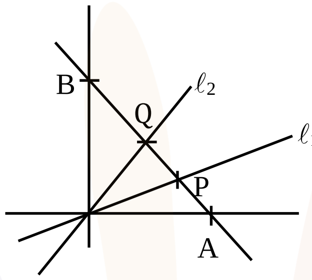
FINAL JEE–MAIN EXAMINATION – APRIL, 2023
Held On Thursday 06th April, 2023
TIME : 09:00 AM to 12:00 PM

SECTION-A

1. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L : 9x + 5y = 45 between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on.

(1) 6x + y = 10 (2) 6x – y = 15 (3) y – 2x = 5 (4) y – x = 5

Sol. **(4)**



L : 9x + 5y = 45

A(5,0) , B : (0,9)

$$\rightarrow P_x = \frac{2\times5+1\times0}{1+2} = \frac{10}{3}$$

$$P_y = \frac{0\times2+9\times1}{1+2} = 3$$

$$P : \left(\frac{10}{3}, 3\right)$$

Similarly $\rightarrow Q_x = \frac{1\times5+2\times0}{1+2} = \frac{5}{3}$

$$Q_y = \frac{1\times0+2\times9}{1+2} = 6$$

$$Q : \left(\frac{5}{3}, 6\right)$$

Now $m_1 = \frac{3-0}{\frac{10}{3}-0} = \frac{9}{10}$

$$m_2 = \frac{6-0}{\frac{5}{3}-0} = \frac{18}{5}$$

Now $L_1 : y\ (m_1 + m_2)x \Rightarrow y = \left(\frac{9}{2}\right)x \Rightarrow 9x = 2y$ ...(2)

from (1) & (2)

$$\begin{aligned} 9x + 5y &= 45 \\ 9x - 2y &= 0 \\ - \quad + \quad & \quad - \\ \hline 7y &= 45 \end{aligned} \Rightarrow y = \frac{45}{7}$$

$$\Rightarrow x = \frac{10}{7}$$

which satisfy y – x = 5 Ans. 4

2. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set $S=\{\lambda\in\mathbb{R}$: the points A, B, C and D are coplanar$\}$. Then $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to :

(1) $\frac{37}{2}$ (2) 13 (3) 25 (4) 41

Sol. **(4)**

A, B, C, D are coplanar

$$\Rightarrow \left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]=0 \qquad \Rightarrow \begin{vmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{vmatrix}=0$$

$\Rightarrow -6\,[6\lambda-12-(\lambda-5)(3-2\lambda)] + 0\ [\ ] + (6-2\lambda)\,[20-4\lambda-21]$

$\Rightarrow -6\,[6\lambda-12+2\lambda^2+15-13\lambda\ ] + (6-2\lambda)\,[-4\lambda-1]=0$

$\Rightarrow -12\lambda^2+42\lambda-18+8\lambda^2-22\lambda-6=0$

$\Rightarrow -4\lambda^2+20\lambda-24=0 \qquad \Rightarrow \lambda^2-5\lambda+6=0$

$(\lambda-3)(\lambda-2)=0$ ⟨ $\lambda=2$, $\lambda=3$

Now $\sum_{\lambda\in s}(\lambda+2)^2 = 16+25=41$

3. Let $I(x)=\int\frac{x^2\left(x\sec^2x+\tan x\right)}{(x\tan x+1)^2}dx$. If I(0) = 0, then $I\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\log_e\frac{(\pi+4)^2}{16}+\frac{\pi^2}{4(\pi+4)}$ (2) $\log_e\frac{(\pi+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$

(3) $\log_e\frac{(\pi+4)^2}{16}-\frac{\pi^2}{4(\pi+4)}$ (4) $\log_e\frac{(\pi+4)^2}{32}+\frac{\pi^2}{4(\pi+4)}$

Sol. **(2)**

$$I(x)=\int\frac{x^2\left(x\sec^2x+\tan x\right)}{(x\tan x+1)^2}dx$$

Let xtan x + 1 = t

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+\int\frac{2x}{x\tan x+1}dx$$

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+2\int\frac{2x}{x\tan x+1}dx$$

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+2\ln|x\sin x+\cos x|+C$$

As I (0) = 0 ⇒ C = 0

$$I\left(\frac{\pi}{4}\right)=\ln\left(\frac{(\pi+4)^2}{32}\right)-\frac{\pi^2}{4(\pi+4)}$$

