FINAL JEE–MAIN EXAMINATION – APRIL, 2019
Held On Monday 08th APRIL, 2019
TIME: 02 : 30 PM To 5 : 30 PM

1. A circuit connected to an ac source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this ?

(1) RC circuit with R = 1 kΩ and C = 1μF

(2) RL circuit with R = 1kΩ and L = 1mH

(3) RL circuit with R = 1 kΩ and L = 10 mH

(4) RC circuit with R = 1kΩ and C = 10 μF

Official Ans. by NTA (4)

Sol. Given phase difference = $\frac{\pi}{4}$

and ω = 100 rad/s

⇒ Reactance (X) = Resistance (R)

now by checking options

Option (1)

R = 1000 Ω and $X_C = \frac{1}{10^{-6}\times 100} = 10^4\Omega$

Option (2)

R = 10^3 Ω and $X_L = 10^{-3} \times 100 = 10^{-1}\Omega$

Option (3)

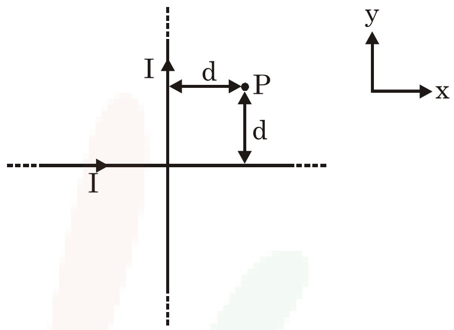
R = 10^3 Ω and $X_L = 10 \times 10^{-3} \times 100 = 1\Omega$

Option (4)

R = 10^3 Ω and $X_C = \frac{1}{10\times 10^{-6}\times 100} = 10^3\Omega$

Clear option (4) matches the given condition

2. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be :



(1) Zero

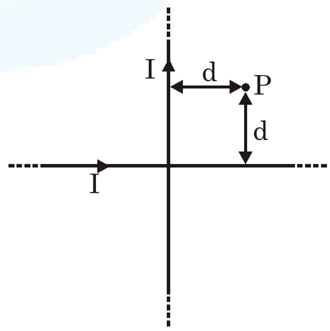
(2) $\frac{+\mu_0 I}{\pi d}(\hat{z})$

(3) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

(4) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

Official Ans. by NTA (1)

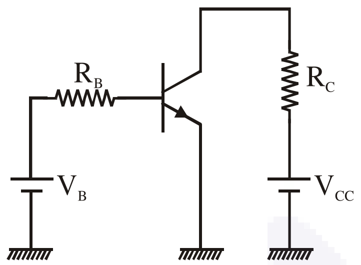
Sol.



Magnetic field at point P

$$\vec{B}_{net} = \frac{\mu_0 i}{2\pi d}(-\hat{k}) + \frac{\mu_0 i}{2\pi d}(\hat{k}) = 0$$

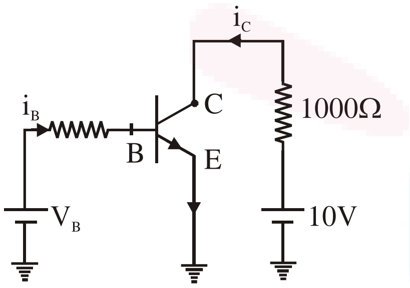
3. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, $R_C = 1k\Omega$ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation ?



(1) 100 μA (2) 7 μA (3) 40 μA (4) 10 μA

Official Ans. by NTA (3)

Sol.



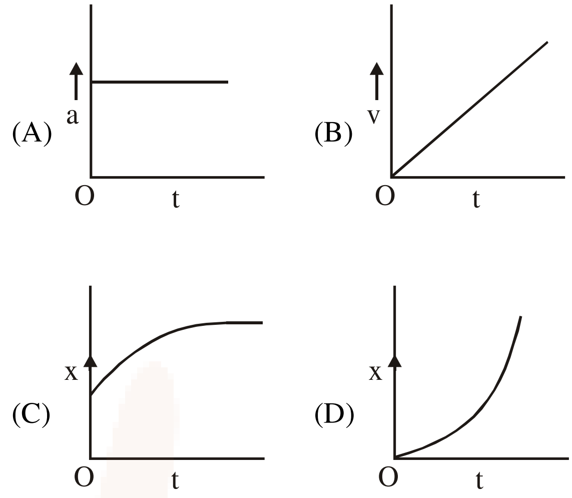
At saturation state, V_{CE} becomes zero

