FINAL JEE–MAIN EXAMINATION – APRIL, 2023

Held On Monday 10th April, 2023

TIME : 03:00 PM to 06:00 PM

SECTION - A

1. If the coefficients of x and x^2 in $(1 + x)^p (1 - x)^q$ are 4 and – 5 respectively, then 2p + 3q is equal to

(1) 60 (2) 63 (3) 66 (4) 69

Sol. (2)

$(1+x)^p(1-x)^q$

$\left(1+px+\frac{p(p-1)}{2!}x^2+\ldots\right)$

$\left(1-qx+\frac{q(q-1)}{2!}x^2-\ldots\right)$

$p-q=4$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$p^2+q^2-p-q-2pq=-10$

$(q+4)^2+q^2-(q+4)-q-2(4+q)q=-10$

$q^2+8q+16-q^2-q-4-q-8q-2q^2=-10$

$-2q=-22$

$q=11$

$p=15$

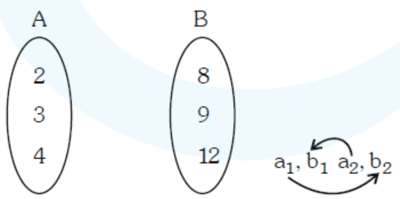
$2(15)+3(11)$

$30+33=63$

2. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R= {$((a_1, b_1), (a_2,b_2)) \in (A \times B, A \times B)$: a_1 divides b_2 and a_2 divides b_1} is

(1) 18 (2) 24 (3) 12 (4) 36

Sol. (4)



a_1 divides b_2

Each elements has 2 choices

$\Rightarrow 3\times 2=6$

a_2 divides b_1

Each elements has 2 choices

$\Rightarrow 3\times 2=6$

Total = 6 × 6 = 36

3. Let time image of the point P(1,2,6) in the plane passing through the points A(1,2,0), B (1, 4, 1) and C (0, 5, 1) be Q (α, β, γ). Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to

(1) 70 (2) 76 (3) 62 (4) 65

Sol. (4)

Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$

Put $(1,4,1)\Rightarrow 2B+C=0$

Put $(0,5,1)\Rightarrow -A+3B+C=0$

Sub : $\overline{B-A=0}\Rightarrow A=B, C=-2B$

$1(x-1)+1(y-2)-2(z-0)=0$

$x+y-2z-3=0$

Image is (α,β,γ)pt $\equiv(1,2,6)$

$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$

$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$

$\alpha=5,\beta=6,\gamma=-2\Rightarrow\alpha^2+\beta^2+\gamma^2$

$=25+36+4=65$

4. The statement ~ [pV(~ (p ∧ q))] is equivalent to

(1) (~ (p ∧ q)) ∧ q (2) ~ (p ∨ q) (3) ~ (p ∧ q) (4) (p ∧ q) ∧ (~p)

Sol. **(4)**

$\sim[pv(\sim(p\wedge q))]$

$\sim p\wedge(p\wedge q)$

5. Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and

$b=\sum_{x\in S}\tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}$ $(\beta-14)^2$ is equal to

(1) 16 (2) 32 (3) 8 (4) 64

Sol. **(2)**

Let $9^{\tan^2 x}=P$

$\frac{9}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1,9$

$9^{\tan^2 x}=1,9^{\tan^2 x}=9$

$\tan^2 x=0,\tan^2 x=1$

$x=0,\pm\frac{\pi}{4}\quad\therefore x\in\left(-\frac{\pi}{2},\frac{p}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$

$=0+2\left(\tan15^\circ\right)^2$

$2(2-\sqrt{3})^2$

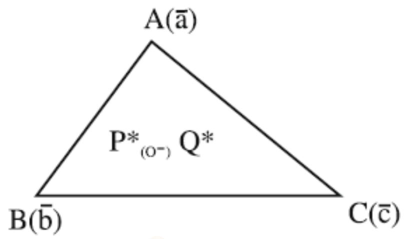
$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

6. If the points P and Q are respectively the circumecenter and the orthocentre of a ΔABC, the $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is equal to