4. The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + ….. is :

(1) 3450 (2) 3420 (3) 3520 (4) 3250

Sol. **(3)**

$$s_n = 5+11+19+29+41+.....+T_n$$
$$S_n = \quad 5+11+19+29+...+T_{n-1}+T_n$$
$$0 = 5+\{\underbrace{6+8+10+12+....}_{(n-1)\text{terms}}\}-T_n$$

$$T_n = 5+\frac{(n-1)}{2}[2\cdot 6+(n-2)\cdot 2]$$

$T_n = 5 + (n - 1)(n + 4) = 5 + n^2 + 3n - 4 = n^2 + 3n + 1$

Now $S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20} n^2 + 3n + 1$

$S_{20} = \frac{20.21.41}{6} + \frac{3.20.21}{2} + 20$

$S_{20} = 2870 + 630 + 20$

$S_{20} = 3520$

5. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to :

(1) 164 (2) 123 (3) 82 (4) 75

Sol. **(2)**

n(total 5) = {1, 4), (2, 3), (3, 2),(4, 1)} = 4

P(success) = $\frac{4}{36} = \frac{1}{9}$

P(at least 4 success) = P (4 success) + P(5 success)

$= {}^5C_4 \cdot \left(\frac{1}{9}\right)^4 \cdot \frac{8}{9} + {}^5C_5\left(\frac{1}{9}\right)^5 = \frac{41}{9^5} = \frac{41}{3^{10}} = \frac{123}{3^{11}} = \frac{k}{3^{11}}$

K = 123

6. Let $A = [a_{ij}]_{2\times 2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and b = |A|. Then $3a^2 + 4b^2$ is equal to :

(1) 14 (2) 4 (3) 3 (4) 7

Sol. **(2)**

$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$

Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = I$

$\begin{bmatrix} \alpha^2+\beta\gamma & \alpha\beta+\beta\delta \\ \alpha\gamma+\gamma\delta & \gamma\beta+\delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow \alpha^2 + \beta\gamma = 1$

$(\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$

$(\alpha + \delta)\gamma = 0$

$\beta\gamma + \delta^2 = 0$

Now $3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$

7. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If d > 0 is its common difference, then : $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is

(1) $\frac{1}{\sqrt{d}}$ (2) 1 (3) $\sqrt{d}$ (4) 0

Sol. (2)

$$\underset{n\to\infty}{\mathrm{Lt}}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2}+\frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3}+\ldots\ldots+\frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}\right)$$

$$= \underset{n\to\infty}{\mathrm{Lt}}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}+\sqrt{a_3}+\ldots\ldots+\sqrt{a_{n-1}}-\sqrt{a_n}}{-d}\right)$$

$$= \underset{n\to\infty}{\mathrm{Lt}}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$= \underset{n\to\infty}{\mathrm{Lt}}\frac{1}{\sqrt{n}}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{\sqrt{d}}\right)$$

$$= \underset{n\to\infty}{\mathrm{Lt}}\frac{1}{\sqrt{d}}\left(\sqrt{\frac{a_1}{n}+d-\frac{d}{n}}-\frac{\sqrt{a_1}}{n}\right)$$

$= 1$

8. If ${}^{2n}C_3 : {}^{n}C_3 : 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is :

(1) 27 :11 (2) 35 : 16 (3) 2 : 1 (4) 65 : 37

Sol. (3)

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = 10 \Rightarrow \frac{2n!(n-3)!}{(2n-3)!n!} = 10$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$$

$$\frac{4(2n-1)}{n-2} = 10 \Rightarrow 8n - 4 = 10n - 20$$

$2n = 16$

Now $\frac{n^2+3n}{n^2-3n+4}$

$$= \frac{64+24}{64-24+4} = \frac{88}{44} = 2$$

Ans. 3

9. Let $A = \{x \in \mathbb{R} : [x+3] + [x+4] \le 3\}$,

$B=\left\{x\in\mathbb{R}:3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$, where [t] denotes greatest integer function. Then,

(1) $A \subset B, A \ne B$ (2) $A \cap B = \phi$ (3) A = B (4) $B \subset C, A \ne B$

Sol. **(3)**

$A = \{x \in \mathbb{R} : [x + 3] + [x + 4] \le 3\}$,

$2[x]+7 \le 3$

$2[x] \le -4$

$[x] \le -2 \Rightarrow x < -1$ ...(A)

$$B = \left\{x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$$

$$3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right)^{x-3} < 3^{-3x}$$

$$3^{2x-3} \left(\frac{\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3} < 3^{-3x}$$

$$\Rightarrow \left(\frac{1}{9}\right)^{x-3} < 3^{-5x+3}$$

$$\Rightarrow 3^{6-2x} < 3^{3-5x}$$

$$\Rightarrow 6 - 2x < 3 - 5x$$

$$\Rightarrow 3 < -3x$$

$$\Rightarrow \boxed{x < -1}$$ ...(B)