$$\Rightarrow i_C = \frac{10V}{1000\Omega} = 10mA$$

now current gain factor $\beta = \frac{i_C}{i_B}$

$$\Rightarrow i_B = \frac{10mA}{250} = 40\mu A$$

4. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

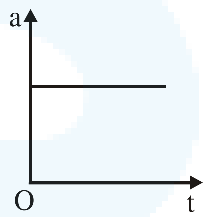


(1) (A), (B), (C) (2) (A)

(3) (A), (B), (D) (4) (B), (C)

Official Ans. by NTA (3)

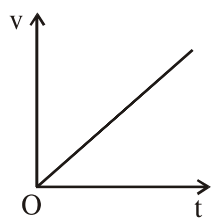
Sol. Given initial velocity u = 0 and acceleration is constant

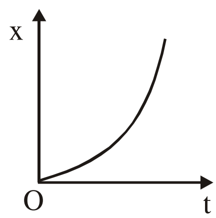


At time t

$v = 0 + at \Rightarrow v = at$

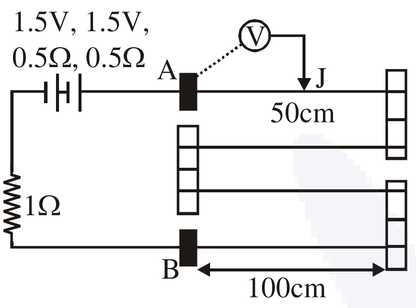
also $x = 0(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$





Graph (A) ; (B) and (D) are correct.

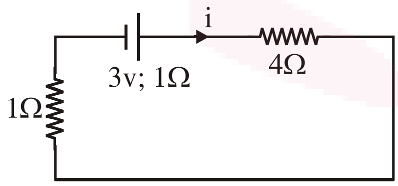
5. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :-



(1) 0.20 V (2) 0.25 V (3) 0.75 V (4) 0.50V

Official Ans. by NTA (2)

Sol.



Resistance of wire AB = 400 × 0.01 = 4Ω

i = $\frac{3}{6} = 0.5A$

Now voltmeter reading = i (Resistance of 50 cm length)

= (0.5A) (0.01 × 50) = 0.25 volt

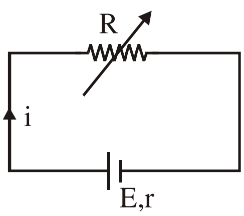
6. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :-

(1) R = 1000 r (2) R = 0.001 r

(3) R = 2r (4) R = r

Official Ans. by NTA (4)

Sol.



Current i = $\frac{E}{r+R}$

Power generated in R

P = i^2R

P = $\frac{E^2R}{(r+R)^2}$

for maximum power $\frac{dP}{dR} = 0$

$$E^2\left[\frac{(r+R)^2 \times 1 - R \times 2(r+R)}{(r+R)^4}\right] = 0$$

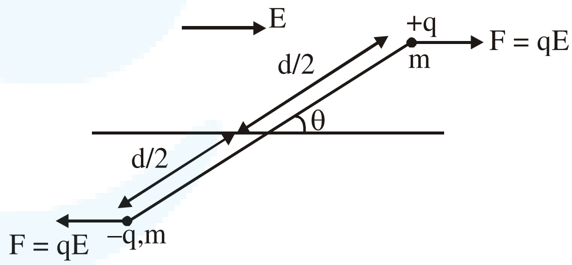
$\Rightarrow$ r = R

7. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is :-

(1) $\sqrt{\frac{qE}{2md}}$ (2) $2\sqrt{\frac{qE}{md}}$ (3) $\sqrt{\frac{2qE}{md}}$ (4) $\sqrt{\frac{qE}{md}}$

Official Ans. by NTA (3)

Sol.



moment of inertia (I) = $m\left(\frac{d}{2}\right)^2 \times 2 = \frac{md^2}{2}$

Now by τ = Iα

(qE) (d sin θ) = $\frac{md^2}{2}.\alpha$

α = $\left(\frac{2qE}{md}\right)\sin\theta$

for small θ

$\Rightarrow$ α = $\left(\frac{2qE}{md}\right)\theta$

$\Rightarrow$ Angular frequency ω = $\sqrt{\frac{2qE}{md}}$

8. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70m, then the minimum height of the transmitting antenna should be :

(Radius of the Earth = 6.4×10^6 m).

