FINAL JEE–MAIN EXAMINATION – APRIL, 2019
Held On Tuesday 09th APRIL, 2019
TIME: 02 : 30 PM To 05 : 30 PM

1. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :-

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb
(3) 3.67×10^{-4} Wb (4) 3.67×10^{-3} Wb

Official Ans. by NTA (2)

Sol. $$\phi_q = \frac{\mu_0 i_1 R^2}{2(R^2+x^2)^{\frac{3}{2}}} \times \pi r^2 = 10^{-3}$$

$$\phi_P = \frac{\mu_0 i_2 r^2}{2(r^2+x^2)^{\frac{3}{2}}} \times \pi R^2$$

$$\frac{\phi_P}{\phi_Q} = \frac{i_2}{i_1} \cdot \frac{(R^2+x^2)^{\frac{3}{2}}}{(r^2+x^2)^{\frac{3}{2}}} = \frac{\phi_P}{10^{-3}}$$

$$\frac{2}{3} = \frac{\phi_P}{10^{-3}}$$

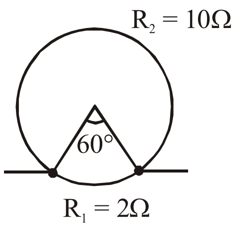
$\phi_P = 6.67 \times 10^{-4}$.

2. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be :-

(1) $\frac{12}{5}\Omega$ (2) $\frac{5}{3}\Omega$ (3) $\frac{5}{2}\Omega$ (4) $\frac{7}{2}\Omega$

Official Ans. by NTA (2)

Sol.



$$R = \frac{\rho \ell^2}{A\ell D} d = \frac{\rho d \ell^2}{m}$$

$R \propto \ell^2$

$R = 12\Omega$ (new resistance of wire)

$R_1 = 2\Omega$ $R_2 = 10\Omega$

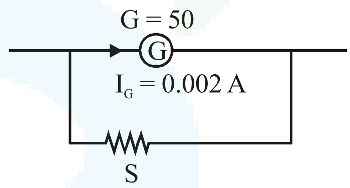
$$R_{eq} = \frac{10 \times 2}{10+2} = \frac{5}{3}\Omega.$$

3. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A ?

(1) 0.2 ohm (2) 0.002 ohm
(3) 0.02 ohm (4) 0.5 ohm

Official Ans. by NTA (1)

Sol.



$S(0.5 - 0.002) = 50 \times 0.002$

$$S = \frac{50 \times 0.002}{(0.5-0.002)} = \frac{0.1}{0.498} = 0.2$$

4. The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \frac{b^2}{4c}$ (2) $a + \frac{b^2}{c}$

(3) $a + \frac{b^2}{2c}$ (4) $a + \frac{b^2}{3c}$

Official Ans. by NTA (4)

Sol. $x = at + bt^2 - ct^3$

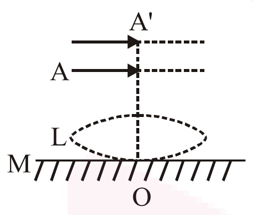
$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

$$a = \frac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$$

$$v_{\left(\text{at } t=\frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)$$

$$= a + \frac{b^2}{3c}.$$

5. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, The pin has to be moved to A', such that OA' = 27 cm, to get its inverted real image at A' itself. The value of μ_1 will be :-



(1) $\sqrt{2}$ (2) $\frac{4}{3}$

(3) $\sqrt{3}$ (4) $\frac{3}{2}$

Official Ans. by NTA (2)

Sol. $\frac{1}{f_1} = \frac{1}{2} \times \frac{2}{18} = \frac{1}{18}$

$\frac{1}{f_2} = \frac{(\mu_1 - 1)}{-18}$

when μ_1 is filled between lens and mirror

$P = \frac{2}{18} - \frac{2}{18}(\mu_1 - 1) = \frac{2 - 2\mu_1 + 2}{18}$

$= F_m = -\left(\frac{18}{2-\mu_1}\right)$

$2 = 6 - 3\mu_1$
$3\mu_1 = 4$
$\mu_1 = 4/3.$

6. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a maganetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :-

