# FINAL JEE–MAIN EXAMINATION – JANUARY, 2023
Held On Tuesday 24th January, 2023
TIME : 03:00 PM to 06:00 PM

## SECTION-A

**61.** Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is $\sigma^2$, then $8\sigma^2$ is equal to

(1) 220
(2) 210
(3) 200
(4) 105

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$\mathbf{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57}$

$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$

$\Rightarrow a_1 + a_6 = 19$

$\Rightarrow 2a_1 + 5d = 19$ and $a_1 + d = 5$

$\Rightarrow a_1 = 2, d = 3$

Numbers : 2, 5, 8, 11, 14, 17

Variance = $\sigma^2$ = mean of squares – square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$8\sigma^2 = 210$

**62.** Let f(x) be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 6
(2) 8
(3) 7
(4) 9

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** $f(x + y) = f(x) \cdot f(y) \ \forall x, y \in N$, $f(1) = 3$

$f(2) = f^2(1) = 3^2$

$f(3) = f(1)\, f(2) = 3^3$

$f(4) = 3^4$

$f(k) = 3^k$

$$\sum_{k=1}^{n} f(k) = 3279$$

$f(1) + f(2) + f(3) + \ldots\ldots\ldots + f(k) = 3279$

$3 + 3^2 + 3^3 + \ldots\ldots\ldots 3^k = 3279$

$$\frac{3(3^k - 1)}{3 - 1} = 3279$$

$$\frac{3^k - 1}{2} = 1093$$

$3^k - 1 = 2186$

$3^k = 2187$

$\boxed{k = 7}$

**63.** The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 4 (2) 0
(3) 3 (4) 2

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

Let $x + \frac{1}{x} = t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$3t(t - 1) + 1(t - 1) = 0$

$(t - 1)(3t + 1) = 0$

$t = 1, -\frac{1}{3}$

$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow$ No solution.

**64.** If $f(x)=\frac{2^{2x}}{2^{2x}+2}$, $x \in R$,

then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+.......+f\left(\frac{2022}{2023}\right)$ is equal to

(1) 2011 (2) 1010
(3) 2010 (4) 1011

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** $f(x)=\frac{4^x}{4^x+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2(4^x)}$

$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}$

$= 1$

$\Rightarrow f(x) + f(1 – x) = 1$

Now $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+......+$

$..........+f\left(1-\frac{3}{2023}\right)+f\left(1-\frac{2}{2023}\right)+f\left(1-\frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum = 1 + 1 + 1 + ……….+ 1 (1011 times)

= 1011

**65.** If $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3)$, $x \in R$, then

(1) 3f(1) + f(2) = f(3)
(2) f(3) – f(2) = f(1)
(3) 2f(0) – f(1) + f(3) = f(2)
(4) f(1) + f(2) + f(3) = f(0)

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3),\ x\in R$

**Let** $f'(1)=a,\ f''(2)=b,\ f'''(3)=c$

$\mathbf{f(x) = x^3 – ax^2 + bx – c}$

$f'(x)=3x^2-2ax+b$

$f''(x)=6x-2a$

$f'''(x)=6$

c = 6, a = 3, b = 6

$f(x) = x^3 – 3x^2 + 6x – 6$

f(1) = –2, f(2) = 2, f(3) = 12, f(0) = –6

2f(0) – f(1) + f(3) = 2 = f(2)

**66.** The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(1) 120
(2) 168
(3) 220
(4) 48

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** Four digit numbers greater than 7000

= 2 × 4 × 3 × 2 = 48

Five digit number = 5! = 120

Total number greater than 7000

= 120 + 48 = 168

**67.** If the system of equations

x + 2y + 3z = 3

4x + 3y – 4z = 4

8x + 4y – λz = 9 + μ

has infinitely many solutions, then the ordered pair (λ, μ) is equal to

(1) $\left(\frac{72}{5},\frac{21}{5}\right)$ (2) $\left(\frac{-72}{5},\frac{-21}{5}\right)$

(3) $\left(\frac{72}{5},\frac{-21}{5}\right)$ (4) $\left(\frac{-72}{5},\frac{21}{5}\right)$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** x + 2y + 3z = 3 …….(i)

4x + 3y – 4z = 4 …….(ii)

8x + 4y – λz = 9 + μ …….(iii)

(i) × 4 – (ii) ⇒ 5y + 16z = 8 ……. (iv)

(ii) × 2 – (iii) ⇒ 2y + (λ–8)z = –1–μ ……(v)

(iv) × 2 – (iii) × 5 ⇒ (32–5(λ–8))z = 16–5(–1–μ)

For infinite solutions ⇒ 72 – 5λ = 0 ⇒ $\lambda=\frac{72}{5}$

21 + 5μ = 0 ⇒ $\mu=\frac{-21}{5}$

$\Rightarrow(\lambda,\mu)\equiv\left(\frac{72}{5},\frac{-21}{5}\right)$

**68.** The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $\frac{-1}{2}(1-i\sqrt{3})$

(2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{-1}{2}(\sqrt{3}-i)$

(4) $\frac{1}{2}(\sqrt{3}+i)$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3=z^3$$

$$\Rightarrow\left(i\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)\right)^3$$

$$=-i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow\frac{-1}{2}(\sqrt{3}-i).$$