(1) 40 m (2) 51 m (3) 32 m (4) 20 m

Official Ans. by NTA (3)

Sol. Range = $\sqrt{2Rh_T}+\sqrt{2Rh_R}$

$50 \times 10^3 = \sqrt{2\times 6400\times 10^3\times h_T}+\sqrt{2\times 6400\times 10^3\times 70}$

by solving h_T = 32 m

9. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to :-

(1) 1 (2) 2

(3) 0.1 (4) 5

Official Ans. by NTA (1)

Sol. mass densities of all nuclei are same so their ratio is 1.

10. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star :-

(1) 305×10^{-9} radian

(2) 152.5×10^{-9} radian

(3) 610×10^{-9} radian

(4) 457.5×10^{-9} radian

Official Ans. by NTA (1)

Sol. Limit of resolution of telescope = $\dfrac{1.22\lambda}{D}$

$\theta = \dfrac{1.22\times 500\times 10^{-9}}{200\times 10^{-2}}$ $=305 \times 10^{-9}$ radian

11. The magnetic field of an electromagnetic wave is given by :-

$$\vec{B}=1.6\times 10^{-6}\cos\left(2\times 10^7 z+6\times 10^{15} t\right)\left(2\hat{i}+\hat{j}\right)\frac{Wb}{m^2}$$

The associated electric field will be :-

(1) $\vec{E}=4.8\times 10^2\cos\left(2\times 10^7 z+6\times 10^{15} t\right)\left(\hat{i}-2\hat{j}\right)\dfrac{V}{m}$

(2) $\vec{E}=4.8\times 10^2\cos\left(2\times 10^7 z-6\times 10^{15} t\right)\left(2\hat{i}+\hat{j}\right)\dfrac{V}{m}$

(3) $\vec{E}=4.8\times 10^2\cos\left(2\times 10^7 z-6\times 10^{15} t\right)\left(-2\hat{j}+\hat{i}\right)\dfrac{V}{m}$

(4) $\vec{E}=4.8\times 10^2\cos\left(2\times 10^7 z+6\times 10^{15} t\right)\left(-\hat{i}+2\hat{j}\right)\dfrac{V}{m}$

Official Ans. by NTA (1)

Allen answer is (4)

Sol. **If we use that direction of light propagation will be along $\vec{E}\times\vec{B}$. Then (4) option is correct.**

Detailed solution is as following.

magnitude of E = CB

$E = 3 \times 10^8 \times 1.6 \times 10^{-6} \times \sqrt{5}$

$E = 4.8 \times 10^2\sqrt{5}$

$\vec{E}$ and $\vec{B}$ are perpendicular to each other

$\Rightarrow \vec{E}.\vec{B} = 0$

$\Rightarrow$ either direction of $\vec{E}$ is $\hat{i}-2\hat{j}$ or $-\hat{i}+2\hat{j}$

from given option

Also wave propagation direction is parallel to

$\vec{E}\times\vec{B}$ which is $-\hat{k}$

$\Rightarrow \vec{E}$ is along $\left(-\hat{i}+2\hat{j}\right)$

12. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :-

(1) 1.9 mm (2) 1.7 mm

(3) 1.5 mm (4) 1.3 mm

Official Ans. by NTA (2)

Sol. Given

$\frac{Y_A}{Y_B} = \frac{7}{4}$ $L_A = 2m$ $A_A = \pi R^2$

$L_B = 1.5m$ $A_B = \pi(2mm)^2$

$$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$$

$$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2mm)^2 . Y_B}{1.5}$$

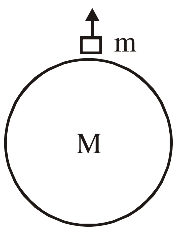
R = 1.74 mm

13. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon ? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon :-

