



FINAL JEE–MAIN EXAMINATION – APRIL, 2019

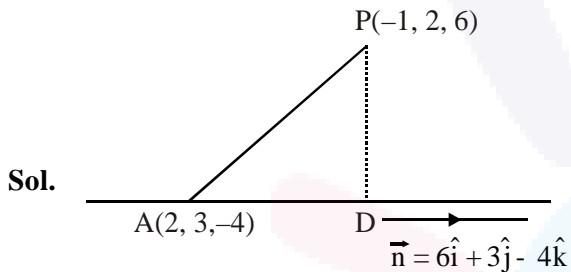
Held On Wednesday 10th APRIL, 2019

TIME: 02 : 30 PM To 05 : 30 PM

1. The distance of the point having position vector  $-\hat{i} + 2\hat{j} + 6\hat{k}$  from the straight line passing through the point  $(2, 3, -4)$  and parallel to the vector,  $6\hat{i} + 3\hat{j} - 4\hat{k}$  is :

- (1) 7 (2)  $4\sqrt{3}$   
 (3)  $2\sqrt{13}$  (4) 6

Official Ans. by NTA (1)



$$AD = \frac{|\vec{AP} \cdot \vec{n}|}{|\vec{n}|} = \sqrt{61}$$

$$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$$

2. If both the mean and the standard deviation of 50 observations  $x_1, x_2, \dots, x_{50}$  are equal to 16, then the mean of  $(x_1 - 4)^2, (x_2 - 4)^2, \dots, (x_{50} - 4)^2$  is :

- (1) 525 (2) 380  
 (3) 480 (4) 400

Official Ans. by NTA (4)

Sol. Mean  $(\mu) = \frac{\sum x_i}{50} = 16$

standard deviation  $(\sigma) = \sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$$

$\Rightarrow$  New mean

$$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8 \sum x_i}{50}$$

$$= (256) \times 2 + 16 - 8 \times 16 = 400$$

3. A perpendicular is drawn from a point on the line  $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$  to the plane  $x + y + z = 3$  such that the foot of the perpendicular Q also lies on the plane  $x - y + z = 3$ . Then the co-ordinates of Q are :

- (1)  $(2, 0, 1)$  (2)  $(4, 0, -1)$   
 (3)  $(-1, 0, 4)$  (4)  $(1, 0, 2)$

Official Ans. by NTA (1)

- Sol. Let point P on the line is  $(2\lambda + 1, -\lambda - 1, \lambda)$  foot of perpendicular Q is given by

$$\frac{x - 2\lambda - 1}{1} = \frac{y + \lambda + 1}{1} = \frac{z - \lambda}{1} = \frac{-(2\lambda - 3)}{3}$$

$\therefore$  Q lies on  $x + y + z = 3$  &  $x - y + z = 3$   
 $\Rightarrow x + z = 3$  &  $y = 0$

$$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda + 3}{3} \Rightarrow \lambda = 0$$

$$\Rightarrow Q \text{ is } (2, 0, 1)$$

4. The tangent and normal to the ellipse  $3x^2 + 5y^2 = 32$  at the point  $P(2, 2)$  meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

- (1)  $\frac{14}{3}$  (2)  $\frac{16}{3}$  (3)  $\frac{68}{15}$  (4)  $\frac{34}{15}$

Official Ans. by NTA (3)

Sol.  $3x^2 + 5y^2 = 32$

$$\left. \frac{dy}{dx} \right|_{(2,2)} = -\frac{3}{5}$$

Tangent :  $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal :  $y - 2 = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area is  $= \frac{1}{2}(QR) \times 2 = QR = \frac{68}{15}$ .



5. Let  $\lambda$  be a real number for which the system of linear equations

$$\begin{aligned} x + y + z &= 6 \\ 4x + \lambda y - \lambda z &= \lambda - 2 \\ 3x + 2y - 4z &= -5 \end{aligned}$$

has infinitely many solutions. Then  $\lambda$  is a root of the quadratic equation.

