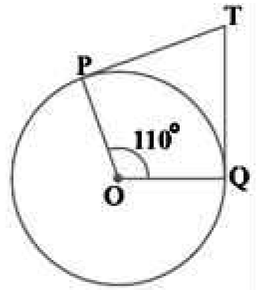


Fig. 10.11

Answer:

Given: TP and TQ are tangents.

Therefore, radius drawn to these tangents from centre of the circle will be perpendicular to the tangents.

Thus, OP ⊥ TP and OQ ⊥ TQ

And therefore,

∠OPT = 90°

∠OQT = 90°

In quadrilateral POQT,

Sum of all interior angles = 360°

∠OPT + ∠POQ +∠OQT + ∠PTQ = 360°

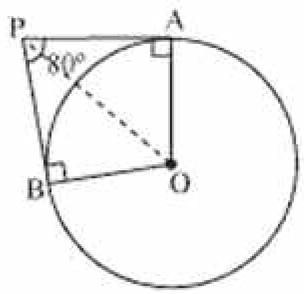
90°+ 110° + 90° + ∠PTQ = 360°

∠PTQ = 70°

Q. 3 If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠ POA is equal to

A. 50° B. 60°
C. 70° D. 80°

Answer: Given: PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ PA and OB ⊥ PB

∠OBP = 90°

∠OAP = 90°

In quadrilateral AOBP,

Sum of all interior angles = 360°

∠OAP + ∠APB +∠PBO + ∠BOA = 360° 90° + 80° +90° + ∠BOA = 360°

∠BOA = 100°

In ΔOPB and ΔOPA,

AP = BP (Tangents from a point)

OA = OB (Radii of the circle)

OP = OP (Common side)

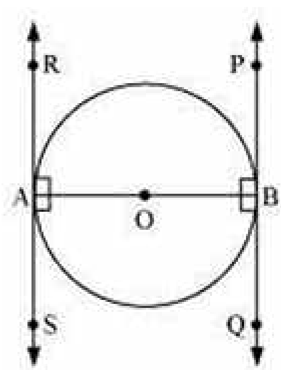
Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

And thus, ∠POB = ∠POA

$< POA = \frac{1}{2} < AOB = \frac{100}{2} = 50°$

Q. 4 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:



Let AB is diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively. Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ RS and OB ⊥ PQ

∠OAR = 90°

∠OAS = 90°

∠OBP = 90°

∠OBQ = 90°

It can be observed that

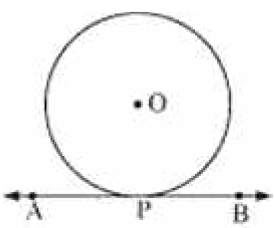
∠OAR = ∠ OBQ (Alternate interior angles)

∠OAS = ∠ OBP (Alternate interior angles)

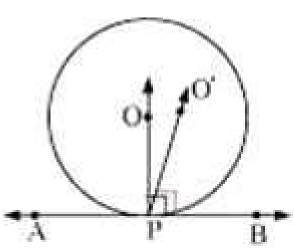
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Q. 5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer: To Prove: A line drawn perpendicular from P will pass from O.
Given: APB is tangent to circle with centre O.
Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



Proof:
Assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O'. Join OP and O'P.



As perpendicular to AB at P passes through O', therefore,

$\angle$ O'PB = 90° ... (1)

O is the centre of the circle and P is the point of contact.
We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.

$\therefore \angle OPB = 90°$ ... (2)

Comparing equations (1) and (2), we obtain

$\angle O'PB = \angle OPB$ ... (3)

From the figure, it can be observed that,

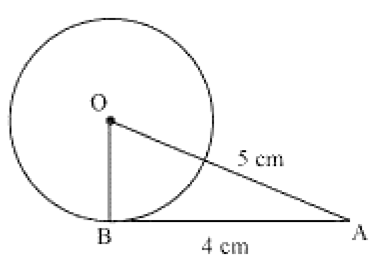
$\angle O'PB < \angle OPB$ ... (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O'P coincides with OP.

Therefore, the perpendicular to AB through P passes through centre O.
Hence, Proved.