(1) $\frac{E}{4}$ (2) $\frac{E}{16}$ (3) $\frac{E}{32}$ (4) $\frac{E}{64}$

Official Ans. by NTA (2)

Sol.



minimum energy required (E) = – (Potential energy of object at surface of earth)

$$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

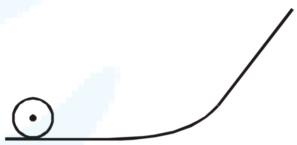
Now $M_{earth} = 64\ M_{moon}$

$$\rho.\frac{4}{3}\pi R_e^3 = 64.\frac{4}{3}\pi R_m^3 \Rightarrow R_e = 4R_m$$

$$\text{Now } \frac{E_{moon}}{E_{earth}} = \frac{M_{moon}}{M_{earth}}.\frac{R_{earth}}{R_{moon}} = \frac{1}{64} \times \frac{4}{1}$$

$$\Rightarrow E_{moon} = \frac{E}{16}$$

14. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by :-



(1) $\frac{14}{15}$ (2) $\frac{4}{5}$ (3) 1 (4) $\frac{2}{\sqrt{5}}$

Official Ans. by NTA (1)

Sol. for solid sphere

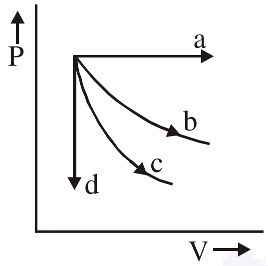
$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph.}$$

for solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{1}{2}mR^2.\frac{v^2}{R^2} = mgh_{cyl.}$$

$$\Rightarrow \frac{h_{sph.}}{h_{cyl.}} = \frac{7/5}{3/2} = \frac{14}{15}$$

15. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :-



(1) d a c b (2) a d c b

(3) a d b c (4) d a b c

Official Ans. by NTA (4)

Sol. isochoric → Process d

isobaric → Process a

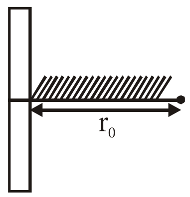
Adiabatic slope will be more than isothermal so

Isothermal → Process b

Adiabatic → Process c

order → d a b c

16. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :-



(1) $v \propto e^{+r/r_0}$ (2) $v \propto \ell n\left(\frac{r}{r_0}\right)$

(3) $v \propto \left(\frac{r}{r_0}\right)$ (4) $v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$

Official Ans. by NTA (4)

Sol. $\frac{1}{2}mV^2 = -q(V_f - V_i)$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$\frac{1}{2}mv^2 = \frac{-q\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_0}{r}\right)$$

$$v \propto \sqrt{\ell n\left(\frac{r}{r_0}\right)}$$

17. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :-

(1) 100 s (2) 20 s (3) 10 s (4) 50 s

Official Ans. by NTA (2)

Sol. $A = A_0e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$$\frac{A_0}{2} = A_0e^{-\gamma(2)}$$

$2 = e^{2\gamma}$

$\ell n2 = 2\gamma$

$$\gamma = \frac{\ell n2}{2}$$

$\because$ $A = A_0e^{-\gamma t}$

$$\ell n\frac{A_0}{A} = \gamma t$$

$$\ell n1000 = \frac{\ell n2}{2}t$$

$$2\left(\frac{3\ell n10}{\ell n2}\right) = t$$

$$\frac{6\ell n10}{\ell n2} = t$$

$t = 19.931$ sec

$t \approx 20$ sec

18. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is :-

(1) –48 V (2) –520 V

(3) 180 V (4) 320 V

Official Ans. by NTA (3)

