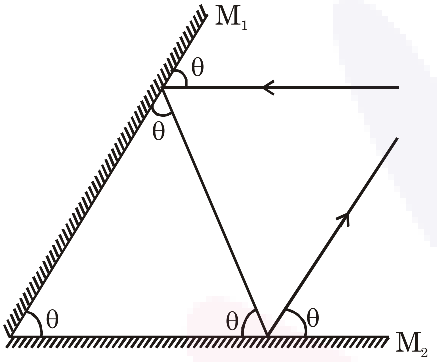
FINAL JEE–MAIN EXAMINATION – JANUARY, 2019
Held On Wednesday 09th JANUARY, 2019
TIME: 2 : 30 PM To 5 : 30 PM

1. Two plane mirrors arc inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be :

(1) 90° (2) 45° (3) 75° (4) 60°

Ans. (4)



Sol. Assuming angles between two mirrors be θ

as per geometry,

sum of anlges of Δ

$3\theta = 180°$

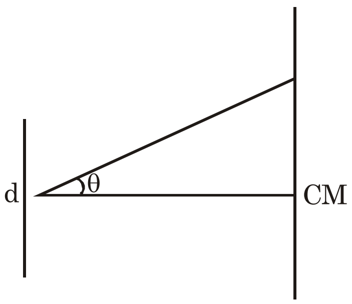
$\theta = 60°$

2. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength λ = 500 nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$is:

(1) 320 (2) 641 (3) 321 (4) 640

Ans. (2)

Sol.



Pam difference

$d\sin\theta = n\lambda$

where d = seperation of slits

λ = wave length

n = no. of maximas

$0.32 \times 10^{-3} \sin 30 = n \times 500 \times 10^{-9}$

n = 320

Hence total no. of maximas observed in angular range $-30° \le \theta \le 30°$ is

maximas = 320 + 1 + 320 = 641

3. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. [f the half-life of A is m_2, the half-life of B is :

(1) $\frac{ln2}{2}$ (2) $2ln2$ (3) $\frac{ln2}{4}$ (4) $4ln2$

Ans. (3)

Sol. Half life of A = $\ell n2$

$t_{1/2} = \frac{\ell n2}{\lambda}$

$\lambda_A = 1$

at t = 0 $R_A = R_B$

$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$

$N_A = N_B$ at t = 0

at t = t $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

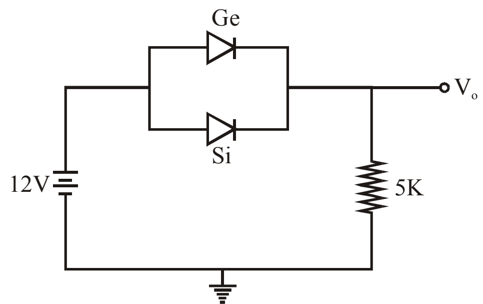
$e^{-(\lambda_B - \lambda_A)t} = e^{-t}$

$\lambda_B - \lambda_A = 3$

$\lambda_B = 3 + \lambda_A = 4$

$t_{1/2} = \frac{\ell n2}{\lambda_B} = \frac{\ell n2}{4}$

4. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by : (assume that the Ge diode has large breakdown voltage)



(1) 0.6 V (2) 0.8 V (3) 0.4 V (4) 0.2 V

Ans. (3)

Sol. Initially Ge & Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V
if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of (0.7 – 0.3) = 0.4 V is observed.

5. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :
(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

Ans. **(2)**

Sol. Frequency of torsonal oscillations is given by

$f = \frac{k}{\sqrt{I}}$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$f_2 = 0.8\ f_1$

$$\frac{m}{M} = 0.375$$

6. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :
[Take R = 8.3 J/ K mole]
(1) 10 kJ (2) 0.9 kJ (3) 6 kJ (4) 14 kJ

Ans. **(1)**

Sol. $Q = nC_v\Delta T$ as gas in closed vessel

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

Q = 10000 J = 10 kJ

7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

Ans. **(3)**

Sol. Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$x = \pm\frac{A}{\sqrt{2}}$$

∴ Correct answer is (3)

8. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to :
(1) 753 Hz (2) 500 Hz
(3) 333 Hz (4) 666 Hz

Ans. **(4)**

Sol. Frequency of the sound produced by flute,

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2 \times 330}{2 \times 0.5} = 660\text{Hz}$$

