



**FINAL JEE–MAIN EXAMINATION – JANUARY, 2023**  
**Held On Tuesday 24th January, 2023**  
**TIME : 09:00 AM to 12:00 PM**

**SECTION-A**

61. The distance of the point (7, -3, -4) from the plane passing through the points (2, -3, 1), (-1, 1, -2) and (3, -4, 2) is :

- (1) 4
- (2) 5
- (3)  $5\sqrt{2}$
- (4)  $4\sqrt{2}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol. Equation of Plane is**

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$$

$$x - z - 1 = 0$$

Distance of P (7, -3, -4) from Plane is

$$d = \left| \frac{7+4-1}{\sqrt{2}} \right| = 5\sqrt{2}$$

62.  $\lim_{t \rightarrow 0} \left( 1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \dots + n^{\frac{1}{\sin^2 t}} \right)^{\sin^2 t}$  is equal to

- (1)  $n^2 + n$
- (2) n
- (3)  $\frac{n(n+1)}{2}$
- (4)  $n^2$

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $\lim_{t \rightarrow 0} \left( 1^{\operatorname{cosec}^2 t} + 2^{\operatorname{cosec}^2 t} + \dots + n^{\operatorname{cosec}^2 t} \right)^{\sin^2 t}$

$$= \lim_{t \rightarrow 0} n \left( \left( \frac{1}{n} \right)^{\operatorname{cosec}^2 t} + \left( \frac{2}{n} \right)^{\operatorname{cosec}^2 t} + \dots + 1 \right)^{\sin^2 t}$$

= n

63. Let  $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v} \cdot \vec{w} = 2$  and  $\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v}$ . Then  $\vec{u} \cdot \vec{w}$  is equal to

- (1) 1
- (2)  $\frac{3}{2}$
- (3) 2
- (4)  $-\frac{2}{3}$

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $\vec{u} = (1, -1, -2), \vec{v} = (2, 1, -1), \vec{v} \cdot \vec{w} = 2$

$$\vec{v} \times \vec{w} = \vec{u} + \lambda \vec{v} \dots\dots\dots(1)$$

Taking dot with  $\vec{w}$  in (1)

$$\vec{w} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{w} + \lambda \vec{v} \cdot \vec{w}$$

$$\Rightarrow 0 = \vec{u} \cdot \vec{w} + 2\lambda$$

Taking dot with  $\vec{v}$  in (1)

$$\vec{v} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{v} + \lambda \vec{v} \cdot \vec{v}$$

$$\Rightarrow 0 = (2 - 1 + 2) + \lambda(6)$$

$$\lambda = -\frac{1}{2}$$

$$\Rightarrow \vec{u} \cdot \vec{w} = -2\lambda = 1$$

64. The value  $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r$  is

- (1)  ${}^{45}C_{23}$
- (2)  ${}^{44}C_{23}$
- (3)  ${}^{45}C_{24}$
- (4)  ${}^{44}C_{22}$

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{23-r}$

$$= {}^{45}C_{23}$$



65. Let a tangent to the curve  $y^2 = 24x$  meet the curve  $xy = 2$  at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

- (1) directrix  $4x = 3$
- (2) directrix  $4x = -3$
- (3) Length of latus rectum  $\frac{3}{2}$
- (4) Length of latus rectum 2

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $y^2 = 24x$   
 $a = 6$   
 $xy = 2$   
 $AB \equiv ty = x + 6t^2 \dots\dots\dots(1)$   
 $AB \equiv T = S_1$   
 $kx + hy = 2hk \dots\dots\dots(2)$   
 From (1) and (2)  
 $\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$   
 $\Rightarrow$  then locus is  $y^2 = -3x$   
 Therefore directrix is  $4x = 3$

66. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$$\begin{aligned} x + y + z &= 1 \\ 2x + Ny + 2z &= 2 \\ 3x + 3y + Nz &= 3 \end{aligned}$$

has unique solution is  $\frac{k}{6}$ , then the sum of value of k and all possible values of N is