Sol. $\vec{E} = (20x + 10)\hat{i}$

$$V_1 - V_2 \; - \int_{-5}^{1}(20x+10)dx$$

$$V_1 - V_2 = -\left(10x^2 + 10x\right)_{-5}^{1}$$

$V_1 - V_2 = 10(25 - 5 - 1 - 1)$

$V_1 - V_2 = 180$ V

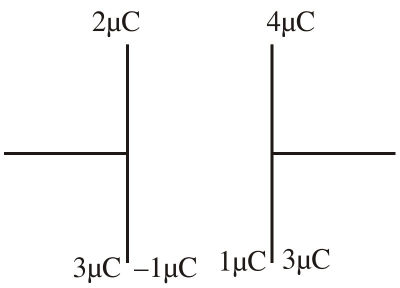
19. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:-

(1) 5V (2) 2V

(3) 3V (4) 1V

Official Ans. by NTA (4)

Sol.



Charges at inner plates are 1μC and –1μC

∴ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\mu C}{1\mu F} = \frac{1 \times 10^{-6} C}{1 \times 10^{-6} \text{Farad}} = 1V$$

20. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to :-

(1) 0.7%

(2) 0.2%

(3) 3.5%

(4) 6.8%

Official Ans. by NTA (4)

Sol. $T = \frac{30 \text{ sec}}{20}$ $\Delta T = \frac{1}{20}$ sec.

L = 55 cm ΔL = 1mm = 0.1 cm

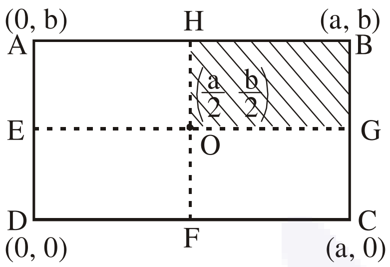
$$g = \frac{4\pi^2 L}{T^2}$$

percentage error in g is

$$\frac{\Delta g}{g} \times 100\% = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right)100\%$$

$$= \left(\frac{0.1}{55} + \frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right)100\% \simeq 6.8\%$$

21. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :-

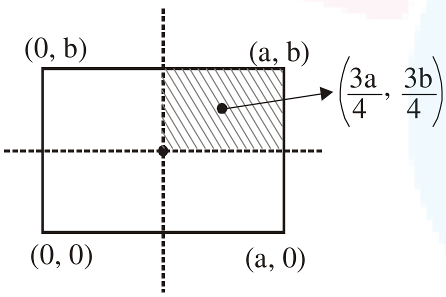


(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Official Ans. by NTA (4)

Sol.



$$x = \frac{M\frac{a}{2} - \frac{M}{4}\times\frac{3a}{4}}{M - \frac{M}{4}}$$

$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4}\times\frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

22. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :
[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K
Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg
Radius of Earth : 6.4×10^6 m
Gravitational acceleration on Earth = $10 ms^{-2}$]

(1) 650 K (2) 3×10^5 K

(3) 10^4 K (4) 800 K

Official Ans. by NTA (3)

Sol. $v_{rms} = \sqrt{\frac{3RT}{m}}$ $v_{escape} = \sqrt{2gR_e}$

$v_{rms} = v_{escape}$

$$\frac{3RT}{m} = 2gR_e$$

$$\frac{3\times1.38\times10^{-23}\times6.02\times10^{26}}{2}\times T$$

$= 2 \times 10 \times 6.4 \times 10^6$

$$T = \frac{4\times10\times6.4\times10^6}{3\times1.38\times6.02\times10^3} = 10\times10^3 = 10^4 k$$

Note : Question gives avogadro Number N_A = 6.02 × 10^{26} /kg but we take N_A = 6.02 × 10^{26} /kmol.

23. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be :-

(1) 20 cm (2) 10 cm

(3) 25 cm (4) 30 cm

Official Ans. by NTA (2)

Sol.

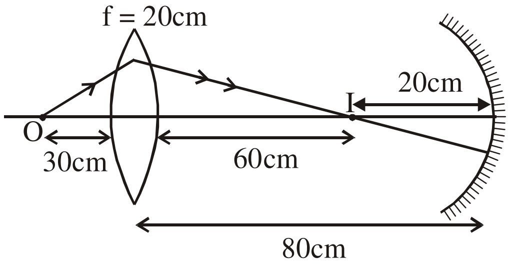


Image formed by lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{30}=\frac{1}{20}$$

v = +60 cm

If image position does not change even when mirror is removed it means image formed by lens is formed at centre of curvature of spherical mirror.