Velocity of observer, $v_0 = 10 \times \frac{5}{18} = \frac{25}{9}\text{m/s}$

∴ frequency detected by observer, f' = $\left[\frac{v + v_0}{v}\right]f$

$$\therefore f' = \left[\frac{\frac{25}{9} + 330}{330}\right]660$$

= 335.56 × 2 = 671.12

∴ closest answer is (4)

9. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 10^8$m/s, $h = 6.6 \times 10^{-34}$ J-s)
(1) 3.75×10^6 (2) 4.87×10^5
(3) 3.86×10^6 (4) 6.25×10^5

Ans. **(4)**

Sol. $f = \dfrac{3\times10^{8}}{8\times10^{-7}} = \dfrac{30}{8}\times10^{14}\,\text{Hz}$

$= 3.75 \times 10^{14}$ Hz

1% of f = 0.0375×10^{14} Hz

$= 3.75 \times 10^{12}$ Hz $= 3.75 \times 10^{6}$ MHz

number of channels $= \dfrac{3.75\times10^{6}}{6} = 6.25 \times 10^{5}$

$\therefore$ correct answer is (4)

10. Two point charges $q_1(\sqrt{10}\,\mu C)$ and $q_2(-25\,\mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is,

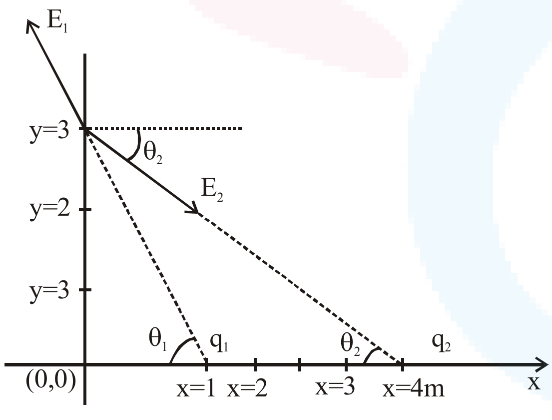
$\left[\text{take } \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\,\text{Nm}^2\text{C}^{-2}\right]$

(1) $(-63\hat{i}+27\hat{j})\times10^{2}$ (2) $(81\hat{i}-81\hat{j})\times10^{2}$

(3) $(63\hat{i}-27\hat{j})\times10^{2}$ (4) $(-81\hat{i}+81\hat{j})\times10^{2}$

Ans. (3)

Sol.



Let $\vec{E}_1$ & $\vec{E}_2$ are the vaues of electric field due to q_1 & q_2 respectively magnitude of

$E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{r^2}$

$E_2 = \dfrac{9\times10^{9}\times(25)\times10^{-6}}{(4^2+3^2)}\,\text{V/m}$

$E_2 = 9 \times 10^{3}$ V/m

$\therefore \vec{E}_2 = 9\times10^{3}\left(\cos\theta_2\hat{i}-\sin\theta_2\hat{j}\right)$

$\because \tan\theta_2 = \dfrac{3}{4}$

$\therefore \vec{E}_2 = 9\times10^{3}\left(\dfrac{4}{5}\hat{i}-\dfrac{3}{5}\hat{j}\right) = \left(72\hat{i}-54\hat{j}\right)\times10^{2}$

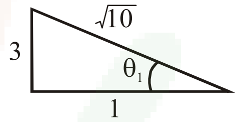
Magnitude of $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{\sqrt{10}\times10^{-6}}{(1^2+3^2)}$

$= \left(9\times10^{9}\right)\times\sqrt{10}\times10^{-7}$

$= 9\sqrt{10}\times10^{2}$

$\therefore \vec{E}_1 = 9\sqrt{10}\times10^{2}\left[\cos\theta_1\left(-\hat{i}\right)+\sin\theta_1\hat{j}\right]$

$\therefore \tan\theta_1 = 3$



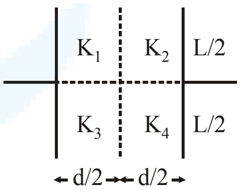
$E_1 = 9\times\sqrt{10}\times10^{2}\left[\dfrac{1}{\sqrt{10}}\left(-\hat{i}\right)+\dfrac{3}{\sqrt{10}}\hat{j}\right]$