A = B

10. One vertex of a rectangular parallelepiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is :

(1) $\frac{12}{5\sqrt{5}}$ (2) $12\sqrt{5}$ (3) $\frac{12}{5}$ (4) $\frac{12}{\sqrt{5}}$

Sol. **(3)**

Equation of OP is $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$a_1 = (0, 0, 0)$ $a_2 = (3, 0, 5)$

$b_1 = (3, 4, 5)$ $b_2 = (0, 0, 1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0} = \frac{y-0}{0} = \frac{z-5}{1}$$

$$S.D = \frac{(\vec{a}_2 \cdot \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\frac{\begin{vmatrix} 3 & 0 & 5 \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}} = \frac{3(4)}{|4\hat{i} - 3\hat{j}|} = \frac{12}{5}$$

11. If the equation of the plane passing through the line of intersection of the planes 2x – y + z = 3, 4x – 3y + 5z + 9 = 0 and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is ax + by + cz + 6 = 0, then a + b + c is equal to :

(1) 15 (2) 14 (3) 13 (4) 12

Sol. **(2)**

Using family of planer

$P : P_1 + \lambda P_2 = 0 \Rightarrow P(2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z = (3 - 9\lambda)$

P is || to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$

Then for λ : $\vec{n}_p \cdot \vec{v}_L = 0$

$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$

$-3 + 5\lambda = 0 \Rightarrow \lambda = \frac{3}{5}$

Hence : P : 22x – 14 y + 20 z = –12

P : 11 x – 7y + 10z + 6 = 0

⇒ a = 11

b = – 7

c = 10

⇒ a + b + c = 14

Ans. 2

12. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$, then the third term from the beginning is :

(1) $30\sqrt{2}$ (2) $60\sqrt{2}$ (3) $30\sqrt{3}$ (4) $60\sqrt{3}$

Sol. **(4)**

$$\frac{T_5}{T_5'} = \frac{{}^nC_4 \cdot \left((2)^{\frac{1}{4}}\right)^{n-4}\left(\frac{1}{3^{\frac{1}{4}}}\right)^4}{{}^nC_4\left(\frac{1}{3^{\frac{1}{4}}}\right)^{n-4}\left(2^{\frac{1}{4}}\right)^4} = \frac{\sqrt{6}}{1}$$

$$2^{\frac{n-8}{4}} \cdot \left(3^{\frac{1}{4}}\right)^{-4-4+n} = \sqrt{6}$$

$$2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$$

$$\frac{n-8}{4} = \frac{1}{2} \Rightarrow n - 8 = 2 \Rightarrow n = 10$$

$$T_3 = {}^{10}C_2\left(2^{\frac{1}{4}}\right)^8\left(\frac{1}{3^{\frac{1}{4}}}\right)^2$$

$$= {}^{10}C_2 . 2^2 . 3^{-\frac{1}{2}} = \frac{10.9}{2} .4. \frac{1}{\sqrt{3}} = 60\sqrt{3}$$

13. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is :

(1) $11-\sqrt{3}$ (2) $9-\sqrt{3}$ (3) $9+\sqrt{3}$ (4) $11+\sqrt{3}$

Sol. **(3)**

$|x^2 - 8x + 15| = 2x - 7$

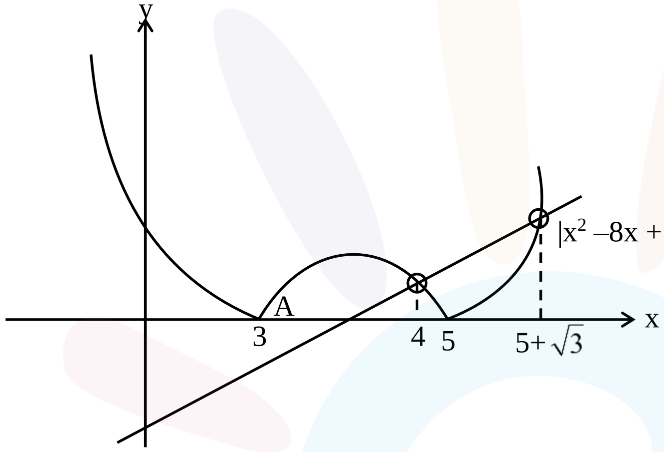
$x^2 - 8x + 15 = 2x - 7$ & $x^2 - 8x + 15 = 7 - 2x$