- (1) 18
- (2) 19
- (3) 20
- (4) 21

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**  $x + y + z = 1$   
 $2x + Ny + 2z = 2$   
 $3x + 3y + Nz = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$= (N - 2)(N - 3)$   
 For unique solution  $\Delta \neq 0$   
 So  $N \neq 2, 3$   
 $\Rightarrow$  P (system has unique solution)  $= \frac{4}{6}$   
 So  $k = 4$   
 Therefore sum  $= 4 + 1 + 4 + 5 + 6 = 20$

67.  $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\frac{\sqrt{8+4\sqrt{3}}}{\sqrt{6+3\sqrt{3}}}\right)$  is equal to

- (1)  $\frac{\pi}{4}$
- (2)  $\frac{\pi}{2}$
- (3)  $\frac{\pi}{3}$
- (4)  $\frac{\pi}{6}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**  $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\frac{\sqrt{8+4\sqrt{3}}}{\sqrt{6+3\sqrt{3}}}\right)$   
 $= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$

68. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that

$$\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$$

Then  $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$  is

equal to

- (1) 4
- (2) 3
- (3) 2
- (4)  $\frac{5}{2}$

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** Let P is  $\vec{0}$ , Q is  $\vec{q}$  and R is  $\vec{r}$   
 A is  $\frac{2\vec{q} + \vec{r}}{3}$ , B is  $\frac{2\vec{r}}{3}$  and C is  $\frac{\vec{q}}{3}$

$$\text{Area of } \Delta PQR \text{ is } = \frac{1}{2} |\vec{q} \times \vec{r}|$$

$$\text{Area of } \Delta ABC \text{ is } \frac{1}{2} |\overline{AB} \times \overline{AC}|$$

$$\overline{AB} = \frac{\vec{r} - 2\vec{q}}{3}, \overline{AC} = \frac{-\vec{r} - \vec{q}}{3}$$

$$\text{Area of } \Delta ABC = \frac{1}{6} |\vec{q} \times \vec{r}|$$

$$\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} = 3$$

69. If A and B are two non-zero  $n \times n$  matrices such that  $A^2 + B = A^2 B$ , then

- (1)  $AB = I$
- (2)  $A^2 B = I$
- (3)  $A^2 = I$  or  $B = I$
- (4)  $A^2 B = BA^2$

**Official Ans. by NTA (4)**

**Ans. (4)**



**Sol.**  $A^2 + B = A^2 B$   
 $(A^2 - 1)(B - 1) = 1 \dots\dots(1)$

$A^2 + B = A^2 B$   
 $A^2(B - 1) = B$   
 $A^2 = B(B - 1)^{-1}$   
 $A^2 = B(A^2 - 1)$   
 $A^2 = BA^2 - B$   
 $A^2 + B = BA^2$   
 $A^2 B = BA^2$

**70.** Let  $y = y(x)$  be the solution of the differential equation  $x^3 dy + (xy - 1) dx = 0, x > 0,$

$y\left(\frac{1}{2}\right) = 3 - e$ . Then  $y(1)$  is equal to

- (1) 1
- (2) e
- (3) 2-e
- (4) 3

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $\frac{dy}{dx} = \frac{1 - xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

If  $f = e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$

$y \cdot e^{-\frac{1}{x}} = \int e^{-\frac{1}{x}} \cdot \frac{1}{x^3} dx$  (put  $-\frac{1}{x} = t$ )

$y \cdot e^{-\frac{1}{x}} = -\int e^t \cdot t dt$

$y = \frac{1}{x} + 1 + Ce^{\frac{1}{x}}$

Where C is constant

Put  $x = \frac{1}{2}$

$3 - e = 2 + 1 + Ce^2$

$C = -\frac{1}{e}$

$y(1) = 1$

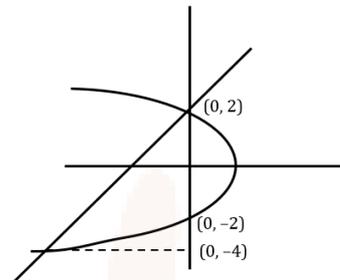
**71.** The area enclosed by the curves  $y^2 + 4x = 4$  and  $y - 2x = 2$  is :