(1) $2\overrightarrow{QP}$ (2) $\overrightarrow{PQ}$ (3) $2\overrightarrow{PQ}$ (4) $\overrightarrow{PQ}$

Sol. **(4)**



$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{a}+\vec{b}+\vec{c}$$

$$\overrightarrow{PG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

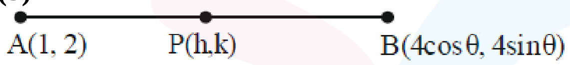
$$\Rightarrow \vec{a}+\vec{b}+\vec{c}=3\overrightarrow{PG}=\overrightarrow{PQ}$$

Ans. (4)

7. Let A be the point (1,2) and B be any point onthe curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α,β) then the length of the line segment AC is

(1) $\frac{6\sqrt{5}}{5}$ (2) $\frac{2\sqrt{5}}{5}$ (3) $\frac{3\sqrt{5}}{5}$ (4) $\frac{4\sqrt{5}}{5}$

Sol. **(3)**



$$\frac{12\cos\theta+2}{5}=h \quad \Rightarrow \quad 12\cos\theta=5h-2$$

sq & add

$$144=(5h-2)^2+(5k-4)^2$$

$$\left(x-\frac{2}{5}\right)^2+\left(y-\frac{4}{5}\right)^2=\frac{144}{25}$$

$$\text{Centre}\equiv\left(\frac{2}{5},\frac{4}{5}\right)\equiv(\alpha,\beta)$$

$$AC=\sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}$$

$$=\sqrt{\frac{9}{25}+\frac{36}{25}}=\frac{\sqrt{45}}{5}=\frac{3\sqrt{5}}{5}$$

8. Let m be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If [x] denotes the greatest integer $\le$ x, then $[\mu 2 + \sigma^2]$ is equal to

(1) 8 (2) 7 (3) 6 (4) 9

Sol. **(1)**

$$\sum f_i = 62$$

$$3k^2+16k-12k-64=0$$

$$k=4 \text{ or } -\frac{16}{3}\text{(rejected)}$$

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2$$

$$= \frac{8\times 1^2 + 15\times 13 + 17\times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 + \mu^2 = \frac{500}{62}$$

$$\left[\sigma^2 + \mu^2\right] = 8$$

9. If S_n = 4+ 11 + 21 + 34 + 50 + ...to n terms, then $\frac{1}{60}$ ($S_{29} - S_9$) is equal to

(1) 220 (2) 227 (3) 226 (4) 223

Sol. **(4)**

$S_n = 4 + 11 + 21 + 34 + 50 + \ldots + n$ terms

Difference are in A.P.

Let $T_n = an^2 + bn + c$

$T_1 = a + b + c = 4$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 21$

By solving these 3 equations

$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$

So $T_n = \frac{3}{2}n^2 + \frac{5}{2}n$

$S_n = \Sigma T_n$

$= \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$

$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{(n)(n+1)}{2}$

$= \frac{n(n+1)}{4}[2n+1+5]$

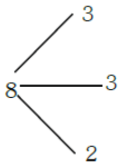
$S_n = \frac{n(n+1)}{4}(2n+6) = \frac{n(n+1)(n+3)}{2}$

$\frac{1}{60}\left(\frac{29\times 30\times 32}{2} - \frac{9\times 10\times 12}{2}\right) = 223$

10. Eight persons are tobe transported from city A to city B in three cars different makes. If each car can accomodate at most three persons, then the number of ways, in which they can be transported, is

(1) 1120 (2) 560 (3) 3360 (4) 1680

Sol. **(4)**



$$\text{Ways} = \frac{8!}{3!3!2!2!} \times 3!$$
$$= \frac{8 \times 7 \times 6 \times 5 \times 4}{4}$$
$$= 56 \times 30$$
$$= 1680$$