(1) 10^{-3} (2) 10^{-1}

(3) 10^{-4} (4) 10^{-2}

Official Ans. by NTA (1)

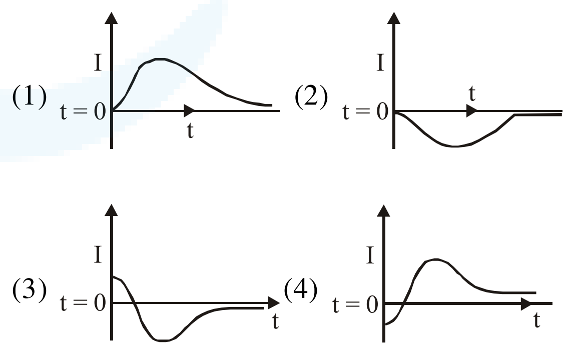
Sol. $\tau = \vec{M} \times \vec{B}$

$C\theta = i\ N\ A\ B$

$10^{-6} \times \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175 \times B$

$B = 10^{-3}$ Tesla.

7. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k > 0)$, as a function of time $(t \geq 0)$. counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by :-



Official Ans. by NTA (2)

Sol. $\phi_{outer} = (\mu_0 nKte^{-\alpha t})4\pi R^2$

$\varepsilon = \frac{-d\phi}{dt} = -Ce^{-\alpha t}[1 - \alpha t]$

$i_{induced} = \frac{-Ce^{-\alpha t}\ [1 - at]}{(\text{Resistance})}$

At t = 0 $i_{induced}$ = –ve.

8. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely ? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)

(1) 10^{-3}K (2) 10^{-4} K (3) 10^{-1} K (4) 10^{-5}K

Official Ans. by NTA (4)

Sol. By law of conservation of energy

$$\frac{1}{2}kx^2 = (m_1s_1 + m_2s_2)\,\Delta T$$

$$\Delta T = \frac{16\times10^{-2}}{4384} = 3.65 \times 10^{-5}.$$

9. A particle 'P' is formed due to a completely inelastic collision of particles 'x' and 'y' having de-Broglie wavelengths 'λ_x' and 'λ_y' respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of 'P' is :-

(1) $\lambda_x + \lambda_y$ (2) $\frac{\lambda_x\lambda_y}{\lambda_x+\lambda_y}$

(3) $\frac{\lambda_x\lambda_y}{|\lambda_x-\lambda_y|}$ (4) $\lambda_x - \lambda_y$

Official Ans. by NTA (3)

Sol. (x)⟶ ⟵(y) = (P)⟶

By momentum conservation

$P_x - P_y = P_P$

$$\frac{h}{\lambda_x}-\frac{h}{\lambda_y}=\frac{h}{\lambda_p}$$

$$\lambda_p = \frac{\lambda_x\lambda_y}{|\lambda_y-\lambda_x|}.$$

10. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is :-

(1) 5 : 3 (2) 2 : 1

(3) 4 : 3 (4) 3 : 1

Official Ans. by NTA (4)

Sol. Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$$\frac{x_1}{x_2} = 3:1.$$

11. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600nm. coming from a distant object, the limit of resolution of the telescope is close to :-

(1) 1.5×10^{-7} rad (2) 2.0×10^{-7} rad

(3) 3.0×10^{-7} rad (4) 4.5×10^{-7} rad

Official Ans. by NTA (3)

Sol. Limit of resolution $= \frac{1.22\,\lambda}{d}$

$$= \frac{1.22\times600\times10^{-9}}{250\times10^{-2}}$$

$= 2.9 \times 10^{-7}$ rad.

12. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular accleration of 20 rad/s^2 must be applied about the axis for a duration of :-

(1) 2 s (2) 5s

(3) 2.5 s (4) 3 s

Official Ans. by NTA (1)

Sol. Given moment of inertia 'I' = 1.5 kgm^2

Angular Acc. "α" = 20 Rad/s^2

$$KE = \frac{1}{2} I\omega^2$$

$$1200 = \frac{1}{2} 1.5 \times \omega^2$$

$$\omega^2 = \frac{1200 \times 2}{1.5} = 1600$$

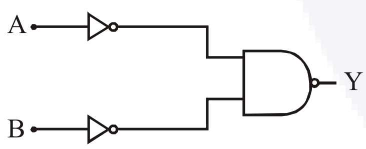
$\omega = 40$ rad/s²

$\omega = \omega_0 + \alpha t$

$40 = 0 + 20\ t$

t = 2sec.

13. The logic gate equivalent to the given logic circuit is :-



(1) OR (2) AND

(3) NOR (4) NAND

Official Ans. by NTA (1)

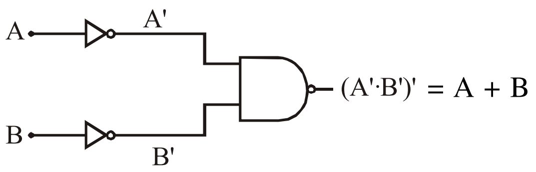
Sol. **Method 1**

Truth table can be formed as

A	B	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the Equivalent is "OR" gate.

Method 2



(OR GATE)

14. 50 W/m² energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1m² surface area will be close to ($c = 3 \times 10^8$ m/s) :-

(1) 15×10^{-8} N

(2) 35×10^{-8} N

(3) 10×10^{-8} N

(4) 20×10^{-8} N

Official Ans. by NTA (4)

Sol. Force on the surface (25% reflecting and rest absorbing)

$$F = \frac{25}{100}\left(\frac{2I}{C}\right) + \frac{75}{100}\left(\frac{I}{C}\right) = \frac{125}{100}\left(\frac{I}{C}\right)$$

$$= \frac{125}{100} \times \left(\frac{50}{3 \times 10^8}\right) = 20.83 \times 10^{-8} \text{ N.}$$

15. The area of a square is 5.29 cm². The area of 7 such squares taking into account the significant figures is :-

(1) 37 cm²

(2) 37.0 cm²

(3) 37.03 cm²

(4) 37.030 cm²

Official Ans. by NTA (4)

Ans. (3)

Sol. Total Area = $A_1 + A_2 + \ldots\ldots A_7$

= A + A + ... 7 times

= 37.03 m².

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits.)

16. The physical sizes of the transmitter and receiver antenna in a communication system are :-

(1) proportional to carrier frequency

(2) inversely proportional to modulation frequency

(3) inversely proportional to carrier frequency

(4) independent of both carrier and modulation frequency

Official Ans. by NTA (3)

Sol. The physical size of antenna of reciver and transmitter both inversely proportional to carrier frequency.

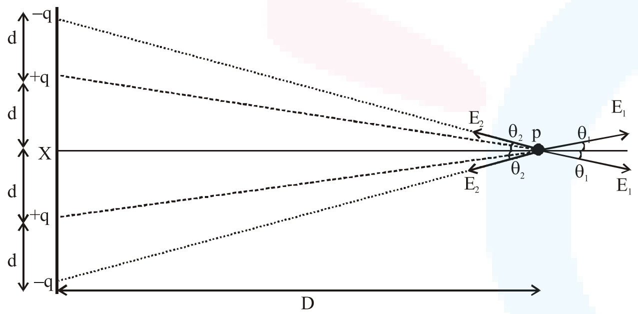
17. Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as :-

(1) $E \propto \frac{1}{D}$ (2) $E \propto \frac{1}{D^3}$

(3) $E \propto \frac{1}{D^2}$ (4) $E \propto \frac{1}{D^4}$

Official Ans. by NTA (4)

Sol.