$E_1 = 9\times10^{2}\left[-\hat{i}+3\hat{j}\right] = \left[-9\hat{i}+27\hat{j}\right]10^{2}$

$\therefore \vec{E} = \vec{E}_1+\vec{E}_2 = \left(63\hat{i}-27\hat{j}\right)\times10^{2}\,\text{V/m}$

$\therefore$ correct answer is (3)

11. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



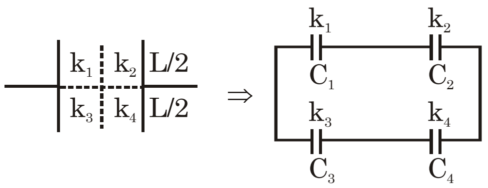
(1) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

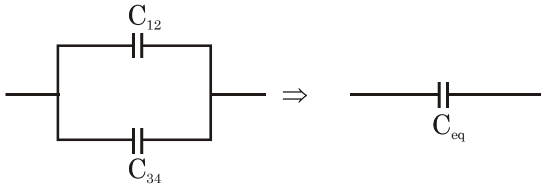
(2) $K = \dfrac{(K_1+K_2)(K_3+K_4)}{(K_1+K_2+K_3+K_4)}$

(3) $K = \dfrac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

(4) $K = \dfrac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$

Ans. (Bonus)

Sol. 



$$C_{12}=\frac{C_1C_2}{C_1+C_2}=\frac{\dfrac{k_1\in_0\dfrac{L}{2}\times L}{d/2}\cdot\dfrac{k_2\left[\in_0\dfrac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\dfrac{\in_0\cdot\dfrac{L}{2}\times L}{d/2}\right]}$$

$$C_{12}=\frac{k_1k_2}{k_1+k_2}\frac{\in_0 L^2}{d}$$

in the same way we get, $C_{34}=\dfrac{k_3k_4}{k_3+k_4}\dfrac{\in_0 L^2}{d}$

$$\therefore\ C_{eq}=C_{12}+C_{34}=\left[\frac{k_1k_2}{k_1+k_2}+\frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d}\ ..(i)$$

Now if $k_{eq}=k$, $C_{eq}=\dfrac{k\in_0 L^2}{d}$ .....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq}=\frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13}=\frac{k_1\in_0 L\dfrac{L}{2}}{d/2}+k_3\in_0\frac{L.\dfrac{L}{2}}{d/2}$$

$$=(k_1+k_3)\frac{\in_0 L^2}{d}$$

$$C_{24}=(k_2+k_4)\ \frac{\in_0 L^2}{d}$$

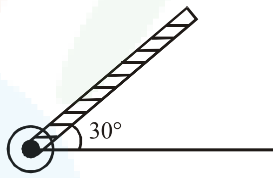
$$C_{eq}=\frac{C_{13}C_{24}}{C_{13}C_{24}}=\frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\in_0 L^2}{d}$$

$$=\frac{k\in_0 L^2}{d}$$

$$k=\frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

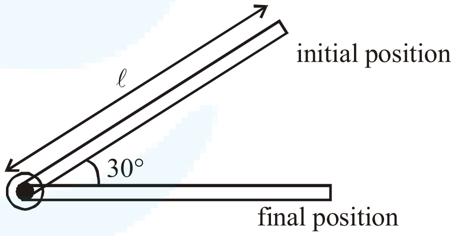
However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)

12. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10ms^{-2}$)



(1) $\sqrt{30}$ (2) $\sqrt{\dfrac{30}{2}}$ (3) $\dfrac{\sqrt{30}}{2}$ (4) $\dfrac{\sqrt{20}}{3}$

Ans. (2)

Sol. 

Work done by gravity from initial to final position is,

$$W=mg\frac{\ell}{2}\sin 30°$$

$$=\frac{mg\ell}{4}$$

According to work energy theorem

$$W=\frac{1}{2}I\omega^2$$

$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$$

$$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times10}{2\times0.5}}$$

$$\omega = \sqrt{30}\ \text{rad/sec}$$

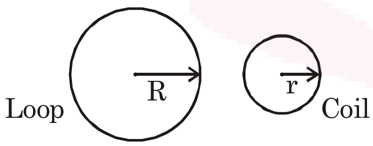
$\therefore$ correct answer is (1)

13. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. R $\frac{B_L}{B_C}$ will be :

(1) $\frac{1}{N}$ (2) N^2 (3) $\frac{1}{N^2}$ (4) N

Ans. **(3)**

Sol.