$x^2 - 10x + 22 = 0$ & $x^2 - 6x + 8 = 0$

$x_1 = 5 + \sqrt{3}$ $x_2 = 5 - \sqrt{3}$ (reject) $x_3 = 4$ $x_4 = 2$ (reject)

Sum of of roots is $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$

Ans. 3

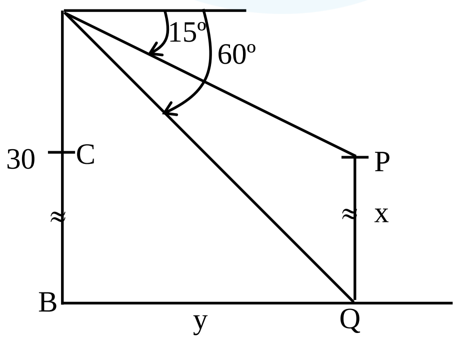


14. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to :

(1) $200(3-\sqrt{3})$ (2) $300(\sqrt{3}+1)$ (3) $300(\sqrt{3}-1)$ (4) $600(\sqrt{3}-1)$

Sol. **(4)**

ΔABQ



$\frac{AB}{BQ} = \tan 60°$

$BQ = \frac{30}{\sqrt{3}} = 10\sqrt{3} = y$

& ΔACP

$\frac{AC}{CP} = \tan 15° \Rightarrow \frac{(30 - x)}{y} = (2 - \sqrt{3})$

$30 - x = 10\sqrt{3}(2 - \sqrt{3})$

$30 - x = 20\sqrt{3} - 30$

$x = 60 - 20\sqrt{3}$

Area = x.y = $20(3-\sqrt{3}).10\sqrt{3}$

$= 600(\sqrt{3}-1)$

Ans. (4)

15. Let $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}, b = \hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c} = -\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}.\vec{d} = 18$, then $[\vec{a}\times\vec{d}]^2$ is equal to :

(1) 760 (2) 640 (3) 720 (4) 680

Sol. **(3)**

$\vec{d} = \lambda(\vec{b}\times\vec{c})$

For λ : $\vec{a}.\vec{d} = 18 \Rightarrow \lambda[\vec{a}\ \vec{b}\ \vec{c}] = 18$

$$\Rightarrow \lambda \begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 18$$

$\Rightarrow \lambda(4-3+8) = 18 \Rightarrow \lambda = 2$

$\Rightarrow \vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$

Hence $|\vec{a}\times\vec{d}|^2 = a^2d^2 - (\vec{a}.\vec{d})^2$

$= 29\cdot 36 - (18)^2 = 18(58-18)$

$= 18\cdot 40 = 720$

Ans. 3

16. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to :

(1) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (2) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$ (3) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$ (4) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

Sol. **(2)**

$2x^y + 3y^x = 20$

$$v_1^{v_2}\left(v_2\frac{1}{v_1} + \ln v_1 . v_2^1\right)$$

$$2x^y\left(y.\frac{1}{x} + \ln x\frac{dy}{dx}\right) + 3y^x\left(x\frac{1}{y}.\frac{dy}{dx} + \ln y.1\right) = 0$$

Put (2, 2)

$$2.4\left(1+\ln 2\frac{dy}{dx}\right) + 3.4\left(1.\frac{dy}{dx} + \ln 2\right) = 0$$

$$\frac{dy}{dx}[8\ln 2 + 12] + 8 + 12\ln 2 = 0$$

$$\frac{dy}{dx} = -\left[\frac{2+3\ln 2}{3+2\ln 2}\right] = -\left[\frac{2+\ln 8}{3+\ln 4}\right]$$

17. If the system of equations

x + y + az = b

2x + 5y + 2z = 6

x + 2y + 3z = 3

has infinitely many solutions, then 2a + 3b is equal to :

(1) 28 (2) 20 (3) 25 (4) 23

Sol. **(4)**

x + y + az = b

2x + 5y + 2z = 6

x + 2y + 3z = 3

For ∞ solution

$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence 2a + 3b = 23

Ans. 4

18. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to:

(1) $(P \vee R) \Rightarrow Q$ (2) $(P \Rightarrow R) \vee (Q \Rightarrow R)$ (3) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ (4) $(P \wedge R) \Rightarrow Q$