- (1)  $\frac{25}{3}$
- (2)  $\frac{22}{3}$
- (3) 9
- (4)  $\frac{23}{3}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.**



$y^2 + 4x = 4$

$y^2 = -4(x - 1)$

$A = \int_{-2}^2 \left( \frac{4 - y^2}{4} - \frac{y - 2}{2} \right) dy = 9$

**72.** Let  $\alpha$  be a root of the equation  $(a - c)x^2 + (b - a)x + (c - b) = 0$  where a, b, c are distinct real numbers such that the matrix

$$\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

is singular. Then the value of

$\frac{(a - c)^2}{(b - a)(c - b)} + \frac{(b - a)^2}{(a - c)(c - b)} + \frac{(c - b)^2}{(a - c)(b - a)}$  is

- (1) 6
- (2) 3
- (3) 9
- (4) 12

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$

$\Rightarrow \alpha^2(c - b) - \alpha(c - a) + (b - a) = 0$

It is singular when  $\alpha = 1$

$\frac{(a - c)^2}{(b - a)(c - b)} + \frac{(b - a)^2}{(a - c)(c - b)} + \frac{(c - b)^2}{(a - c)(b - a)}$

$\frac{(a - b)^3 + (b - c)^3 + (c - a)^3}{(a - b)(b - c)(c - a)}$   
 $= 3 \frac{(a - b)(b - c)(c - a)}{(a - b)(b - c)(c - a)} = 3$



73. The distance of the point  $(-1, 9, -16)$  from the plane  $2x + 3y - z = 5$  measured parallel to the line

$$\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12} \text{ is}$$

- (1)  $13\sqrt{2}$  (2) 31  
 (3) 26 (4)  $20\sqrt{2}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** Equation of line

$$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$$

G.P on line  $(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$

point of intersection of line & plane

$$6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5$$

$$\lambda = 2$$

Point  $(5, 1, 8)$

$$\text{Distance} = \sqrt{36 + 64 + 576} = 26$$

74. For three positive integers  $p, q, r$ ,  $x^{pq^2} = y^{qr} = z^{p^2r}$  and  $r = pq + 1$  such that  $3, 3 \log_y x, 3 \log_z y, 7 \log_x z$  are in A.P. with common difference  $\frac{1}{2}$ . Then

$r - p - q$  is equal to

- (1) 2 (2) 6  
 (3) 12 (4) -6

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.**  $pq^2 = \log_x \lambda$

$$qr = \log_y \lambda$$

$$p^2 r = \log_z \lambda$$

$$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \dots\dots(1)$$

$$\log_x z = \frac{pq^2}{p^2 r} = \frac{q^2}{pr} \dots\dots(2)$$

$$\log_z y = \frac{p^2 r}{qr} = \frac{p^2}{q} \dots\dots(3)$$

$$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr} \text{ in A.P}$$

$$\frac{3r}{pq} - 3 = \frac{1}{2}$$

$$r = \frac{7}{6} pq \dots\dots(4)$$

$$r = pq + 1$$

$$pq = 6 \dots\dots(5)$$

$$r = 7 \dots\dots(6)$$

$$\frac{3p^2}{q} = 4$$

After solving  $p = 2$  and  $q = 3$

75. Let  $p, q \in \mathbb{R}$  and  $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$ ,

$i = \sqrt{-1}$  Then  $p + q + q^2$  and  $p - q + q^2$  are roots of the equation.

