FINAL JEE–MAIN EXAMINATION – JANUARY, 2019
Held On Thursday 10th JANUARY, 2019
TIME: 02 : 30 PM To 05 : 30 PM

1. Two forces P and Q of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle is :

(1) 30° (2) 60° (3) 90° (4) 120°

Ans. (4)

Sol. $4F^2 + 9F^2 + 12F^2 \cos\theta = R^2$

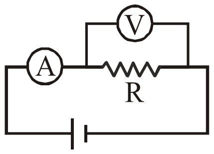
$4F^2 + 36F^2 + 24F^2 \cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2 \cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$\cos\theta = -\frac{12F^2}{24F^2} = -\frac{1}{2}$

2. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :



(1) 350Ω (2) 570Ω (3) 35 Ω (4) 600 Ω

Ans. (2)

Sol. $0.95\,R = \frac{R R_\upsilon}{R + R_\upsilon}$

$0.95 \times 30 = 0.05\,R_\upsilon$

$R_\upsilon = 19 \times 30 = 570\,\Omega$

3. An unknown metal of mass 192 g heated to a temperature of 100ºC was immersed into a brass calorimeter of mass 128 g containing 240 g of water a temperature of 8.4°C Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C (Specific heat of brass is 394 J kg^{-1} K^{-1})

(1) 1232 J kg^{-1} K^{-1} (2) 458 J kg^{-1} K^{-1}

(3) 654 J kg^{-1} K^{-1} (4) 916 J kg^{-1} K^{-1}

Ans. (4)

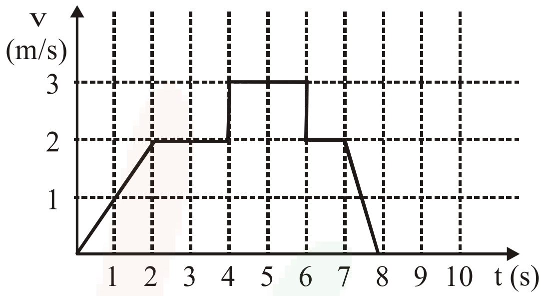
Sol. $192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

$+ 240 \times 4200 \times (21.5 - 8.4)$

$\Rightarrow S = 916$

4. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?



(1) 6 m (2) 9 m (3) 3 m (4) 10 m

Ans. (2)

S = Area under graph

$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9$ m

5. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is :

(1) 437.5 J (2) 637.5 J

(3) 740 J (4) 540 J

Ans. (1)

$L\frac{di}{dt} = 25$

$L \times \frac{15}{1} = 25$

$L = \frac{5}{3}$ H

$\Delta E = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2) = \frac{5}{6} \times 525 = 437.5$ J

6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :

(1) 11×10^{-5} W (2) 11×10^{-4} W

(3) 11×10^{5} W (4) 11×10^{-3} W

Ans. (1)

$P = I^2R$

$4.4 = 4 \times 10^{-6}$ R

$R = 1.1 \times 10^6\ \Omega$

$$P' = \frac{11^2}{R} = \frac{11^2}{1.1} \times 10^{-6} = 11 \times 10^{-5} W$$

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

(1) 4260 ± 80 cm^3 (2) 4300 ± 80 cm^3

(3) 4264.4 ± 81.0 cm^3 (4) 4264 ± 81 cm^3

Ans. (1)

$$\frac{\Delta V}{V} = 2\frac{\Delta d}{d} + \frac{\Delta h}{h} = 2\left(\frac{0.1}{12.6}\right) + \frac{0.1}{34.2}$$

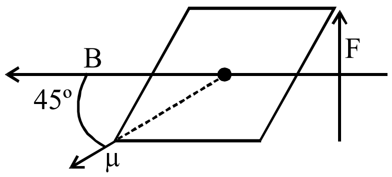
$$V = 12.6 \times \frac{\pi}{4} \times 314.2$$

8. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic neeedle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is :

(1) 3.6×10^{-5} N (2) 6.5×10^{-5} N

(3) 1.3×10^{-5} N (4) 1.8×10^{-5} N

Ans. (2)



$$\mu B \sin 45° = F\frac{\ell}{2}\sin 45°$$

$F = 2\mu B$

9. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot ?