11. If A = $\frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then |adj(adj(2A))| is equal to

(1) 2^{16} (2) 2^{8} (3) 2^{12} (4) 2^{20}

Sol. **(1)**

$$|A| = \frac{1}{5!6!7!} 5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$
$$R_3 \to R_3 \to R_2$$
$$R_2 \to R_2 \to R_1$$
$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$
$$|\text{adjadj}(2A)| = |2A|^{(n-1)^2}$$
$$= |2A|^4$$
$$= \left(2^3 |A|\right)^4$$
$$= 2^{12} |A|^4 \Rightarrow 2^{16}$$

12. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(1) 13 (2) 20 (3) 10 (4) 5

Sol. **(4)**

$(22)^{2022} + (2022)^{22}$

divided byy 3

$(21+1)^{2022} + (2022)^{22}$

$= 3k + 1$

$(\alpha = 1)$

Divided by 7

$(21+1)^{2022} + (2023-1)^{22}$

$7k + 1 + 1$ $(\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

13. Let g(x) = f(x) + f(1-x) and $f^{n}(x) > 0$, $x \in (0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to

(1) $\frac{5\pi}{4}$ (2) π (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

Sol. **(2)**

$$g(x) = f(x) + f(1-x) \& f''(x) > 0, x \in (0,1)$$

$$g'(x) = f'(x) - f'(1-x) = 0$$

$$\Rightarrow f'(x) = f'(1-x)$$

$$x = 1 - x$$

$$x = \frac{1}{2}$$

$$g'(x) = 0$$

$$\text{at} x = \frac{1}{2}$$

$$g''(x) = f''(x) + f''(1-x) > 0$$

g is concave up

hence $\alpha = \frac{1}{2}$

$$\tan^{-1} 2\alpha + \tan^{-1}\frac{1}{\alpha} + \tan^{-1}\frac{\alpha+1}{\alpha}$$

$$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$$

14. For $\alpha, \beta, \gamma, \delta \in N$, if $\int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx = \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$, where $e = \sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to

(1) 4 (2) – 4 (3) – 8 (4) 1

Sol. **(1)**

$$x = e^{\ln x}$$

$$\int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx = \int\left[e^{2(x\ln x - x)} + e^{-2(x\ln x - x)}\right]\ln x\,dx$$

$$x\ln x - x = t$$

$$\ln x \cdot dx = dt$$

$$\int\left(e^{2t} + e^{-2t}\right)dt$$

$$\frac{e^{2t}}{2} - \frac{e^{-2t}}{2} + C$$

$$= \frac{1}{2}\left(\frac{x}{e}\right)^{2x} - \frac{1}{2}\left(\frac{e}{x}\right)^{2x} + C$$

$$\alpha = \beta = \gamma = \delta = 2$$

$$\alpha + 2\beta + 3\gamma - 4\delta = 4$$

15. Let f be a continuous function satisying $\int_0^{t^2}(f(x)+x^2)dx=\frac{4}{3}t^3, \forall t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to

(1) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$ (2) $\pi\left(1-\frac{\pi^3}{16}\right)$ (3) $-\pi\left(1+\frac{\pi^3}{16}\right)$ (4) $\pi^2\left(1-\frac{\pi^3}{16}\right)$

Sol. **(2)**

$\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3, \forall t>0$

$\left(f\left(t^2\right)+t^4\right)=2t$

$f\left(t^2\right)=2t-t^4$

$t=\frac{\pi}{2}\Rightarrow f\left(\frac{\pi^2}{4}\right)=\frac{2\pi}{2}-\frac{\pi^4}{16}$

$=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right)$

16. Let a dic be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

(1) 60 (2) 30 (3) 90 (4) 15

Sol. **(1)**

$P(\text{odd number7times})=P(\text{odd number9times})$

${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}={}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

${}^nC_7={}^nC_9$

$\Rightarrow n=16$

Required

$P={}^{16}C_2\times\left(\frac{1}{2}\right)^{16}$

$=\frac{16\cdot 15}{2}\times\frac{1}{2^{16}}=\frac{15}{2^{13}}$

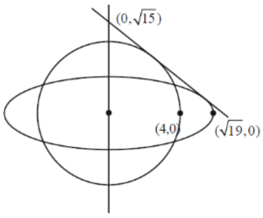
$\Rightarrow\frac{60}{2^{15}}\Rightarrow k=60$

17. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis ofthe ellipse at the angle.