Q. 6 The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4cm. Find the radius of the circle.

Answer:



Let us consider a circle centred at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

OB ⊥ AB (Radius ⊥ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

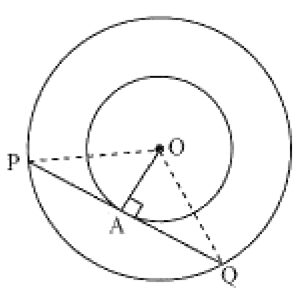
$16 + BO^2 = 25$ $BO^2 = 9$

$BO = 3$

Hence, the radius of the circle is 3 cm.

Q. 7 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:



Let the two concentric circles be centred at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

OA ⊥ PQ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3^2 + AP^2 = 5^2$

$9 + AP^2 = 25$

$AP^2 = 16$

$AP = 4$

In ΔOPQ,

Since OA ⊥ PQ,

AP = AQ (Perpendicular from the centre of the circle bisects the chord)

PQ = 2AP = 2 × 4 = 8

Therefore, the length of the chord of the larger circle is 8 cm.

Q. 8 A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that
AB + CD = AD + BC

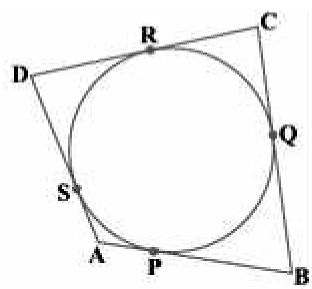


Fig. 10.12

Answer:

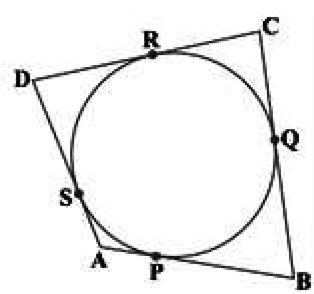


Fig. 10.12

To Prove: AB + CD = AD + BC
Proof:
In the given figure, it can be observed that AB touches the circle at point P; BC touches the circle at point Q; CD touches the circle at point R; DA touches the circle at point S.

Then, we can say from a theorem that " ***Length of tangents drawn from an external point to the circle are same*** ",
Therefore,

DR = DS (Tangents on the circle from point D) ….. (1)

CR = CQ (Tangents on the circle from point C) …... (2)

BP = BQ (Tangents on the circle from point B) ….... (3)

AP = AS (Tangents on the circle from point A) ….... (4)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC
Hence, Proved.

Q. 9 In Fig. 10.13, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠ AOB = 90°

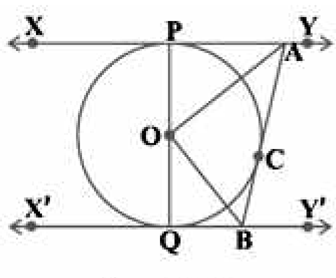


Fig. 10.13

Answer: Let us join point O to C.

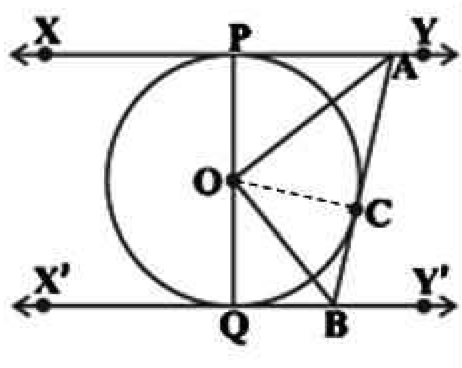


Fig. 10.13

In ΔOPA and ΔOCA,

OP = OC (Radius of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

$\Delta OPA \cong \Delta OCA$ (SSS congruence criterion)

$\angle POA = \angle COA$ … (*i*)
Similarly, $\Delta OQB \cong \Delta OCB$

$\angle QOB = \angle COB$ … (*ii*)

Since POQ is a diameter of the circle, it is a straight line.

Therefore, $\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

From equations (*i*) and (*ii*), it can be observed that $2\angle COA + 2\angle COB = 180°$

$\angle COA + \angle COB = 180° / 2$

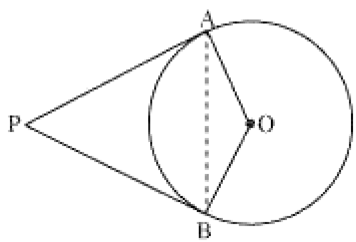
$\angle COA + \angle COB = 90°$

$\angle AOB = 90°$

Hence Proved.