(1)  $\lambda^2 - 3\lambda - 4 = 0$       (2)  $\lambda^2 - \lambda - 6 = 0$   
 (3)  $\lambda^2 + 3\lambda - 4 = 0$       (4)  $\lambda^2 + \lambda - 6 = 0$

Official Ans. by NTA (2)

Sol.  $D = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

6. The smallest natural number  $n$ , such that the

coefficient of  $x$  in the expansion of  $\left(x^2 + \frac{1}{x^3}\right)^n$

is  ${}^n C_{23}$ , is :

(1) 35                      (2) 38  
 (3) 23                      (4) 58

Official Ans. by NTA (2)

Sol.  $T_r = \sum_{r=0}^n {}^n C_r x^{2n-2r} \cdot x^{-3r}$

$$2n - 5r = 1 \Rightarrow 2n = 5r + 1$$

for  $r = 15$ ,  $n = 38$

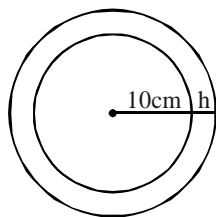
smallest value of  $n$  is 38.

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm<sup>3</sup>/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1)  $\frac{1}{9\pi}$       (2)  $\frac{5}{6\pi}$       (3)  $\frac{1}{18\pi}$       (4)  $\frac{1}{36\pi}$

Official Ans. by NTA (3)

Sol.  $V = \frac{4}{3}\pi((10+h)^3 - 10^3)$



$$\frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$$

$$-50 = 4\pi(10+5)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{1}{18} \text{ cm/min}$$

8. If  $5x + 9 = 0$  is the directrix of the hyperbola  $16x^2 - 9y^2 = 144$ , then its corresponding focus is :

(1)  $\left(-\frac{5}{3}, 0\right)$                       (2) (5, 0)

(3) (-5, 0)                      (4)  $\left(\frac{5}{3}, 0\right)$

Official Ans. by NTA (3)

Sol.  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$$a = 3, b = 4 \text{ \& } e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$$

corresponding focus will be  $(-ae, 0)$  i.e.,  $(-5, 0)$ .

9. The sum  $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \dots$   
 $+ \frac{1^3+2^3+3^3+\dots+15^3}{1+2+3+\dots+15} - \frac{1}{2}(1+2+3+\dots+15)$

(1) 1240                      (2) 1860  
 (3) 660                      (4) 620

Official Ans. by NTA (4)

Sol.  $\text{Sum} = \sum_{n=1}^{15} \frac{1^3+2^3+\dots+n^3}{1+2+\dots+n} - \frac{1}{2} \cdot \frac{15 \cdot 16}{2}$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2 - (n-1))}{6} - 60$$

$$= \frac{15 \cdot 16 \cdot 17}{6} - 60 = 620$$



10. If the line  $ax + y = c$ , touches both the curves

$x^2 + y^2 = 1$  and  $y^2 = 4\sqrt{2}x$ , then  $|c|$  is equal to :

- (1)  $1/2$  (2)  $2$   
 (3)  $\sqrt{2}$  (4)  $\frac{1}{\sqrt{2}}$

**Official Ans. by NTA (3)**

**Sol.** Tangent to  $y^2 = 4\sqrt{2}x$  is  $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to  $x^2 + y^2 = 1$

$$\Rightarrow \left| \frac{\sqrt{2}/m}{\sqrt{1+m^2}} \right| = 1 \Rightarrow m = \pm 1$$

$\Rightarrow$  Tangent will be  $y = x + \sqrt{2}$  or  $y = -x - \sqrt{2}$   
 compare with  $y = -ax + C$

$$\Rightarrow a = \pm 1 \text{ \& } C = \pm\sqrt{2}$$

11. If  $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$ ,

where  $-1 \leq x \leq 1, -2 \leq y \leq 2, x \leq \frac{y}{2}$ ,

then for all  $x, y, 4x^2 - 4xy \cos \alpha + y^2$  is equal to

- (1)  $4 \sin^2 \alpha - 2x^2y^2$  (2)  $4 \cos^2 \alpha + 2x^2y^2$   
 (3)  $4 \sin^2 \alpha$  (4)  $2 \sin^2 \alpha$