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Sol. **(3)**

$\frac{x^2}{19}+\frac{y^2}{15}=1$



Let tang be

$y = mx \pm \sqrt{19m^2 + 15}$

$mx - y \pm \sqrt{19m^2 + 15} = 0$

Parallel from (0, 0) = 4

$\left|\frac{\pm\sqrt{19m^2 + 15}}{\sqrt{m^2 + 1}}\right| = 4$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$m = \pm\frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ with x-axis

Required angle $\frac{\pi}{3}$.

18. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$, Let $\vec{d}$-be a vector which is perpendicular to both $\vec{a}$, and $\vec{b}$, and $\vec{c}.\vec{d} = 12$. The $\left(-\hat{i} + \hat{j} - \hat{k}\right).\left(\vec{c} \times \vec{d}\right)$-is equal to

(1) 24 (2) 42 (3) 48 (4) 44

Sol. **(4)**

$\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{k}$

$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$

$\vec{d} = \lambda(35\hat{i} - 13\hat{j} - 21\hat{k})$

$\lambda(35 + 13 - 42) = 12$

$\lambda = 2$

$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$

$(\hat{i} + \hat{j} - \hat{k})(\vec{c} \times \vec{d})$

$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$

19. Let S = $\left\{z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$. Then which of the following is NOT correct ?

(1) $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

(2) $(x, y) = \left(0, -\frac{1}{2}\right)$

(3) x = 0

(4) $y + x^2 + y^2 \neq -\frac{1}{4}$

Sol. **(2)**

$\frac{2z-3i}{4z+2i} \in R$

$\frac{2(x+iy)-3i}{4(x+iy)+2i} = \frac{2x+(2y-3)i}{4x+(4y+2)i} \times \frac{4x-(4y+2)i}{4x-(4y+2)i}$

$4x(2y-3) - 2x(4y+2) = 0$

$x = 0 \qquad y \neq -\frac{1}{2}$

Ans. = 2

20. Let the line $\frac{x}{1} = \frac{6-y}{2} = \frac{z+8}{5}$ intersect the lines $\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1}$ and $\frac{x+3}{6} = \frac{3-y}{3} = \frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane 2x – 2y + z = 14 is

(1) 3

(2) $\frac{10}{3}$

(3) 4

(4) $\frac{11}{3}$

Sol. **(3)**

$\frac{x}{1} = \frac{y-6}{-2} = \frac{z+8}{5} = \lambda \qquad \ldots(1)$

$\frac{x-5}{4} = \frac{y-7}{3} = \frac{z+2}{1} = \mu \qquad \ldots(2)$

$\frac{x+3}{6} = \frac{y-3}{-3} = \frac{z-6}{1} = \gamma \qquad \ldots(3)$

Intersection of (1) & (2) "A"

$(\lambda, -2\lambda+6, 5\lambda-8) \& (4\mu+5, 3\mu+7, \mu-2)$

$\lambda = 1, \mu = -1$

$A(1, 4, -3)$

Intersection (1) & (3) "B"

$(\lambda, -2\lambda+6, 5\lambda-8) \& (6\gamma-3, -3\gamma+3, \gamma+6)$

$\lambda = 3$

$\gamma = 1$

$B(3, 0, 7)$

Mod point of A & B $\Rightarrow$ (2, 2, 2)

Perpendicular distance from the plane

$2x - 2y + z = 14$

$\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right| = 4$

SECTION - B

21. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _______.