(1) 2750 kHz (2) 2000 kHz

(3) 2250 kHz (4) 2900 kHz

Ans. (2)

$$f_{carrier} = \frac{250}{0.1} = 2500\,KHZ$$

∴ Range of signal = 2250 Hz to 2750 Hz

Now check all options : for 2000 KHZ

f_{mod} = 200 Hz

∴ Range = 1800 KHZ to 2200 KHZ

10. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then :

(1) $T_h = 0.5\ T_c$ (2) $T_h = 2\ T_c$

(3) $T_h = 1.5\ T_c$ (4) $T_h = T_c$

Ans. (4)

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

$$T_h = 2\pi\sqrt{\frac{mR^2}{(2\mu)B}}$$

$$T_C = 2\pi\sqrt{\frac{1/2mR^2}{\mu B}}$$

11. The electric field of a plane polarized electromagnetic wave in free space at time t= 0 is given by an expression

$$\vec{E}(x,y) = 10\hat{j}\ \cos\ [(6x + 8z)]$$

The magnetic field $\vec{B}$ (x, z, t) is given by : (c is the velocity of light)

(1) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos\left[\left(6x-8z+10ct\right)\right]$

(2) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos\left[\left(6x+8z-10ct\right)\right]$

(3) $\frac{1}{c}\left(6\hat{k}+8\hat{i}\right)\cos\left[\left(6x+8z-10ct\right)\right]$

(4) $\frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos\left[\left(6x+8z+10ct\right)\right]$

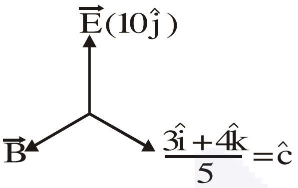
Ans. (2)

$$\vec{E}=10\hat{j}\cos\left[\left(6\hat{i}+8\hat{k}\right)\cdot\left(x\hat{i}+z\hat{k}\right)\right]$$

$$= 10\hat{j}\cos[\vec{K}\cdot\vec{r}]$$

$\therefore$ $\vec{K}=6\hat{i}+8\hat{k}$; direction of waves travel.

i.e. direction of 'c'.



$\therefore$ Direction of $\hat{B}$ will be along

$$\hat{C}\times\hat{E}=\frac{-4\hat{i}+3\hat{k}}{5}$$

Mag. of $\vec{B}$ will be along $\hat{C}\times\hat{E}=\frac{-4\hat{i}+3\hat{k}}{5}$

Mag. of $\vec{B}$ $=\frac{E}{C}=\frac{10}{C}$

$$\therefore\ \vec{B}=\frac{10}{C}\left(\frac{-4\hat{i}+3\hat{k}}{5}\right)=\frac{\left(-8\hat{i}+6\hat{k}\right)}{C}$$

12. Condiser the nuclear fission

$Ne^{20} \rightarrow 2He^4 + C^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement :

(1) 8.3 MeV energy will be released

(2) energy of 12.4 MeV will be supplied

(3) energy of 11.9 MeV has to be supplied

(4) energy of 3.6 MeV will be released

Ans. (3)

$Ne^{20} \rightarrow 2He^4 + C^{12}$

$8.03 \times 20 \qquad 2 \times 7.07 \times 4 + 7.86 \times 12$

$\therefore E_B = (BE)_{react} - (BE)_{product} = 9.72$ MeV

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is :

(1) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (2) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$

(3) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (4) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

Ans. (3)

$|\vec{A}+\vec{B}|=2a\cos\theta/2$ ___(1)

$|\vec{A}-\vec{B}|=2a\cos\frac{(\pi-\theta)}{2}=2a\sin\theta/2$ ___(2)