**Official Ans. by NTA (3)**

**Sol.**  $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos \alpha$$

$$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\Rightarrow \left( \cos \alpha - \frac{xy}{2} \right)^2 = (1-x^2) \left( 1 - \frac{y^2}{4} \right)$$

$$x^2 + \frac{y^2}{4} - xy \cos \alpha = 1 - \cos^2 \alpha = \sin^2 \alpha$$

12. If  $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$ , where  $c$  is a constant of integration, then  $g(-1)$  is equal to :

- (1)  $-\frac{5}{2}$  (2)  $1$   
 (3)  $-\frac{1}{2}$  (4)  $-1$

**Official Ans. by NTA (1)**

**Sol.** Let  $x^2 = t$   $2x dx = dt$

$$\Rightarrow \frac{1}{2} \int t^2 \cdot e^{-t} dt = \frac{1}{2} \left[ -t^2 \cdot e^{-t} + \int 2t \cdot e^{-t} dt \right]$$

$$= \frac{1}{2} \left( -t^2 \cdot e^{-t} \right) + \left( -t \cdot e^{-t} + \int 1 \cdot e^{-t} dt \right)$$

$$= -\frac{t^2 e^{-t}}{2} - t e^{-t} - e^{-t} = \left( -\frac{t^2}{2} - t - 1 \right) e^{-t}$$

$$= \left( -\frac{x^4}{2} - x^2 - 1 \right) e^{-x^2} + C$$

$$g(x) = -1 - x^2 - \frac{x^4}{2} + k e^{x^2}$$

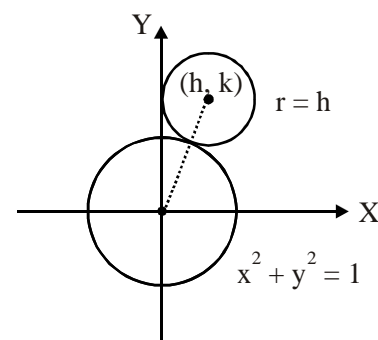
for  $k = 0$

$$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

13. The locus of the centres of the circles, which touch the circle,  $x^2 + y^2 = 1$  externally, also touch the y-axis and lie in the first quadrant, is :

- (1)  $y = \sqrt{1+4x}, x \geq 0$   
 (2)  $x = \sqrt{1+4y}, y \geq 0$   
 (3)  $x = \sqrt{1+2y}, y \geq 0$   
 (4)  $y = \sqrt{1+2x}, x \geq 0$

**Official Ans. by NTA (4)**



**Sol.**

$$\sqrt{h^2 + k^2} = |h| + 1$$

$$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$$

$$\Rightarrow y^2 = 1 + 2x$$

$$\Rightarrow y = \sqrt{1+2x}; x \geq 0.$$



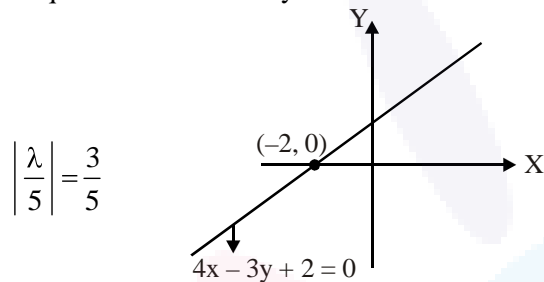
14. Lines are drawn parallel to the line  $4x - 3y + 2 = 0$ , at a distance  $\frac{3}{5}$  from the origin.

Then which one of the following points lies on any of these lines ?