Sol. **(26664)**

$2,1,2,3$

$- \quad - \quad \underline{1} \quad \frac{3!}{2!}=3$

$- \quad - \quad \underline{2} \quad 3!=6$

$- \quad - \quad \underline{3} \quad \frac{3!}{2!}=3$

Sum of digits of unit place $=3\times1+6\times2+3\times3=24$

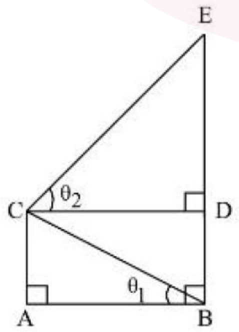
Required sum

$=24\times1000+24\times100+24\times10+24\times1$

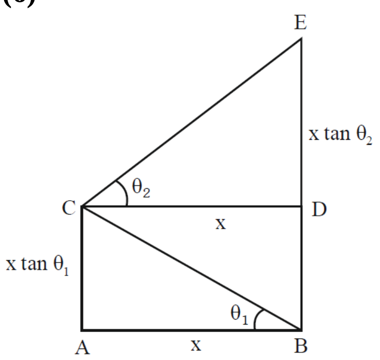
$=24\times1111$

= 26664

22. In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}$ (BE) = 4 (AB). If the area of ΔCAB is $2\sqrt{3}-3$ unit², when $\frac{\theta_2}{\theta_1}$ is the largest, thenthe perimeter (in unit) of ΔCED is equal to _______.



Sol. **(6)**



$\sqrt{3}BE=4AB$

$Ar(\Delta CAB)=2\sqrt{3}-3$

$\frac{1}{2}x^2\tan\theta_1=2\sqrt{3}-3$

$BE=BD+DE$

$=x(\tan\theta_1+\tan\theta_2)$

$BE=AB(\tan\theta_1+\cot\theta_1)$

$$\frac{4}{\sqrt{3}}\tan\theta_1+\cot\theta_1 \Rightarrow \tan\theta_1=\sqrt{3},\frac{1}{\sqrt{3}}$$

$\theta_1=\frac{\pi}{6}\quad \theta_2=\frac{\pi}{3}$

$\theta_1=\frac{\pi}{3}\quad \theta_2=\frac{\pi}{6}$

as $\frac{\theta_2}{\theta_1}$ is largest $\therefore \theta_1=\frac{\pi}{6}\theta_2=\frac{\pi}{3}$

$$\therefore x^2=\frac{(2\sqrt{3}-3)\times 2}{\tan\theta_1}=\frac{\sqrt{3}(2-\sqrt{3})\times 2}{\tan\frac{\pi}{6}}$$

$x^2=12-6\sqrt{3}=(3-\sqrt{3})^2$

$x=3-\sqrt{3}$

Perimeter of ΔCED

$=CD+DE+CE$

$=3\sqrt{3}+(3-\sqrt{3})\sqrt{3}+(3-\sqrt{3})\times 2=6$

Ans. (6)

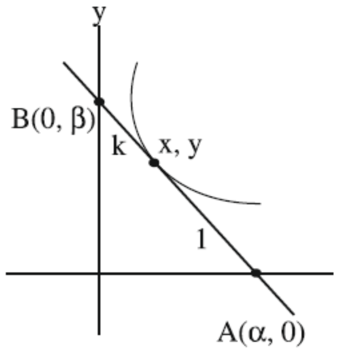
23. Let the tangent at any point P on a curve passing through the points (1,1) and $\left(\frac{1}{10},100\right)$, intersect positive x-axis and y-axis at the points A and B respectively . If PA : PB =1 : k and y = y (x) is the solution of the differential equation $e^{\frac{dy}{dx}}=kx+\frac{k}{2}$, y(0) = k, then 4y then 4y(1) – 5$\log e^3$ is equal to________.