$\Rightarrow n\left(2a\cos\frac{\theta}{2}\right)=2a\frac{\sin\theta}{2}$

$\Rightarrow \tan\frac{\theta}{2}=n$

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is :

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

Ans. (4)

$v=\omega\sqrt{A^2-x^2}$ ___(1)

$a = -\omega^2 x$ ___(2)

$|v| = |a|$ ___(3)

$\omega\sqrt{A^2-x^2}=\omega^2 x$

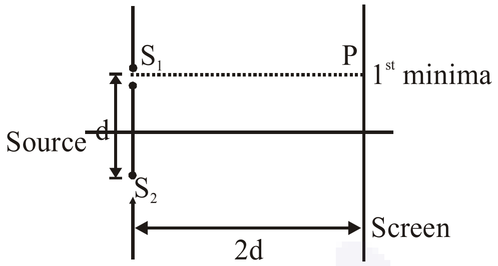
$A^2 - x^2 = \omega^2 x^2$

$5^2 - 4^2 = \omega^2(4^2)$

$\Rightarrow 3 = \omega \times 4$

$T = 2\pi/\omega$

15. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?



(1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{(5-\sqrt{2})}$

(3) $\frac{\lambda}{(\sqrt{5}-2)}$ (4) $\frac{\lambda}{2(\sqrt{5}-2)}$

Ans. (4)

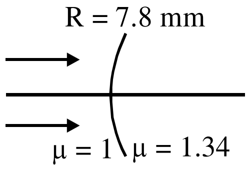
$$\sqrt{5}d-2d=\frac{\lambda}{2}$$

16. The eye can be regarded as a single refracting surface . The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

(1) 2 cm (2) 1 cm

(3) 3.1 cm (4) 4.0 cm

Ans. (3)



$$\frac{1.34}{V}-\frac{1}{\infty}=\frac{1.34-1}{7.8}$$

$\therefore$ V = 30.7 mm

17. Half mole of an ideal monoatomic gas is heated at constant pressure of 1atm from 20 ºC to 90ºC. Work done by gas is close to : (Gas constant R = 8.31 J /mol.K)

(1) 73 J (2) 291 J (3) 581 J (4) 146 J

Ans. (2)

$$WD = P\Delta V = nR\Delta T = \frac{1}{2} \times 8.31 \times 70$$

18. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2.The work function of the metal is 5eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be : [1 eV = 1.6×10^{-19}J]

(1) 10^{10} and 5 eV (2) 10^{14} and 10 eV

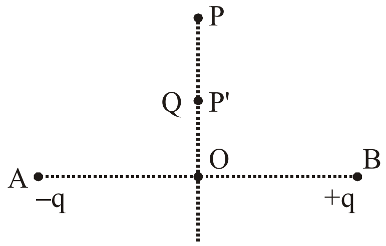
(3) 10^{12} and 5 eV (4) 10^{11} and 5 eV

Ans. (4)

$$I=\frac{nE}{At}$$

$$16\times10^{-3}=\left(\frac{n}{t}\right)_{Photon}\frac{10\times1.6\times10^{-19}}{10^{-4}}=10^{12}$$

19. Charges –q and +q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and y >> 2a. The charge Q experiences and electrostatic force F. If Q is now moved along the equatorial line to P' such that OP'=$\left(\frac{y}{3}\right)$, the force on Q will be close to : $\left(\frac{y}{3}>>2a\right)$



(1) $\frac{F}{3}$ (2) 3F (3) 9F (4) 27F

Ans. **(4)**

Sol. Electric field of equitorial plane of dipole

$$= -\frac{K\vec{P}}{r^3}$$

$\therefore$ At P, F = $-\frac{K\vec{P}}{r^3}Q.$

At P^1 , F^1 = $-\frac{K\vec{P}Q}{(r/3)^3} = 27F.$

20. Two stars of masses 3 × 10^{31} kg each, and at distance 2 × 10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is : (Take Gravitational constant G = 6.67 ×10^{-11} Nm^2 kg^{-2})