L = $2\pi R$ L = N $\times$ $2\pi r$

R = Nr

$B_L = \frac{\mu_0 i}{2R}$ $B_C = \frac{\mu_0 Ni}{2r}$

$$B_C = \frac{\mu_0 N^2 i}{2R}$$

$$\frac{B_L}{B_C} = \frac{1}{N^2}$$

14. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is:

(1) 1.28×10^4 km (2) 6.4×10^3 km

(3) 3.2×10^3 km (4) 1.6×10^3 km

Ans. **(3)**

Sol. $U_{surface} + E_1 = U_h$

KE of satelite is zero at earth surface & at height h

$$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re+h)}$$

$$E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e+h}\right)$$

$$E_1 = \frac{GM_e m}{(R_e+h)} \times \frac{h}{R_e}$$

Gravitational attraction $F_G = ma_C = \frac{mv^2}{(R_e+h)}$

$$E_2 \Rightarrow \frac{mv^2}{(R_e+h)} = \frac{GM_e m}{(R_e+h)^2}$$

$$mv^2 = \frac{GM_e m}{(R_e+h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e+h)}$$

$E_1 = E_2$

$$\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200\text{km}$$

15. The energy associated with electric field is (U_E) and with magnetic field is (U_B) for an electromagnetic wave in free space. Then :

(1) $U_E = \frac{U_B}{2}$ (2) $U_E < U_B$

(3) $U_E = U_B$ (4) $U_E > U_B$

Ans. **(3)**

Sol. Average energy density of magnetic field,

$u_B = \frac{B_0^2}{2\mu_0}$, B_0 is maximum value of magnetic field.

Average energy density of electric field,

$$u_E = \frac{\varepsilon_0 \epsilon_0^2}{2}$$

now, $\epsilon_0 = CB_0$, $C^2 = \frac{1}{\mu_0 \epsilon_0}$

$$u_E = \frac{\epsilon_0}{2} \times C^2 B_0^2$$

$= \frac{\epsilon_0}{2} \times \frac{1}{\mu_0 \epsilon_0} \times B_0^2 = \frac{B_0^2}{2\mu_0} = u_B$

$u_E = u_B$

since energy density of electric & magnetic field is same, energy associated with equal volume will be equal.

$u_E = u_B$

16. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is :

(1) 2.26×10^3 J (2) 3.39×10^3 J

(3) 5.65×10^2 J (4) 5.17×10^2 J

Ans. (4)

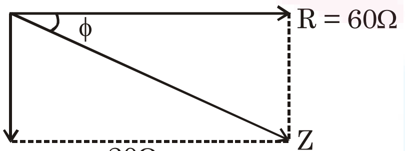
Sol. $R = 60\Omega$ $f = 50\text{Hz}, \omega = 2\pi f = 100\pi$

$x_C = \frac{1}{\omega C} = \frac{1}{100\pi \times 120 \times 10^{-6}}$

$x_C = 26.52\ \Omega$

$x_L = \omega L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$

$x_C - x_L = 20.24 \approx 20$



$x_C - x_L = 20\Omega$

$z = \sqrt{R^2 + (x_C - x_L)^2}$

$z = 20\sqrt{10}\Omega$

$\cos\phi = \frac{R}{z} = \frac{3}{\sqrt{10}}$

$P_{avg} = VI \cos\phi,\ I = \frac{v}{z}$

$= \frac{v^2}{z}\cos\phi$

$= 8.64$ watt

$Q = P.t = 8.64 \times 60 = 5.18 \times 10^2$

17. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

(1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^5}{G}}$

(3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

Ans. (4)

Sol. $F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^x h^y C^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

on comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ...(ii)

on solving (i) & (ii) $x = y = \frac{1}{2},\ z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

18. The magnetic field associated with a light wave is given, at the origin, by $B = B_0 [\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons ? ($c = 3 \times 10^8 ms^{-1}$, $h = 6.6 \times 10^{-34}$ J-s)

(1) 7.72 eV (2) 8.52 eV

(3) 12.5 eV (4) 6.82 eV

Ans. (1)

Sol. $B = B_0\sin(\pi \times 10^7C)t + B_0\sin(2\pi \times 10^7C)t$

since there are two EM waves with different frequency, to get maximum kinetic energy we take the photon with higher frequency

$B_1 = B_0\sin(\pi \times 10^7C)t \quad \nu_1 = \frac{10^7}{2} \times C$

$B_2 = B_0\sin(2\pi \times 10^7C)t \quad \nu_2 = 10^7C$

where C is speed of light $C = 3 \times 10^8$ m/s

$\nu_2 > \nu_1$

so KE of photoelectron will be maximum for photon of higher energy.