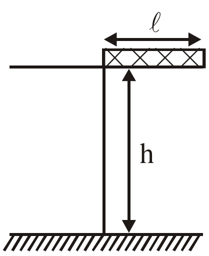
Radius of curvature of mirror = 80 – 60 = 20 cm

$\Rightarrow$ focal length of mirror f = 10 cm

for virtual image, object is to be kept between focus and pole.

$\Rightarrow$ maximum distance of object from spherical mirror for which virtual image is formed, is 10cm.

24. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time τ = 0.01s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :-



(1) 0.02 (2) 0.28

(3) 0.5 (4) 0.3

Official Ans. by NTA (3)

Sol. Angular impulse = change in angular momentum

$\tau\,\Delta t = \Delta L$

$$mg\frac{\ell}{2}\times .01 = \frac{m\ell^2}{3}\omega$$

$$\omega=\frac{3g\times 0.01}{2\ell}$$

$$=\frac{3\times 10\times .01}{2\times 0.3}$$

$$=\frac{1}{2}=0.5\text{ rad/s}$$

time taken by rod to hit the ground

$$t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 5}{10}}=1\text{ sec.}$$

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

25. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :-

(1) $S^{3/2}I^{1/2}h^{0}$ (2) $S^{1/2}I^{1/2}h^{0}$

(3) $S^{1/2}I^{1/2}h^{-1}$ (4) $S^{1/2}I^{3/2}h^{-1}$

Official Ans. by NTA (2)

Sol. $p = k\, s^aI^bh^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a(ML^2)^b(ML^2T^{-1})^c$

a + b + c = 1

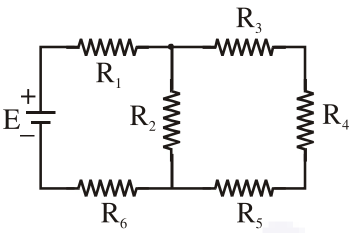
2 b + 2c = 1

–2a – c = –1

$a = \frac{1}{2}$ $b = \frac{1}{2}$ c = 0

$s^{1/2}I^{1/2}h^0$

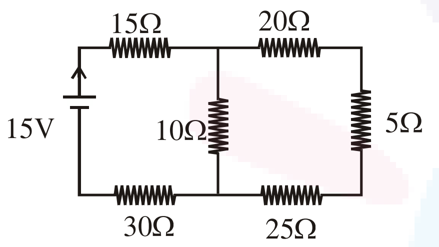
26. In the figure shown, what is the current (in Ampere) drawn from the battery ? You are given:
$R_1 = 15\Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\Omega$, $R_5 = 25\Omega$, $R_6 = 30\ \Omega$, $E = 15$ V

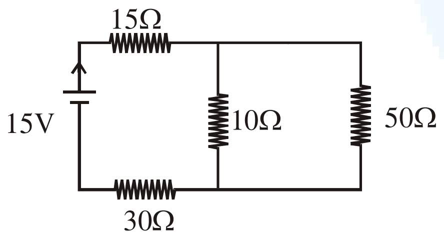


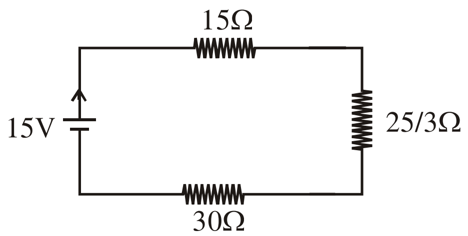
(1) 7/18 (2) 13/24 (3) 9/32 (4) 20/3

Official Ans. by NTA (3)

Sol.







$$R_{eq} = 15 + \frac{25}{3} + 30 = \frac{45+25+90}{3} = \frac{160}{3}$$

$$I = \frac{E}{R_{eq}} = \frac{15\times 3}{160} = \frac{9}{32}\text{amp.}$$

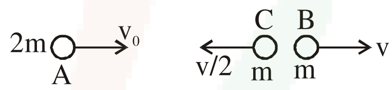
27. A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively :-

(1) $2\lambda_A, \lambda_A$ (2) $\lambda_A, 2\lambda_A$

(3) $\lambda_A, \frac{\lambda_A}{2}$ (4) $\frac{\lambda_A}{2}, \lambda_A$

Official Ans. by NTA (4)

Sol.



let mass of B and C is m each.