Sol. **(5)**

$Y-y=\frac{dy}{dx}(X-x)$

$Y=0$

$X=\frac{-ydx}{dy}+x$



$\frac{k\alpha+0}{k+1}=x,\ \alpha=\frac{k+1}{k}x$

$\frac{k+1}{k}x=-y\frac{dx}{dy}+x$

$x+\frac{x}{k}=-y\frac{dx}{dy}+x$

$$x\frac{dy}{dx}+ky=0$$

$$\frac{dy}{dx}+\frac{k}{x}y=0$$

$$y\cdot x^{k}=C$$

$$C=1$$

$$100\cdot\left(\frac{1}{10}\right)^{k}=1$$

$$K=2$$

$$\frac{dy}{dx}=\ln(2x+1)$$

$$y=\frac{(2x+1)}{2}(\ln(2x+1)-1)+c$$

$$2=\frac{1}{2}(0-1)+C$$

$$C=2+\frac{1}{2}=\frac{5}{2}$$

$$y(1)=\frac{3}{2}(\ell\ln 3-1)+\frac{5}{2}$$

$$=\frac{3}{2}\ln 3+1$$

$$4y(1)=6\ln 3+4$$

$$4y(1)-5\ln 3=4+\ln 3$$

24. Suppose a_1, a_2, 2, a_3, a_4 be in an arithemetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to________.

Sol. **(16)**

$$\frac{(a-2d)}{4},\frac{(a-d)}{2},a,2(a+d),4(a+2d)$$

$$a=2$$

$$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$$

$$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$$

$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$$

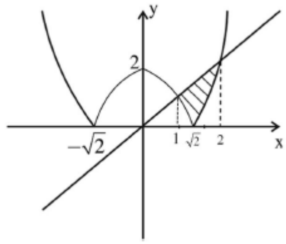
$$d=1$$

$$\Rightarrow a_4=4(a+2d)$$

$$=16$$

25. If the area of the region $\{(x,y) : |x^2 - 2| \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to ______.

Sol. **(27)**



$$A = \int_1^{\sqrt{2}} \left(x - \left(2 - x^2\right)\right)dx + \int_{\sqrt{2}}^{2} \left(x - \left(x^2 - 2\right)\right)dx$$

$$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$$

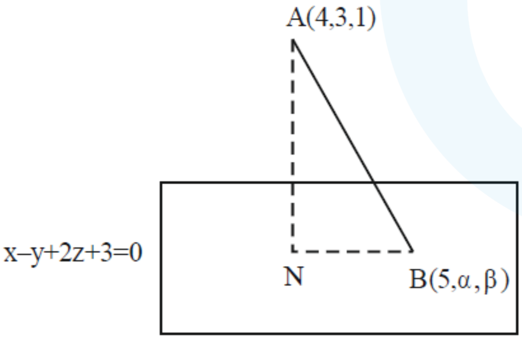
$$= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2}$$

$$6A = -16\sqrt{2} + 27 \therefore 6A + 16\sqrt{2} = 27$$

Ans. 27

26. Let the food of perpendicular from the point A (4, 3, 1) on the plane P : x –y + 2z + 3 = 0 be N. If B(5, α, β), α, $\beta \in Z$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to ________.

Sol. **(7)**



$$AN = \sqrt{6}$$

$$5 - \alpha + 2\beta + 3 = 0$$

$$\Rightarrow \alpha = 8 + 2\beta$$

N is given by

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$$

$$x = 3, y = 4, z = -1$$

N

$$(3, 4, -1)$$

$$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$$

$$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$$

$$\text{Area of } \Delta ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$$

$$\frac{1}{2}\times\sqrt{6}\times BN=3\sqrt{2}$$
$$BN=2\sqrt{3}$$
$$4+(2\beta+4)^2+(\beta+1)^2=12$$
$$(2\beta+4)^2+(\beta+1)^2-8=0$$
$$5\beta^2+18\beta+9=0$$
$$(5\beta+3)(\beta+3)=0$$
$$\beta=-3$$
$$\alpha=2$$
$$\alpha^2+\beta^2+\alpha\beta=9+4-6=7$$

27. Let S be the set of values of λ, for which the system of equations
$6\lambda x - 3y + 3z = 4\lambda^2$,
$2x + 6\lambda y + 4z = 1$,
$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{l\in S}|\lambda|$ is equal to ________.