- (1)  $\left(-\frac{1}{4}, \frac{2}{3}\right)$       (2)  $\left(\frac{1}{4}, \frac{1}{3}\right)$   
 (3)  $\left(-\frac{1}{4}, -\frac{2}{3}\right)$       (4)  $\left(\frac{1}{4}, -\frac{1}{3}\right)$

**Official Ans. by NTA (1)**

**Sol.** Required line is  $4x - 3y + \lambda = 0$



$$\Rightarrow \lambda = \pm 3.$$

So, required equation of line is  $4x - 3y + 3 = 0$  and  $4x - 3y - 3 = 0$

(1)  $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$

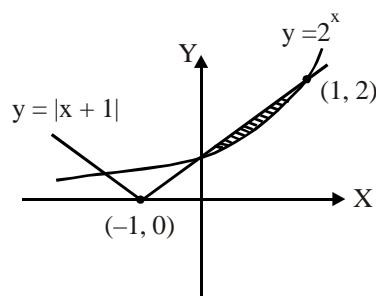
15. The area (in sq. units) of the region bounded by the curves  $y = 2^x$  and  $y = |x + 1|$ , in the first quadrant is :

- (1)  $\frac{3}{2} - \frac{1}{\log_e 2}$       (2)  $\frac{1}{2}$   
 (3)  $\log_e 2 + \frac{3}{2}$       (4)  $\frac{3}{2}$

**Official Ans. by NTA (1)**

**Sol.** Required Area

$$\int_0^1 ((x+1) - 2^x) dx$$



$$= \left( \frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right)_0^1$$

$$= \left( \frac{1}{2} + 1 - \frac{2}{\ln 2} \right) - \left( 0 + 0 - \frac{1}{\ln 2} \right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

16. If the plane  $2x - y + 2z + 3 = 0$  has the distances

$\frac{1}{3}$  and  $\frac{2}{3}$  units from the planes  $4x - 2y + 4z + \lambda = 0$

and  $2x - y + 2z + \mu = 0$ , respectively, then the maximum value of  $\lambda + \mu$  is equal to :

- (1) 15      (2) 5  
 (3) 13      (4) 9

**Official Ans. by NTA (3)**

**Sol.**  $4x - 2y + 4z + 6 = 0$

$$\frac{|\lambda - 6|}{\sqrt{16 + 4 + 16}} = \frac{|\lambda - 6|}{6} = \frac{1}{3}$$

$$|\lambda - 6| = 2$$

$$\lambda = 8, 4$$

$$\frac{|\mu - 3|}{\sqrt{4 + 4 + 1}} = \frac{2}{3}$$

$$|\mu - 3| = 2$$

$$\mu = 5, 1$$

$\therefore$  Maximum value of  $(\mu + \lambda) = 13$ .

17. If  $z$  and  $w$  are two complex numbers such that

$$|zw| = 1 \text{ and } \arg(z) - \arg(w) = \frac{\pi}{2}, \text{ then :}$$

- (1)  $\bar{z}w = i$       (2)  $\bar{z}w = -i$   
 (3)  $z\bar{w} = \frac{1-i}{\sqrt{2}}$       (4)  $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

**Official Ans. by NTA (2)**

**Sol.**  $|z| \cdot |w| = 1$   $z = re^{i(\theta + \pi/2)}$  and  $w = \frac{1}{r} e^{i\theta}$

$$\bar{z} \cdot w = e^{-i(\theta + \pi/2)} \cdot e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$z \cdot \bar{w} = e^{i(\theta + \pi/2)} \cdot e^{-i\theta} = e^{i(\pi/2)} = i$$



18. Let a, b and c be in G. P. with common ratio r, where  $a \neq 0$  and  $0 < r \leq \frac{1}{2}$ . If 3a, 7b and 15c are the first three terms of an A. P., then the 4<sup>th</sup> term of this A. P. is :

(1)  $\frac{7}{3}a$  (2) a

(3)  $\frac{2}{3}a$  (4) 5a

**Official Ans. by NTA (2)**

**Sol.**  $b = ar$   
 $c = ar^2$   
 3a, 7b and 15 c are in A.P.  
 $\Rightarrow 14b = 3a + 15c$   
 $\Rightarrow 14(ar) = 3a + 15 ar^2$   
 $\Rightarrow 14r = 3 + 15r^2$   
 $\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r-1)(5r-3) = 0$   
 $r = \frac{1}{3}, \frac{3}{5}$ .