$$(1) x^2 + 4x - 1 = 0 \quad (2) x^2 - 4x + 1 = 0$$

$$(3) x^2 + 4x + 1 = 0 \quad (4) x^2 - 4x - 1 = 0$$

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$

$$2^{200} \left( \cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)^{200} = 2^{199}(p + iq)$$

$$2 \left( -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = p + iq$$

$$p = -1, q = -\sqrt{3}$$

$$\alpha = p + q + q^2 = 2 - \sqrt{3}$$

$$\beta = p - q + q^2 = 2 + \sqrt{3}$$

$$\alpha + \beta = 4$$

$$\alpha \cdot \beta = 1$$

equation  $x^2 - 4x + 1 = 0$

76. The relation  $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$

is: \_\_\_\_

- (1) transitive but not reflexive  
 (2) symmetric but not transitive  
 (3) reflexive but not symmetric  
 (4) neither symmetric nor transitive

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** Reflexive :  $(a, a) \Rightarrow \gcd \text{ of } (a, a) = 1$

Which is not true for every  $a \in \mathbb{Z}$ .

Symmetric:

$$\text{Take } a = 2, b = 1 \Rightarrow \gcd(2, 1) = 1$$

$$\text{Also } 2a = 4 \neq b$$

$$\text{Now when } a = 1, b = 2 \Rightarrow \gcd(1, 2) = 1$$

$$\text{Also now } 2a = 2 = b$$

$$\text{Hence } a = 2b$$



$\Rightarrow R$  is not Symmetric

Transitive:

Let  $a = 14, b = 19, c = 21$

$\gcd(a, b) = 1$

$\gcd(b, c) = 1$

$\gcd(a, c) = 7$

Hence not transitive

$\Rightarrow R$  is neither symmetric nor transitive.

77. The compound statement

$(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$  is equivalent to

(1)  $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

(2)  $(\sim Q) \vee P$

(3)  $((\sim P) \vee Q) \wedge (\sim Q)$

(4)  $(\sim P) \vee Q$

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.** Let  $r = (\sim(P \wedge Q)) \vee ((\sim P) \wedge Q); s = ((\sim P) \wedge (\sim Q))$

P	Q	$\sim(P \wedge Q)$	$(\sim P) \wedge Q$	r	s	$r \rightarrow s$
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

Option (A) :  $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

is equivalent to (not of only P)  $\wedge$  (not of only Q)  
= (Both P, Q) and (neither P nor Q)

78. Let  $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ ; Then at  $x = 0$

- (1) f is continuous but not differentiable
- (2) f is continuous but  $f'$  is not continuous
- (3) f and  $f'$  both are continuous
- (4)  $f'$  is continuous but not differentiable

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** Continuity of  $f(x) : f(0^+) = h^2 \cdot \sin \frac{1}{h} = 0$

$f(0^-) = (-h)^2 \cdot \sin\left(\frac{-1}{h}\right) = 0$

$f(0) = 0$

$f(x)$  is continuous

$f'(0^+) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \frac{h^2 \cdot \sin\left(\frac{1}{h}\right) - 0}{h} = 0$

$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \frac{h^2 \cdot \sin\left(\frac{1}{-h}\right) - 0}{-h} = 0$

$f(x)$  is differentiable.

$f'(x) = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$

$f'(x) = \begin{cases} 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\Rightarrow f'(x)$  is not continuous (as  $\cos\left(\frac{1}{x}\right)$  is highly

oscillating at  $x = 0$ )

79. The equation  $x^2 - 4x + [x] + 3 = x[x]$ , where  $[x]$  denotes the greatest integer function, has:

- (1) exactly two solutions in  $(-\infty, \infty)$
- (2) no solution
- (3) a unique solution in  $(-\infty, 1)$
- (4) a unique solution in  $(-\infty, \infty)$

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.**  $x^2 - 4x + [x] + 3 = x[x]$

$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$

$\Rightarrow (x-1)(x-3) = [x] \cdot (x-1)$

$\Rightarrow x = 1$  or  $x - 3 = [x]$

$\Rightarrow x - [x] = 3$

$\Rightarrow \{x\} = 3$  (Not Possible)