Sol. **(24)**

$$\Delta=\begin{vmatrix}6\lambda & -3 & 3\\ 2 & 6\lambda & 4\\ 3 & 2 & 3\lambda\end{vmatrix}=0$$
$$2\lambda\left(9\lambda^2-4\right)+(3\lambda-6)+(2-9\lambda)=0$$
$$18\lambda^3-14\lambda-4=0$$
$$(\lambda-1)(3\lambda+1)(3\lambda+2)=0$$
$$\Rightarrow \lambda=1,-1/3,-2/3$$

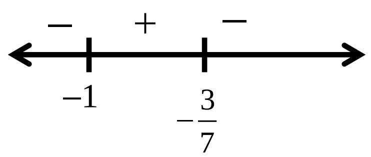
For each values of $\lambda,\Delta_1=\begin{vmatrix}6\lambda & -3 & 4\lambda^2\\ 2 & 6\lambda & 1\\ 3 & 2 & \lambda\end{vmatrix}\neq 0$

$$12\left(1+\frac{1}{3}+\frac{2}{3}\right)=24$$

28. If the domain of the function $f(x)=\sec^{-1}\left(\frac{2x}{5x+3}\right)$ is [α, β) U (γ, δ], then |3α+10(β+γ)+21δ| is equal to _______.

Sol. **(24)**

$$f(x)=\sec^{-1}\frac{2x}{5x+3}$$
$$\left|\frac{2x}{5x+3}\right|$$
$$\left|\frac{2x}{5x+3}\right|\geq 1\Rightarrow |2x|\geq|5x+3|$$
$$(2x)^2-(5x+3)^2\geq 0$$
$$(7x+3)(-3x-3)\geq 0$$



$$\therefore \text{ domain}\left[-1,\frac{-3}{5}\right)\cup\left(\frac{-3}{5},\frac{-3}{7}\right]$$

$$\alpha=-1,\beta=\frac{-3}{5},\gamma=\frac{-3}{5},\delta=\frac{-3}{7}$$

$$3\alpha+10(\beta+\gamma)+21\delta=-3$$

$$-3+10\left(\frac{-6}{5}\right)+\left(\frac{-3}{7}\right)21=-24$$

29. Let the quadratic curve passing through the point (–1,0) and touching the line y = x at (1,1) be y = f(x). Then the x-intercept of the normal to the curve at the point (α, α + 1) in the first quadrant is ________.

Sol. **(11)**

$$f(x)=(x+1)(ax+b)$$

$$1=2a+2b$$

$$f'(x)=(ax+b)+a(x+1)$$

$$1=(3a+b)$$

$$\Rightarrow b=1/4, a=1/4$$

$$f(x)=\frac{(x+1)^2}{4}$$

$$f'(x)=\frac{x}{2}+\frac{1}{2} \quad \alpha+1=\frac{(\alpha+1)^2}{4}, \alpha>-1$$

$$\alpha+1=4$$

$$\alpha=3$$

normal at (3, 4)

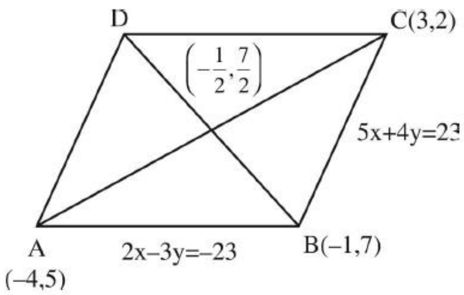
$$y-4=-\frac{1}{2}(x-3)$$

$$y=0 \quad x=8+3$$

Ans. 11

30. Let the equations of two adjacent sides of a parallelogram ABCD be 2x–3y =–23 and 5x + 4y = 23. If the equation of its one diagonal AC is 3x + 7y = 23 and the distance of A from the other diagonal is d, then 50 d^2 is equal to ________.

Sol. **(529)**



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y-\frac{7}{2}=\frac{\frac{7}{2}}{-\frac{1}{2}}\left(x+\frac{1}{2}\right)$$

$\Rightarrow \quad 7x+y=0$

Distance of A from diagonal BD

$=d=\frac{23}{\sqrt{50}}$

$\Rightarrow \quad 50d^2=(23)^2$

$50d^2=529$