$\nu_2 = 10^7C$ Hz

$h\nu = \phi + KE_{max}$

energy of photon

$E_{ph} = h\nu = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}$J

$= \frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}} eV = 12.375 eV$

$KE_{max} = E_{ph} - \phi$

$= 12.375 - 4.7 = 7.675\ eV \approx 7.7\ eV$

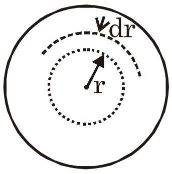
19. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r)=\dfrac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :

(1) $\dfrac{a}{2}\log\left(1-\dfrac{Q}{2\pi aA}\right)$ (2) $a\log\left(1-\dfrac{Q}{2\pi aA}\right)$

(3) $a\log\left(\dfrac{1}{1-\dfrac{Q}{2\pi aA}}\right)$ (4) $\dfrac{a}{2}\log\left(\dfrac{1}{1-\dfrac{Q}{2\pi aA}}\right)$

Ans. **(4)**

Sol.



$$Q = \int \rho dv$$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= \int_0^R \frac{A}{r^2}e^{-2r/a}\left(4\pi r^2 dr\right)$$

$$= 4\pi A\int_0^R e^{-2r/a}dr$$

$$= 4\pi A\left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$$

$$= 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a}-1\right)$$

$$Q = 2\pi aA(1 - e^{-2R/a})$$

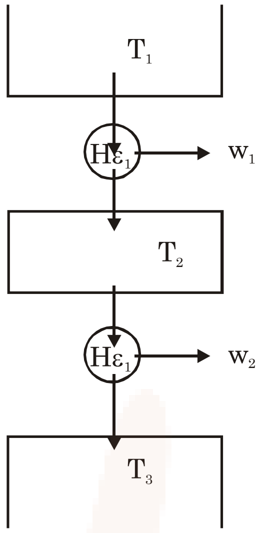
$$R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$

20. Two Carrnot engines A and B are operated in series. The first one, A, receives heat at T_1(= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T_3(= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal :

(1) 400 K (2) 600 K (3) 500 K (4) 300 K

Ans. **(3)**

Sol.



$w_1 = w_2$

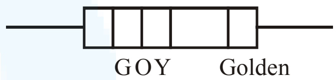
$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

$T_2 = 500$ K

21. A carbon resistance has a following colour code. What is the value of the resistance ?

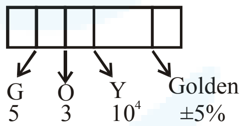


(1) 1.64 MΩ ± 5% (2) 530 kΩ ± 5%

(3) 64 kΩ ± 10% (4) 5.3 MΩ ± 5%

Ans. **(2)**

Sol.



$R = 53 \times 10^4 \pm 5\% = 530\text{ k}\Omega \pm 5\%$

22. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds ?

(1) 850 J (2) 900 J

(3) 950 J (4) 875 J

Ans. **(2)**

Sol. $x = 3t^2 + 5$

$$v = \frac{dx}{dt}$$

$v = 6t + 0$

at t = 0 v = 0

t = 5 sec v = 30 m/s

W.D. = ΔKE

$$\text{W.D.} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(2)(30)^2 = 900\text{J}$$

23. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is :

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$

(3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

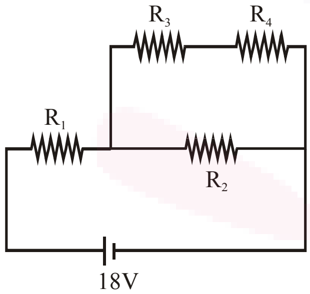
Ans. (3)

Sol. $v_x = -a\omega\sin\omega t \Rightarrow v_y = a\omega\cos\omega t$

$v_z = a\omega \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

24. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5V, then the value R_2 will be :