Only one solution  $x = 1$  in  $(-\infty, \infty)$

80. Let  $\Omega$  be the sample space and  $A \subseteq \Omega$  be an event. Given below are two statements :

(S1) : If  $P(A) = 0$ , then  $A = \phi$

(S2) : If  $P(A) = 1$ , then  $A = \Omega$

Then

- (1) only (S1) is true
- (2) only (S2) is true
- (3) both (S1) and (S2) are true
- (4) both (S1) and (S2) are false

**Official Ans. by NTA (4)**

**Ans. (4)**



**Sol.**  $\Omega$  = sample space

A = be an event

$$A = \left\{ \frac{1}{2} \right\}, \Omega = [0, 1]$$

If  $P(A) = 0 \Rightarrow A \neq \phi$

If  $P(\bar{A}) = 1 \Rightarrow \bar{A} \neq \Omega$

Then both statement are false

**SECTION-B**

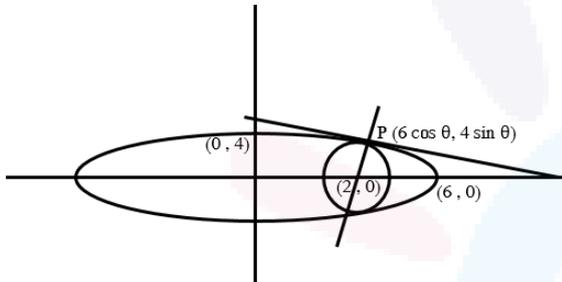
**81.** Let C be the largest circle centred at (2, 0) and inscribed in the ellipse =  $\frac{x^2}{36} + \frac{y^2}{16} = 1$ .

If (1,  $\alpha$ ) lies on C, then  $10\alpha^2$  is equal to \_\_\_\_\_

**Official Ans. by NTA (118)**

**Ans. (118)**

**Sol.**



**Equation of normal of ellipse**  $\frac{x^2}{36} + \frac{y^2}{16} = 1$  at

any point P (6 cos  $\theta$ , 4 sin  $\theta$ ) is

$3 \sec \theta x - 2 \operatorname{cosec} \theta y = 10$  this normal is also the normal of the circle passing through the point (2, 0) So,

$6 \sec \theta = 10$  or  $\sin \theta = 0$  (Not possible)

$\cos \theta = \frac{3}{5}$  and  $\sin \theta = \frac{4}{5}$  so point P =  $\left( \frac{18}{5}, \frac{16}{5} \right)$

So the largest radius of circle

$$r = \frac{\sqrt{320}}{5}$$

So the equation of circle  $(x-2)^2 + y^2 = \frac{64}{5}$

Passing it through (1,  $\alpha$ )

Then  $\alpha^2 = \frac{59}{5}$

$10\alpha^2 = 118$

**82.** Suppose  $\sum_{r=0}^{2023} r^2 {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$ . Then the value of  $\alpha$  is \_\_\_\_\_

**Official Ans. by NTA (1012)**

\_\_\_\_\_ **Ans. (1012)**

**Sol.** using result

$$\sum_{r=0}^n r^2 {}^n C_r = n(n+1) \cdot 2^{n-2}$$

Then  $\sum_{r=0}^{2023} r^2 {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$

=  $2023 \times \alpha \times 2^{2022}$  So,

$\Rightarrow \alpha = 1012$

**83.** The value of  $12 \int_0^3 |x^2 - 3x + 2| dx$  is \_\_\_\_\_

**Official Ans. by NTA (22)**

**Ans. (22)**

**Sol.**  $12 \int_0^3 |x^2 - 3x + 2| dx$

=  $12 \int_0^3 \left( x - \frac{3}{2} \right)^2 - \frac{1}{4} dx$

If  $x - \frac{3}{2} = t$

$dx = dt$

=  $24 \int_0^{3/2} \left( t^2 - \frac{1}{4} \right) dt$

=  $24 \left[ -\int_0^{1/2} \left( t^2 - \frac{1}{4} \right) dt + \int_{1/2}^{3/2} \left( t^2 - \frac{1}{4} \right) dt \right] = 22$