Q. 10 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Answer:



To Prove: $\angle APB + \angle BOA = 180°$

Proof: Let us consider a circle centred at point O.

Let P be an external point from which two tangents PA and PB are drawn to the circle which touches the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends

$\angle AOB$ at centre O of the circle.

It can be observed that

$OA \perp PA$ (radius of circle is always perpendicular to tangent)

Therefore, $\angle OAP = 90°$

Similarly, $OB \perp PB$

$\angle OBP = 90°$

In quadrilateral OAPB,

Sum of all interior angles = 360°

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$90° + \angle APB + 90° + \angle BOA = 360°$

$\angle APB + \angle BOA = 180°$

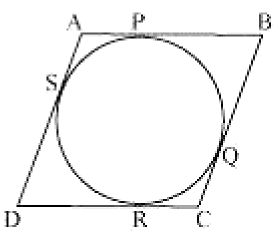
Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Q. 11 Prove that the parallelogram circumscribing a circle is a rhombus.

Answer: Since ABCD is a parallelogram,

AB = CD ... (1)

BC = AD ... (2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain 2AB = 2BC

AB = BC ...(3)

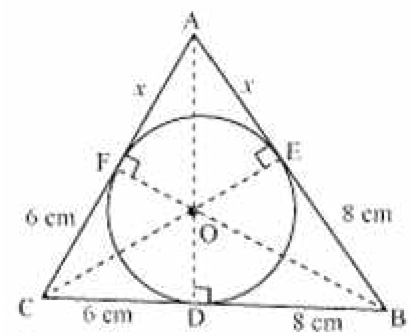
Comparing equations (1), (2), and (3), we obtain

AB = BC = CD = DA

Hence, ABCD is a rhombus.

Q. 12 A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

Answer:



To Find: AB and AC

Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be x.

In ΔABC,

From the theorem which states that the lengths of two tangents drawn from an external point to a circle are equal.
So,

CF = CD = 6cm (Tangents on the circle from point C)

BE = BD = 8cm (Tangents on the circle from point B)

AE = AF = x (Tangents on the circle from point A)

AB = AE + EB = $x + 8$

BC = BD + DC = 8 + 6 = 14

CA = CF + FA = $6 + x$

By heron's formula

$$area\, of\ \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

where, a, b and c are sides of triangle and s is semi perimeter.

$2S$ = AB + BC + CA

$= x + 8 + 14 + 6 + x$

$= 28 + 2x$

$S = \frac{28+2x}{2} = 14 + x$

$area\ of\ \Delta\ ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\{14+x\}\{x\}(8)(6)}$

$= 4\sqrt{3(14x+x^2)}$

Also, area of triangle = $\frac{1}{2} \times base \times height$

And from the given figure it is clear that Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$Area\ of\ \Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$

$$Area\ of\ \Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$$

$$Area\ of\ \Delta OCA = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow 4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$

$\Rightarrow 4\sqrt{3(14x+x^2)} = 56 + 4x$

$= \sqrt{3(14x+x^2)} = 14 + x$

$\Rightarrow 3x(14+x) = (14+x)^2$

$\Rightarrow$ 3x = 14 + x

$\Rightarrow$ 2x = 14

$\Rightarrow$ x = 7

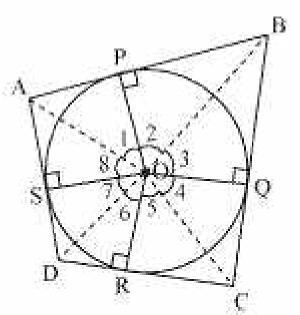
Therefore, $x = 7$

Hence,
AB $= x + 8 = 7 + 8 = 15$ cm

CA $= 6 + x = 6 + 7 = 13$ cm
Hence, AB = 15 cm and AC = 13 cm

Q. 13 Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer:



To Prove: ∠ AOB + ∠ COD = 180°, ∠ BOC + ∠ DOA = 180°
Given: ABCD is circumscribing the circle.
Proof: Let ABCD be a quadrilateral circumscribing a circle centred at O such that it touches the circle at point P, Q, R, S.

Join the vertices of the quadrilateral ABCD to the centre of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

And thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$ Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$

$\angle AOB + \angle COD = 180°$

Similarly, we can prove that $\angle BOC + \angle DOA = 180°$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.