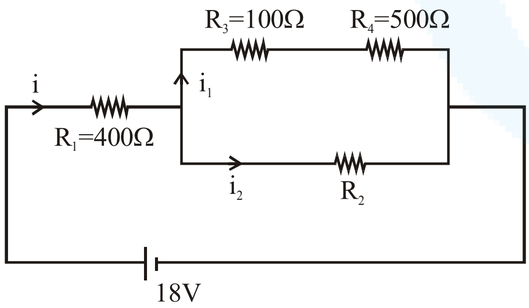


(1) 300 Ω (2) 230 Ω

(3) 450 Ω (4) 550 Ω

Ans. (1)

Sol.



$V_4 = 5V$

$i_1 = \frac{V_4}{R_4} = 0.01\ A$

$V_3 = i_1R_3 = 1V$

$V_3 + V_4 = 6V = V_2$

$V_1 + V_3 + V_4 = 18V$

$V_1 = 12\ V$

$i = \frac{V_1}{R_1} = 0.03\text{Amp.}$

$i_2 = 0.02$ Amp $V_2 = 6V$

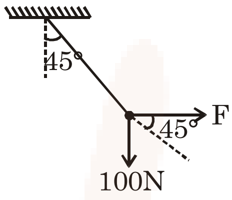
$R_2 = \frac{V_2}{i_2} = \frac{6}{0.02} = 300\Omega$

25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10\ ms^{-2}$)

(1) 200 N (2) 100 N (3) 140 N (4) 70 N

Ans. (2)

Sol.



at equation

$\tan 45° = \frac{100}{F}$

$F = 100\ N$

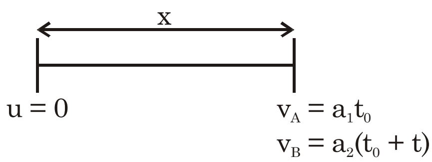
26. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$

(3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

Ans. (4)

Sol. For A & B let time taken by A is t_0



from ques.

$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t$ ....(i)

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$

$\Rightarrow \left(\sqrt{a_2} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t$ ....(ii) putting t_0 in equation

$$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1}-\sqrt{a_2}} - a_2t$$

$$= \left(\sqrt{a_1}+\sqrt{a_2}\right)\sqrt{a_2}t - a_2t \Rightarrow v = \sqrt{a_1a_2}t$$

$$\Rightarrow \sqrt{a_1a_2}t + a_2t - a_2t$$

27. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

(1) 25 A (2) 50 A

(3) 35 A (4) 45 A

Ans. **(4)**

Sol. $\eta = \frac{P_{out}}{P_{in}} = \frac{V_SI_S}{V_PI_P}$

$$\Rightarrow 0.9 = \frac{23 \times I_S}{230 \times 5}$$

$$\Rightarrow I_S = 45A$$

28. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :

(1) 9.6 m (2) 4.8 m (3) 2.9 m (4) 6.0 m

Ans. **(2)**

Sol. In flow volume = outflow volume

$$\Rightarrow \frac{0.74}{60} = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$$

$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$$

$$\Rightarrow h \approx 4.8m$$

29. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :

(1) 5.755 m (2) 5.725 mm

(3) 5.740 m (4) 5.950 mm

Ans. **(2)**

Sol. $LC = \frac{\text{Pitch}}{\text{No. of division}}$

$LC = 0.5 \times 10^{-2}$ mm

+ve error $= 3 \times 0.5 \times 10^{-2}$ mm

$= 1.5 \times 10^{-2}$ mm $= 0.015$ mm

Reading = MSR + CSR – (+ve error)

$= 5.5$ mm $+ (48 \times 0.5 \times 10^{-2}) - 0.015$

$= 5.5 + 0.24 - 0.015 = 5.725$ mm

30. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $= 1.6 \times 10^{-19}$C)

(1) 2.0×10^{-24} kg

(2) 1.6×10^{-19} kg

(3) 1.6×10^{-27} kg

(4) 9.1×10^{-31} kg

Ans. **(1)**

Sol. $\frac{mv^2}{R} = qvB$

$mv = qBR$ ....(i)

Path is straight line

it $qE = qvB$

$E = vB$ ....(ii)

From equation (i) & (ii)

$$m = \frac{qB^2R}{E}$$

$m = 2.0 \times 10^{-24}$ kg