Electric field at p = $2E_1\cos\theta_1 - 2E_1\cos\theta_2$

$$= \frac{2Kq}{(d^2+D^2)} \times \frac{D}{(d^2+D^2)^{1/2}} - \frac{2Kq}{[(2d)^2+D^2]} \times \frac{D}{[(2d)^2+D^2]^{1/2}}$$

$$= 2KqD\left[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation ∵ d << D

$$= \frac{2KqD}{D^3}\left[1-\frac{3}{2}\frac{d^2}{D^2} - \left(1-\frac{3\times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9kqd^2}{D^4}$$

18. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then :-

(1) A has one vibrational mode and B has two

(2) Both A and B have a vibrational mode each

(3) A is rigid but B has a vibrational mode

(4) A has a vibrational mode but B has none

Official Ans. by NTA (4)

Sol. For A

$R = C_p - C_v = 7$

$$C_v = \frac{fR}{2} = 22 \Rightarrow f = \frac{44}{7} = 6.3$$

$f \simeq 6$ → 5 (Rotation + Translational), 1 (Vibration)

For B

$R = C_p - C_v = 9$

$$C_v = \frac{fR}{2} = 21 \Rightarrow f = \frac{42}{9}$$

$f \simeq 5$ → 5 (Rotation + Translational), 0 (Vibration)

19. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4-20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1 ?

(1) 40 (2) 100

(3) 25 (4) 50

Official Ans. by NTA (4)

Sol. $\vec{r} = 15t^2\hat{i} + (4-20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} + (-40t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$$

$$|\vec{a}| = 50 m/s^2.$$

20. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T :

(1) T/R^2 is a constant

(2) TR is a constant

(3) T^2/R^3 is a constant

(4) T/R is a constant

Official Ans. by NTA (4)

Sol. $m = \int_0^R \rho\, 4\pi r^2 dr$

$m = 4\pi KR$

$v \propto \sqrt{4\pi K}$

$$\frac{T}{R} = \frac{2\pi}{\sqrt{4\pi K}}.$$

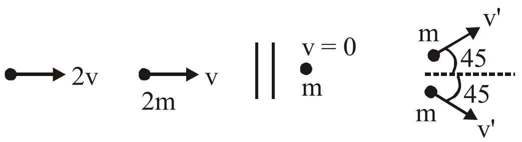
21. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the origianl direction.

The speed of each of the moving particle will be :-

(1) $v/(2\sqrt{2})$ (2) $2\sqrt{2}v$

(3) $\sqrt{2}v$ (4) $v/\sqrt{2}$

Official Ans. by NTA (2)

Sol. 

Linear momentum conservation

$$m\, 2v + 2m\, v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$$

$v' = 2\sqrt{2}\,v.$

22. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :-

(1) 0.5 (2) 0.7 (3) 0.6 (4) 0.8

Official Ans. by NTA (3)

Sol. In 1st situation

$V_b \rho_b g = V_s \rho_w g$

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad ...(i)$$

here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil

finally in equilibrium condition

$$V_b \rho_b\, g = \frac{V_b}{2}\rho_O g + \frac{V_b}{2}\rho_w g$$

$2\rho_b = \rho_0 + \rho_w$

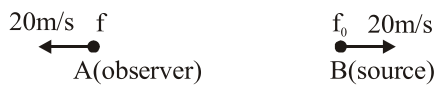
$$\Rightarrow \frac{\rho_O}{\rho_w} = \frac{3}{5} = 0.6$$

23. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B ? (speed of sound in air = 340 ms^{-1}) :-

(1) 2250 Hz (2) 2060 Hz

(3) 2150 Hz (4) 2300 Hz

Official Ans. by NTA (1)

Sol. 

Applying Doppler effect for sound

$f = \frac{v + v_0}{v - v_s} f_0$ (v_0 & v_s is taken ⊕ when approaching each other)

$2000 = \frac{340 + (-20)}{340 - (-20)} f_0$

$f_0 = 2250$ Hz.

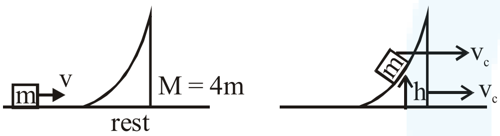
24. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :-

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{g}$

(3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{2g}$

Official Ans. by NTA (3)

Sol.