**84.** The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is \_\_\_\_\_

**Official Ans. by NTA (60)**

\_\_\_\_\_ **Ans. (60)**



**Sol. Even digits occupy at even places**

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = \frac{24 \times 120}{4 \times 12} = 60$$

**85.** Let  $\lambda \in \mathbb{R}$  and let the equation E be  $|x|^2 - 2|x| + |\lambda - 3| = 0$ . Then the largest element in the set S =

{  $x + \lambda$  :  $x$  is an integer solution of E } is \_\_\_\_\_

**Official Ans. by NTA (5)**

**Ans. (5)**

**Sol.**  $|x|^2 - 2|x| + |\lambda - 3| = 0$

$$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$$

$$(|x| - 1)^2 + |\lambda - 3| = 1$$

At  $\lambda = 3$ ,  $x = 0$  and  $2$ ,

at  $\lambda = 4$  or  $2$ , then

$x = 1$  or  $-1$

So maximum value of  $x + \lambda = 5$

**86.** A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

**Official Ans. by NTA (546)**

**Ans. (546)**

**Sol.** For at most two language courses

$$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$$

**87.** Let a tangent to the Curve  $9x^2 + 16y^2 = 144$  intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is \_\_\_\_\_

**Official Ans. by NTA (7)**

**Ans. (7)**

**Sol.** Equation of tangent at point P( $4 \cos \theta, 3 \sin \theta$ ) is

$$\frac{x \cos \theta}{4} + \frac{y \sin \theta}{3} = 1$$

So A is  $(4 \sec \theta, 0)$  and point B is  $(0, 3 \operatorname{cosec} \theta)$

$$\text{Length AB} = \sqrt{16 \sec^2 \theta + 9 \operatorname{cosec}^2 \theta}$$

$$= \sqrt{25 + 16 \tan^2 \theta + 9 \cot^2 \theta} \geq 7$$

**88.** The value of  $\frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$  is \_\_\_\_\_.

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.**  $I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx \dots\dots\dots(1)$

Using  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx \dots\dots\dots(2)$$

Adding (1) & (2)

$$2I = \frac{8}{\pi} \int_0^{\frac{\pi}{2}} 1 dx$$

$$I = 2$$

**89.** The shortest distance between the lines  $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$  and  $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$  is equal to \_\_\_\_\_

**Official Ans. by NTA (14)**

**Ans. (14)**

**Sol. Shortest distance between the lines**

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}} = \frac{16 + 12 + 168}{|-4\hat{i} + 6\hat{j} - 12\hat{k}|} = \frac{196}{14} = 14$$



90. The 4<sup>th</sup> term of GP is 500 and its common ratio is  $\frac{1}{m}$ ,  $m \in \mathbb{N}$ . Let  $S_n$  denote the sum of the first  $n$  terms of this GP. If  $S_6 > S_5 + 1$  and  $S_7 < S_6 + \frac{1}{2}$ , then the number of possible values of  $m$  is \_\_\_\_\_

**Official Ans. by NTA (12)**

**Ans. (12)**

**Sol.**  $T_4 = 500$  where  $a =$  first term,

$$r = \text{common ratio} = \frac{1}{m}, m \in \mathbb{N}$$

$$ar^3 = 500$$

$$\frac{a}{m^3} = 500$$

$$S_n - S_{n-1} = ar^{n-1}$$

$$S_6 > S_5 + 1 \quad \text{and} \quad S_7 - S_6 < \frac{1}{2}$$

$$S_6 - S_5 > 1 \quad \frac{a}{m^6} < \frac{1}{2}$$

$$ar^5 > 1 \quad m^3 > 10^3$$

$$\frac{500}{m^2} > 1 \quad m > 10 \dots\dots(2)$$

$$m^2 < 500 \dots\dots(1)$$

From (1) and (2)

$$m = 11, 12, 13, \dots\dots, 22$$

So number of possible values of  $m$  is 12