Revision Notes

Class-12 Mathematics

Chapter 1 – Relations and Functions

Relation

- It defines **relationship** between two set of values let say from set A to set B.
- Set A is then called domain and set B is then called codomain.
- If $(a, b) \in R$, it shows that a is related to b under the relation R

Types of Relations

1. Empty Relation:

- In this there is **no relation** between any element of a set**.**
- It is also known as void relation
- For example: if set A is $\{2,4,6\}$ then an empty relation can be $R = \{x, y\}$ where $x + y > 11$

2. Universal Relation:

- In this **each element** of a set is **related** to **every element** of that set.
- For example: if set A is $\{2,4,6\}$ then a universal relation can be $R = \{x, y\}$ where $x + y > 0$

3. Trivial Relation: Empty relation and universal relation is sometimes called trivial relation.

4. Reflexive Relation:

- In this each element of set (say) A is related to itself i.e., a relation R in set A is called **reflexive** if $(a, a) \in R$ for every $a \in A$.
- For example: if $\text{Set } A = \{1,2,3\}$ then relation $R = \{(1,1),(1,2),(2,2),(2,1),(3,3)\}$ is reflexive since each element of set A is related to itself.

5. Symmetric Relation:

- A relation R in set A is called **symmetric** if $(a,b)\in R$ and $(b,a)\in R$ for every $a,b\in A$.
- For example: if $\text{Set } A=\{1,2,3\}$ then relation $R=\{(1,2),(2,1),(2,3),(3,2),(3,1),(1,3)\}$ is symmetric.

6. Transitive Relation:

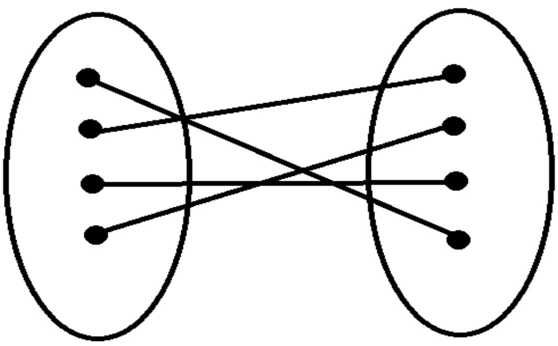
- A relation R in set A is called **transitive** if $(a,b)\in R$ and $(b,c)\in R$ then (a,c) also belongs to R for every $a,b,c\in A$.
- For example: if $\text{Set } A=\{1,2,3\}$ then relation $R=\{(1,2),(2,3),(1,3)(2,3),(3,2),(2,2)\}$ is transitive.

7. Equivalence Relation:

- A relation R on a set A is equivalence if R is **reflexive, symmetric and transitive.**
- For example: $R=\{(L_1,L_2):\text{line } L_1 \text{ is parallel line } L_2\}$,
  This relation is reflexive because every line is parallel to itself
  Symmetric because if L_1 parallel to L_2 then L_2 is also parallel to L_1
  Transitive because if L_1 parallel to L_2 and L_2 parallel to L_3 then L_1 is also parallel to L_3

Functions

- A function f from a set A to a set B is a rule which associates each element of set A to a **unique element** of set B.



- **Set A is domain** and **set B is codomain** of the function

- **Range** is the set of all possible resulting value given by the function.
- For example: x^2 is a function where values of x will be the domain and value given by x^2 is range.

Types of Function:

1. One-One Function:

- A function f from set A to set B is called one-one function if no two distinct elements of A have the same image in B.
- Mathematically, a function f from set A to set B if $f(x) = f(y)$ implies that $x = y$ for all $x, y \in A$.
- One-one function is also called an injective **function**.
- For example: If a function f from a set of real number to a set of real number, then $f(x) = 2x$ is a one-one function.

2. Onto Function:

- A function f from set A to set B is called onto function if each element of set B has a preimage in set A or range of function f is equal to the codomain i.e., set B.
- Onto function is also called **surjective function**.
- For example: If a function f from a set of natural number to a set of n Natural number, then $f(x) = x - 1$ is onto function.

3. Bijective Function:

- A function f from set A to set B is called bijective function if it is **both one-one function and onto function.**
- For example: If a function f from a set of real number to a set of real number, then $f(x) = 2x$ is one-one function and onto function.

Composition of function and invertible function

- **Composition of function:** Let $f: A \to B$ and $g: B \to C$ then the composite of g and f, written as $g \circ f$ is a function from A to C such that $(g \circ f)(a) = g(f(a))$ for all $a \in A$. **(Not in the current syllabus)**
- **Properties of composition of function:** Let $f: A \to B$, $g: B \to C$ and $h: C \to A$ then

a. Composition is **associative** i.e., $h(gf) = (hg)f$

b. If f and g are one-one then $g \circ f$ is also **one-one**

c. If f and g are onto then $g \circ f$ is also **onto**

d. **Invertible function:** If f is bijective then there is a function $f^{-1} : B \to A$ such that $\left(f^{-1}f\right)(a) = a$ for all $a \in A$ and $\left(f^{-1}f\right)(b) = b$ for all $b \in B$

- f^{-1} is the **inverse** of the function f and is **always unique**.

Binary Operations

- A binary operation are **mathematical operations** such as addition, subtraction, multiplication and division performed between two operands.
- A binary operation on a set A is defined as operations performed between two elements of set A and the result also belongs to set A. Then set A is called **binary composition**.
- It is denoted by $*$
- For example: Binary addition of real numbers is a binary composition
- since on adding two real number the result will always a real number.

Properties of Binary Composition:

- A binary operation $*$ on the set X is **commutative**, i.e., $a*b = b*a$, for every $a,b \in X$
- A binary operation $*$ on the set X is **associative**, i.e., $a*(b*c) = (a*b)*c$, for every $a,b,c \in X$
- There exists **identity** for the binary operation $* : A \times A \to A$, i.e., $a*e = e*a = a$ for all $a,e \in A$
- A binary operation $* : A \times A \to A$ is said to be **invertible** with respect to the operation $*$ if there exist an element b in A such that $a*b = b*a = e$ e is identity element in A then b is the inverse of a and is denoted by a^{-1}.