Sol. **(1)**

$(P \Rightarrow Q) \wedge (R \Rightarrow Q)$

We known that $P \Rightarrow Q \equiv \sim P \vee Q$

$\Rightarrow (\sim P \vee Q) \wedge (\sim R \vee Q)$

$\Rightarrow (\sim P \wedge \sim R) \vee Q$

$\Rightarrow \sim (P \vee R) \vee Q$

$\Rightarrow (P \vee R) \Rightarrow Q$

19. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)\,dx$ is equal to :

(1) $10 \log_e 2 - 6$ (2) $10 \log_e 2 + 6$ (3) $5 \log_e 2 - 3$ (4) $5 \log_e 2 + 3$

Sol. **(1)**

$$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \quad \ldots(1)$$

$$x \to \frac{1}{x}$$

$$5f\left(\frac{1}{x}\right) + 4\,f(x) = x + 3 \quad \ldots(2)$$

$(1) \times 5 - (2) \times 4$

$$\Rightarrow f(x) = \frac{5}{9x} - \frac{4}{9}x + \frac{1}{3}$$

$$\Rightarrow 18\int_1^2 f(x)\,dx = 18\left(\frac{5}{9}\ln 2 - \frac{4}{9} \times \frac{3}{2} + \frac{1}{3}\right)$$

$= 10 \ln 2 - 6$

20. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to :

(1) 12 (2) 10 (3) 11 (4) 9

Sol. **(2)**

Combine var. $= \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1 + n_2)}$

$13 = \frac{15\cdot 14 + 15\cdot \sigma^2}{30} + \frac{15\cdot 15(12-14)^2}{30\times 30}$

$13 = \frac{14+\sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

SECTION-B

21. Let the tangents to the curve $x^2 + 2x - 4y + 9 = 0$ at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to _______.

Sol. **(292)**

C : $x^2 + 2x - 4y + 9 = 0$

C : $(x + 1)^2 = 4(y - 2)$

$T_{P(1,3)}$: $x.1 + (x + 1) - 2(y + 3) + 9 = 0$

: $2x - 2y + 4 = 0$

T_P : $x - y + 2 = 0$

A : (0, 2)

Line || to $x-3y = 6$ passes (1, 3) is $x - 3y + 8 = 0$

Meet parabola : $y^2 = 4x$

$\Rightarrow y^2 = 4(3y - 8)$

$\Rightarrow y^2 - 12y + 32 = 0$

$\Rightarrow (y - 8)(y - 4) = 0$

$\Rightarrow$ point of intersection are

(4, 4) & $\underset{B}{(16,8)}$ lies on $2x - 3y = 8$

Hence A : (0, 2)

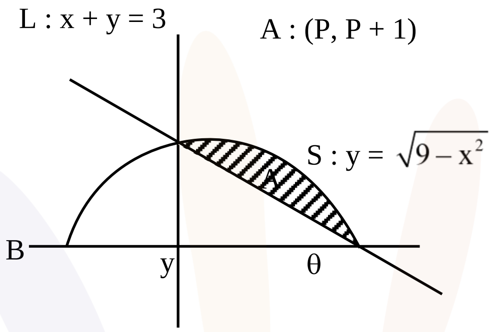
B : (16, 8)

$(AB)^2 = 256 + 36 = 292$

22. Let the point (p, p + 1) lie inside the region $E = \left\{(x, y) : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to _____.

Sol. **(3)**

$3 - x \le y \le \sqrt{9 - x^2}; 0 \le x \le 3$



$L(A) > 0 \Rightarrow P + P + 1 - 3 > 0 \Rightarrow P > 1 \quad \ldots(1)$

$S(A) < 0 \Rightarrow P + 1 - \sqrt{9 - P^2} < 0$

$\Rightarrow P + 1 < \sqrt{9 - P^2}$

$\Rightarrow P + 2P + 1 < 9 - P^2$

$\Rightarrow 2P^2 + 2P - 8 < 0$

$\Rightarrow P^2 + P - 4 < 0$

$\Rightarrow P \in \left(\frac{-1 - \sqrt{17}}{2}, \frac{-1 + \sqrt{17}}{2}\right) \quad \ldots(2)$

$(1) \cap (2)\ P \in \left(1, \frac{\sqrt{17} - 1}{2}\right) \equiv (a, b)$

$b^2 + b - a^2 = 4 - 1 = 3$

23. Let y = y(x) be a solution of the differential (xcos x)dy + (xysin x + ycos x – 1)dx = 0, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to _____.