Applying Linear momentum conservation

$mv = (m + M)v_c$

$v_c = \frac{v}{5}$

applying work energy theorem

$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2} mv^2$

solve, $h = \frac{2v^2}{5g}$

25. A He^+ ion is in its first excited state. Its ionization energy is :-

(1) 6.04 eV (2) 13.60 eV

(3) 54.40 eV (4) 48.36 eV

Official Ans. by NTA (2)

Sol. Energy levels in Hydrogen like atom is given by

$E = -13.6 \frac{z^2}{n^2}$ eV

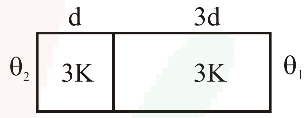
As He^+ is 1st excited state

∴ z = 2, n = 2

E = –13.6 eV

As total energy of He^+ in 1st excited state is –13.6 eV, ionisation energy should be +13.6eV.

26. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and '3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is :-

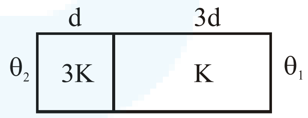


(1) $\frac{\theta_2 + \theta_1}{2}$ (2) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$

(3) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (4) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$

Official Ans. by NTA (2)

Sol.



Let the temperature of interface be "θ"

$i_1 = i_2$ {Steady state conduction}

$\frac{3KA(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$

$\theta = \frac{9\theta_2}{10} + \frac{\theta_1}{10}$

27. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system , when the beads reach the opposite ends of the rod, will be :-

(1) $\frac{M\omega_0}{M + 3m}$ (2) $\frac{M\omega_0}{M + m}$

(3) $\frac{M\omega_0}{M+2m}$ (4) $\frac{M\omega_0}{M+6m}$

Official Ans. by NTA (4)

Sol. Applying angular momentum conservation, about axis of rotation

$L_i = L_f$

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + m\left(\frac{L}{2}\right)^2 \times 2\right)\omega$$

$$\Rightarrow \omega = \frac{M\omega_0}{M+6m}.$$

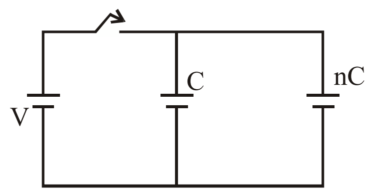
28. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :-

(1) $\frac{V}{K+n}$ (2) V

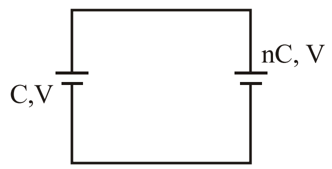
(3) $\frac{(n+1)V}{(K+n)}$ (4) $\frac{nV}{K+n}$

Official Ans. by NTA (3)

Sol.



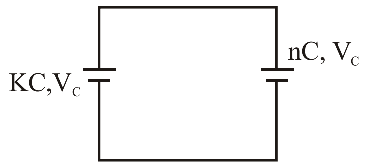
After fully charging, battery is disconnected



Total charge of the system = CV + nCV

= (n + 1)CV

After the insertion of dielectric of constant K



New potential (common)

$$V_C = \frac{\text{total charge}}{\text{total capacitance}}$$

$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}.$$

29. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25fs (femto second), it's approximate resistivity is :-

($m_e = 9.1 \times 10^{-31}$ kg)

(1) 10^{-5} Ωm (2) 10^{-6} Ωm

(3) 10^{-7} Ωm (4) 10^{-8} Ωm

Official Ans. by NTA (4)

Sol. $\rho = \frac{m}{ne^2\tau}$

$= 1.67 \times 10^{-8}\Omega m$

30. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :-

(1) 320m/s, 120 Hz (2) 180m/s, 80 Hz

(3) 180m/s, 120 Hz (4) 320m/s, 80 Hz

Official Ans. by NTA (4)

Sol. $3\left(\frac{v}{2\ell}\right) = 240$

$3\frac{v}{2\times 2} = 240$

v = 320 m/s

fundamental frequency = $\frac{v}{2\ell} = \frac{320}{2\times 2}$ = 80 Hz.