Only acceptable value is  $r = \frac{1}{3}$ , because

$$r \in \left(0, \frac{1}{2}\right]$$

$$\therefore c, d = 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$$

$$\therefore 4^{\text{th}} \text{ term} = 15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$$

19. The integral  $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$  equal to :

- (1)  $3^{7/6} - 3^{5/6}$
- (2)  $3^{5/3} - 3^{1/3}$
- (3)  $3^{4/3} - 3^{1/3}$
- (4)  $3^{5/6} - 3^{2/3}$

**Official Ans. by NTA (1)**

**Sol.**  $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x \cdot \sin x} dx$   
 $= \int \frac{\tan^{2/3} x \cdot \sec^2 x \cdot dx}{\tan^2 x}$   
 $= \int \frac{\sec^2 x}{\tan^{4/3} x} \cdot dx \quad \{\tan x = t, \sec^2 x dx = dt\}$   
 $= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3(t^{-1/3})$   
 $\Rightarrow I = -3 \tan(x)^{-1/3}$   
 $\Rightarrow I = \frac{3}{(\tan x)^{1/3}} \Big|_{\pi/6}^{\pi/3} = -3 \left[ \frac{1}{(\sqrt{3})^{1/3}} - (\sqrt{3})^{1/3} \right]$   
 $= 3 \left( 3^{1/3} - \frac{1}{3^{1/6}} \right) = 3^{7/6} - 3^{5/6}$

20. Let  $y = y(x)$  be the solution of the differential equation,  $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$ ,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , such that  $y(0) = 1$ . Then :

- (1)  $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$
- (2)  $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$
- (3)  $y\left(\frac{\pi}{4}\right) - y\left(\frac{-\pi}{4}\right) = \sqrt{2}$
- (4)  $y\left(\frac{\pi}{4}\right) + y\left(\frac{-\pi}{4}\right) = \frac{\pi^2}{2} + 2$

**Official Ans. by NTA (2)**



**Sol.**  $\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

I.F =  $e^{\int \tan x dx} = e^{\ln \sec x} = \sec x$

$\therefore y \cdot \sec x = \int (2x + x^2 \tan x) \sec x dx$

$= \int 2x \sec x dx + \int x^2 (\sec x \tan x) dx$

$y \sec x = x^2 \sec x + \lambda$

$\Rightarrow y = x^2 + \lambda \cos x$

$y(0) = 0 + \lambda = 1 \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

**21.** Let  $a_1, a_2, a_3, \dots$  be an A. P. with  $a_6 = 2$ . Then the common difference of this A. P., which maximises the produce  $a_1 a_4 a_5$ , is :

(1)  $\frac{6}{5}$  (2)  $\frac{8}{5}$

(3)  $\frac{2}{3}$  (4)  $\frac{3}{2}$

**Official Ans. by NTA (2)**

**Sol.** Let  $a$  is first term and  $d$  is common difference then,  $a + 5d = 2$  (given) ... (1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

$f'(d) = 0 \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0$  at  $d = 8/5$

$\Rightarrow d = \frac{8}{5}$

**22.** The angles A, B and C of a triangle ABC are in A.P. and  $a : b = 1 : \sqrt{3}$ . If  $c = 4$  cm, then the area (in sq. cm) of this triangle is :

(1)  $4\sqrt{3}$  (2)  $\frac{2}{\sqrt{3}}$

(3)  $2\sqrt{3}$  (4)  $\frac{4}{\sqrt{3}}$

**Official Ans. by NTA (3)**

**Sol.**  $\angle B = \frac{\pi}{3}$ , by sine Rule

$\sin A = \frac{1}{2}$

$\Rightarrow A = 30^\circ, a = 2, b = 2\sqrt{3}, c = 4$

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$  sq. cm

**23.** Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

**Official Ans. by NTA (3)**

**Sol.**  $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\Rightarrow n = 7$ .

**24.** Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :

(1) 210 (2) 190

(3) 170 (4) 180

**Official Ans. by NTA (3)**

**Sol.** Total cases = number of diagonals  
 $= {}^{20}C_2 - 20 = 170$



25. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to :}$$

- (1) 6 (2) 1  
(3) 0 (4) -4

**Official Ans. by NTA (3)**

**Sol.** By expansion, we get  
 $-5x^3 + 30x - 30 + 5x = 0$   
 $\Rightarrow -5x^3 + 35x - 30 = 0$   
 $\Rightarrow x^3 - 7x + 6 = 0$ , All roots are real  
 So, sum of roots = 0