(1) 1.4 ×10^5 m/s (2) 24 ×10^4 m/s

(3) 3.8 ×10^4 m/s (4) 2.8 ×10^5 m/s

Ans. **(4)**

By energy convervation between 0 & ∞.

$$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV^2 = 0 + 0$$

[M is mass of star m is mass of meteroite)

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{r}} = 2.8 \times 10^5 m/s$$

21. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz)

(1) 7 (2) 5 (3) 6 (4) 4

Ans. **(1)**

Sol. For closed organ pipe, resonate frequency is odd multiple of fundamental frequency.

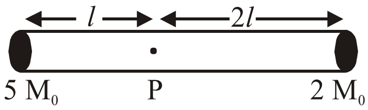
$\therefore$ (2n + 1) $f_0 \le$ 20,000

(f_o is fundamental frequency = 1.5 KHz)

$\therefore$ n = 6

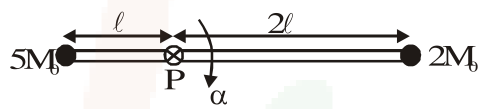
$\therefore$ Total number of overtone that can be heared is 7. (0 to 6).

22. A rigid massless rod of length 3l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$ (3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

Ans. **(3)**



Applying torque equation about point P.

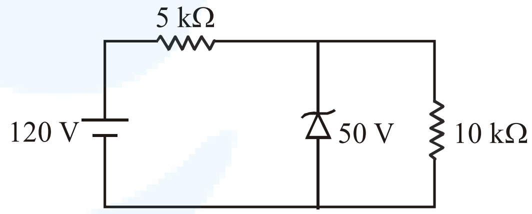
$2M_0$ (2l) – 5 M_0 gl = Iα

I = $2M_0$ (2l)^2 + 5M_0 l^2 = 13 M_0l^{2}d

$$\therefore \quad \alpha = -\frac{M_0 g\ell}{13M_0\ell^2} \Rightarrow \alpha = -\frac{g}{13\ell}$$

$\therefore \quad \alpha = \frac{g}{13\ell}$ anticlockwise

23. For the circuit shown below, the current through the Zener diode is :



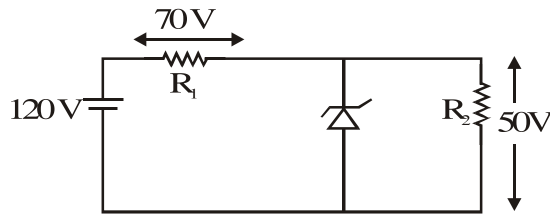
(1) 5 mA (2) Zero (3) 14 mA (4) 9 mA

Ans. **(4)**

Assuming zener diode doesnot undergo breakdown, current in circuit = $\frac{120}{15000} = 8mA$

$\therefore$ Voltage drop across diode = 80 V > 50 V.

The diode undergo breakdown.



Current is R_1 = $\frac{70}{5000} = 14mA$

Current is R_2 = $\frac{50}{10000} = 5mA$

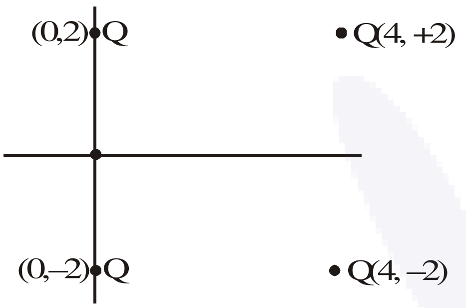
$\therefore$ Current through diode = 9mA

24. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

(1) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (2) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(3) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\varepsilon_0}$

Ans. (2)



Potential at origin = $\frac{KQ}{2}+\frac{KQ}{2}+\frac{KQ}{\sqrt{20}}+\frac{KQ}{\sqrt{20}}$

(Potential at ∞ = 0)