By momentum conservation

$$2mv_0 = mv - \frac{mv}{2}$$

$v = 4v_0$

$p_A = 2mv_0$ $p_B = 4mv_0$ $p_c = 2mv_0$

De-Broglie wavelength $\lambda = \frac{h}{p}$

$$\lambda_A = \frac{h}{2mv_0}\ ;\ \lambda_B = \frac{h}{4mv_0}\ ;\ \lambda_C = \frac{h}{2mv_0}$$

28. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.

If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :-

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - \frac{v_2}{2}$

(3) $v_4 - v_2$ (4) $v_4 + v_2$

Official Ans. by NTA (3)

Sol. Applying linear momentum conservation

$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$

$m_1v_1 + 0.5\ m_1v_2 = m_1(0.5\ v_1) + 0.5\ m_1v_4$

$0.5\ m_1v_1 = 0.5\ m_1(v_4 - v_2)$

$v_1 = v_4 - v_2$

29. Let $|\vec{A}_1| = 3$, $|\vec{A}_2| = 5$ and $|\vec{A}_1 + \vec{A}_2| = 5$. The value of $(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$ is :-

(1) –112.5 (2) –106.5 (3) –118.5 (4) –99.5

Official Ans. by NTA (3)

Sol. $|\vec{A}_1| = 3 \qquad |\vec{A}_2| = 5 \qquad |\vec{A}_1 + \vec{A}_2| = 5$

$$|\vec{A}_1 + \vec{A}_2| = \sqrt{|\vec{A}_1|^2 + |\vec{A}_2|^2 + 2|\vec{A}_1||\vec{A}_2|\cos\theta}$$

$$5 = \sqrt{9 + 25 + 2\times3\times5\cos\theta}$$

$$\cos\theta = -\frac{9}{2\times3\times5} = -\frac{3}{10}$$

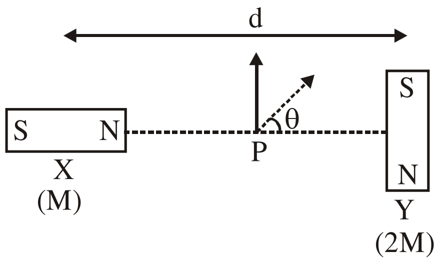
$$(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$$

$$= 6|\vec{A}_1|^2 + 9\vec{A}_1.\vec{A}_2 - 4\vec{A}_1\vec{A}_2 - 6|\vec{A}_2|^2$$

$$54 + 5\times3\times5\left(-\frac{3}{10}\right) - 6\times25$$

$$= 54 - 150 - \frac{45}{2} = -118.5$$

30. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle θ = 45° with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant ? (d is much larger than the dimensions of the dipole)



(1) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

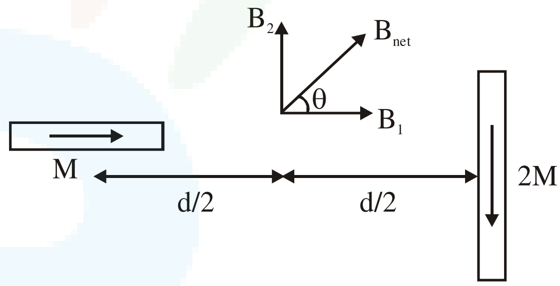
(2) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

(3) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(4) 0

Official Ans. by NTA (4)

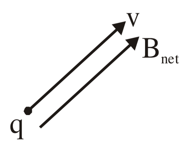
Sol.



$$B_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}; \qquad B_2 = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}$$

$B_1 = B_2$

$\Rightarrow B_{net}$ is at 45° (θ = 45°)



velocity of charge and B_{net} are parallel so by $\vec{F} = q(\vec{v}\times\vec{B})$ force on charge particle is zero.