26. Let  $f(x) = \log_e(\sin x)$ , ( $0 < x < \pi$ ) and  $g(x) = \sin^{-1}(e^{-x})$ , ( $x \geq 0$ ). If  $\alpha$  is a positive real number such that  $a = (fog)'(\alpha)$  and  $b = (fog)(\alpha)$ , then :

- (1)  $a\alpha^2 - b\alpha - a = 0$   
 (2)  $a\alpha^2 + b\alpha - a = -2\alpha^2$   
 (3)  $a\alpha^2 + b\alpha + a = 0$   
 (4)  $a\alpha^2 - b\alpha - a = 1$

**Official Ans. by NTA (4)**

**Sol.**  $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$   
 $(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

27. If the tangent to the curve  $y = \frac{x}{x^2 - 3}$ ,  $x \in \mathbb{R}$ ,

( $x \neq \pm\sqrt{3}$ ), at a point  $(\alpha, \beta) \neq (0, 0)$  on it is parallel to the line  $2x + 6y - 11 = 0$ , then :

- (1)  $|6\alpha + 2\beta| = 19$   
 (2)  $|2\alpha + 6\beta| = 11$   
 (3)  $|6\alpha + 2\beta| = 9$   
 (4)  $|2\alpha + 6\beta| = 19$

**Official Ans. by NTA (1)**

**Sol.**  $\frac{dy}{dx}\bigg|_{(\alpha, \beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given that :

$$\frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}$$

$$\Rightarrow \alpha = 0, \pm 3 \quad (\alpha \neq 0)$$

$$\Rightarrow \beta = \pm \frac{1}{2}. \quad (\beta \neq 0)$$

$$|6\alpha + 2\beta| = 19$$

28. The number of real roots of the equation

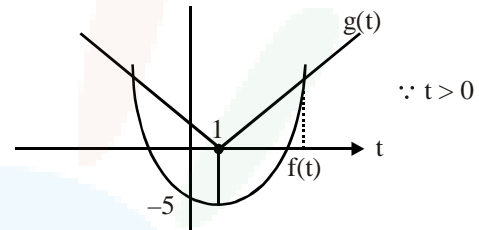
$$5 + |2^x - 1| = 2^x (2^x - 2) \text{ is :}$$

- (1) 2 (2) 3  
(3) 4 (4) 1

**Official Ans. by NTA (4)**

**Sol.** Let  $2^x = t$   
 $5 + |t - 1| = t^2 - 2t$   
 $\Rightarrow |t - 1| = (t^2 - 2t - 5)$   
 $\qquad\qquad\qquad g(t) \qquad\qquad f(t)$

From the graph



So, number of real root is 1.

29. If  $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$ , then  $a + b$  is equal to :-

- (1) -7 (2) -4  
(3) 5 (4) 1

**Official Ans. by NTA (1)**

**Sol.**  $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$

$$1 - a + b = 0 \quad \dots(i)$$

$$2 - a = 5 \quad \dots(ii)$$

$$\Rightarrow a + b = -7.$$

30. The negation of the boolean expression

$\sim s \vee (\sim r \wedge s)$  is equivalent to :

- (1)  $r$  (2)  $s \wedge r$   
(3)  $s \vee r$  (4)  $\sim s \wedge \sim r$

**Official Ans. by NTA (2)**

**Sol.**  $\sim(\sim s \vee (\sim r \wedge s))$

$$s \wedge (r \vee \sim s)$$

$$(s \wedge r) \vee (s \wedge \sim s)$$

$$(s \wedge r) \vee (c)$$

$$(s \wedge r)$$