Sol. **(2)**

$(x \cos x)\, dy + (xy\sin x + y\cos x - 1)\, dx = 0, \ 0 < x < \frac{\pi}{2}$

$\frac{dy}{dx} + \left(\frac{x \sin x + \cos x}{x \cos x}\right) y = \frac{1}{x \cos x}$

IF = x secx

$y.x \sec x = \int \frac{x \sec x}{x \cos x} dx = \tan x + c$

Since $y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$ Hence $c = \sqrt{3}$

Hence $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$

24. Let $a \in \mathbb{Z}$ and [t] be the greatest integer $\leq$ t. Then the number of points, where the function f(x) = [a + 13 sin x], $x \in (0, \pi)$ is not differentiable, is _____.

Sol. **(25)**

$f(x) = [a + 13 \sin x] = a + [13 \sin x]$ in $(0, \pi)$

$x \in (0, \pi)$

$\Rightarrow 0 < 13 \sin x \leq 13$

$\Rightarrow [13 \sin x] = \{0, 1, 2, 3, \ldots 12, 13,\}$

↓ ↓ ↓

2 2 1

Total point of N.D. = 25.

25. If the area of the region $S = \{(x, y) : 2y - y^2 \leq x^2 \leq 2y,\ x \geq y\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number n is equal to _____.

Sol. **(5)**

$x^2 + y^2 - 2y \geq 0$ & $x^2 - 2y \leq 0$, $x \geq y$

Hence required area = $\frac{1}{2} \times 2 \times 2 - \int_0^2 \frac{x^2}{2} dx - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{7}{6} - \frac{\pi}{4} \Rightarrow n = 5$

26. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _____.

Sol. **3483638676**

Total – (one child receive no orange + two child receive no orange)

$= 3^{20} - ({}^3C_1 (2^{20} - 2) + {}^3C_2 1^{20}) = 3483638676$

27. Let the image of the point P (1, 2, 3) in the plane 2x – y + z = 9 be Q. If the coordinates of the point R are (6, 10, 7). then the square of the area of the triangle PQR is _____.

Sol. **(594)**

Let $Q(\alpha, \beta, \gamma)$ be the image of P, about the plane

$2x - y + z = 9$

$\frac{\alpha - 1}{2} = \frac{\beta - 2}{-1} = \frac{\gamma - 3}{1} = 2$

$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$

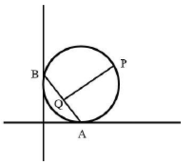
Then area of triangle PQR is = $\frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$= \left|-12\hat{i} - 3\hat{j} + 21\hat{k}\right| = \sqrt{144 + 9 + 441} = \sqrt{594}$

Square of area = 594

28. A circle passing through the point P(α. β) in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then value of αβ is _____.

Sol. **(121)**



Let equation of circle is $(x - a)^2 + (y - a)^2 = a^2$
which is passing through P (α,β)
then $(\alpha - a)2 + (\beta - a)^2 = a^2$
$\alpha^2 + \beta^2 - 2\alpha a - 2\beta\alpha + a^2 = 0$
Here equation of AB is $x + y = a$
Let Q (α',β') be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1} = \frac{\beta'-\beta}{1} = \frac{-(\alpha+\beta-a)}{2}$$

$$PQ^2 = (\alpha' - \alpha) + (\beta' - \beta) = \frac{1}{4}(\alpha + \beta - a)^2 + \frac{1}{4}(\alpha + \beta - a)^2$$

$$121 = \frac{1}{2}(\alpha+\beta-a)^2$$

$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$
$242 = 2\alpha\beta$
$\Rightarrow \alpha\beta = 121$

29. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is ______.

Sol. **(5005)**

$$\left(x^4 - \frac{1}{x^3}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r \left(x^4\right)^{15-r} \left(\frac{-1}{x^3}\right)^r$$

$60 - 7r = 18$
$r = 6$
Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$

30. Let A = {1, 2, 3, 4, …., 10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation $R = \{(a, b) \in A \times A: 2(a - b)^2 + 3(a - b) \in B\}$ is _____.

Sol. **(18)**
A = {1,2,3,......10}
B = {0,1, 2,3, 4}
$R = \{(a, b) \in A \times A: 2(a - b)^2 + 3(a - b) \in B\}$
Now $2(a - b)^2 + 3(a - b) = (a - b)(2(a - b) + 3)$
$\Rightarrow a = b$ or $a - b = -2$
When $a = b \Rightarrow$ 10 order pairs
When $a - b = -2 \Rightarrow$ 8 order pairs
Total = 18