$= KQ\left(1+\frac{1}{\sqrt{5}}\right)$

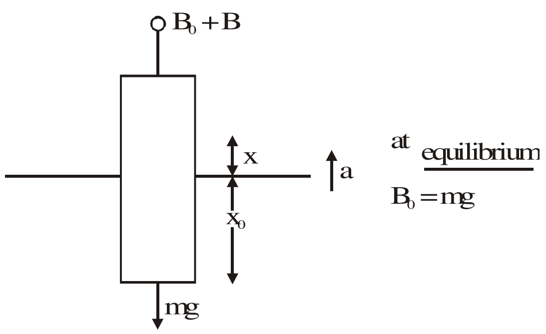
$\therefore$ Work required to put a fifth charge Q at origin is equal to $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

25. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3)

(1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1}

(3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1}

Ans. (Bonus)



Extra Boyant force = δAxg

B_0 + B × mg = ma

B = ma

$a = \left(\frac{\delta Ag}{m}\right)^x$

$w^2 = \frac{\delta Ag}{m}$

$w = \sqrt{\frac{10^3 \times \pi(2.5)^2 \times 10^{-4} \times 10}{310 \times 10^{-6} \times 10^3}}$

$= \sqrt{63.30}$ = 7.95

26. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

(1) 692 pJ (2) 60 pJ

(3) 508 pJ (4) 560 pJ

Ans. (3)

Intial energy of capacitor

$U_i = \frac{1}{2}\frac{v^2}{c}$

$= \frac{1}{2} \times \frac{120 \times 120}{12} = 600$ J

Since battery is disconnected so charge remain same.

Final energy of capacitor

$U_f = \frac{1}{2}\frac{v^2}{c}$

$= \frac{1}{2} \times \frac{120 \times 120}{12 \times 6.5} = 92$

W + U_f = U_i

W = 508 J

27. Two kg of a monoatomic gas is at a pressure of 4 × 10^4 N/m^2 . The density of the gas is 8 kg /m^3. What is the order of energy of the gas due to its thermal motion ?

(1) 10^3 J (2) 10^5 J

(3) 10^6 J (4) 10^4 J

Ans. (4)

Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right)$

$$= \frac{N}{N_A}\frac{3}{2}RT$$

$$= \frac{3}{2}(nRT)$$

$$= \frac{3}{2}PV$$

$$= \frac{3}{2}P\left(\frac{m}{8}\right)$$

$$= \frac{3}{2}\times 4\times 10^4 \times \frac{2}{8}$$

$= 1.5 \times 10^4$

order will 10^4

28. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

(1) 15 J (2) 10 J (3) 12 J (4) 9 J

Ans. (1)

Work done = $\vec{F}\cdot\vec{d}$

= 12J

work energy theorem

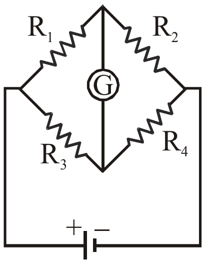
$w_{net} = \Delta K.E.$

$12 = K_f - 3$

$K_f = 15J$

29. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω, respectively.

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3, would be :



(1) Red, Green, Brown
(2) Brown, Blue, Brown
(3) Grey, Black, Brown
(4) Brown, Blue, Black

Ans. (2)

$R_1 = 32 \times 10 = 320$

for wheat stone bridge

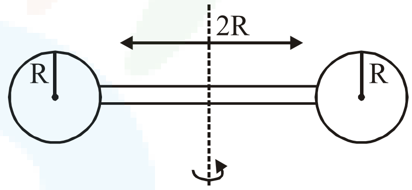
$$\Rightarrow \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

$$\frac{320}{R_3} = \frac{80}{40}$$

$R_3 = 160$

Brown Blue Brown

30. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

Ans. (3)

For Ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2$$

$$= \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

Irod = for rod $\frac{M(2R)^2}{R} = \frac{MR^2}{3}$

$I_{system} = I_{Ball} + I_{rod}$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15}MR^2$$