**69.** The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is

(1) $\sqrt{6}$

(2) $2\sqrt{2}$

(3) 2

(4) 4

**Official Ans. by NTA (2)**

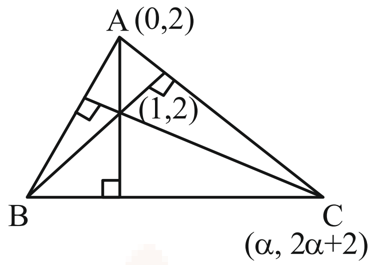
**Ans. (2)**

**Sol.** $AB:(\lambda+1)x+\lambda y=4$

$AC:\lambda x+(1-\lambda)y+\lambda=0$

Vertex A is on y-axis

$\Rightarrow x=0$




So $y=\frac{4}{\lambda}, y=\frac{\lambda}{\lambda-1}$

$\Rightarrow\frac{4}{\lambda}=\frac{\lambda}{\lambda-1}$

$\Rightarrow\lambda=2$

AB : $3x+2y=4$

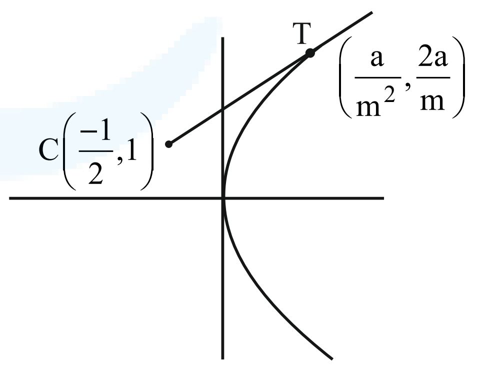
AC : $2x-y+2=0$

$\Rightarrow A(0,2)$ Let C $(\alpha, 2\alpha+2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right)=-1$

$\left(\frac{2\alpha}{\alpha-1}\right)\left(-\frac{3}{2}\right)=-1\ \Rightarrow\alpha=-\frac{1}{2}$

So $C\left(-\frac{1}{2},1\right)$




Let Equation of tangent be $y=mx+\frac{3}{2m}$

$m^2+2m-3=0$

$\Rightarrow m=1,-3$

So tangent which touches in first quadrant at T is

$T\equiv\left(\frac{a}{m^2},\frac{2a}{m}\right)$

$\equiv\left(\frac{3}{2},3\right)$

$\Rightarrow CT=\sqrt{4+4}=2\sqrt{2}$

**70.** The set of all values of a for which $\lim\limits_{x\to a}([x-5]-[2x+2]) = 0$, where $[\propto]$ denotes the greater integer less than or equal to $\propto$ is equal to

(1) (– 7.5, –6.5) (2) (– 7.5, –6.5]

(3) [– 7.5, –6.5] (4) [– 7.5, –6.5)

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol.** $\lim\limits_{x\to a}([x-5]-[2x+2])=0$

$\lim\limits_{x\to a}([x]-5-[2x]-2)=0$

$\lim\limits_{x\to a}([x]-[2x])=7$

$[a]-[2a]=7$

$a \in I,\quad a=-7$

$a \notin I,\quad a=I+f$

Now, $[a]-[2a]=7$

$-I - [2f] = 7$

Case-I: $f \in \left(0, \frac{1}{2}\right)$

$2f \in (0, 1)$

$-I = 7$

$I = -7 \Rightarrow a \in (-7, -6.5)$

Case-II: $f \in \left(\frac{1}{2}, 1\right)$

$2f \in (1, 2)$

$-I - 1 = 7$

$I = -8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

**71.** If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then $\alpha$ is equal to

(1) 30 (2) 60

(3) 15 (4) 10

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** $S=0.({}^{30}C_0)^2+1.({}^{30}C_1)^2+2.({}^{30}C_2)^2 + \ldots\ldots + 30.({}^{30}C_{30})^2$

$S=30.({}^{30}C_0)^2+29.({}^{30}C_1)^2+28.({}^{30}C_2)^2 + \ldots\ldots + 0.({}^{30}C_0)^2$

$2S = 30.({}^{30}C_0{}^2+{}^{30}C_1{}^2+ \ldots\ldots + {}^{30}C_{30}{}^2)$

$S = 15\cdot{}^{60}C_{30} = 15\cdot\frac{60!}{(30!)^2}$

$\frac{15\cdot 10!}{(30!)^2} = \frac{\alpha\cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

**72.** Let the plane containing the line of intersection of the planes

P1: $x + (\lambda + 4)y + z = 1$ and

P2 : $2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is

(1) $5\sqrt{6}$

(2) $4\sqrt{6}$

(3) $2\sqrt{6}$

(4) $3\sqrt{6}$

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** Equation of plane passing through point of intersection of P1 and P2

P = P1 + kP2

$(x+(\lambda+4)y+z-1)+k(2x+y+z-2)=0$

Passing through (0, 1, 0) and (1, 0, 1)

$(\lambda + 4 - 1) + k(1 - 2) = 0$

$(\lambda + 3) - k = 0$ ….(1)

Also passing (1, 0, 1)

$(1 + 1 - 1) + k(2 + 1 - 2) = 0$

$1 + k = 0$

$k = -1$

put in (1)

$\lambda + 3 + 1 = 0$

$\lambda = -4$

Then point $(2\lambda, \lambda, -\lambda)$

$(-8, -4, 4)$

$d = \left|\frac{-16-4+4-2}{\sqrt{6}}\right|$

$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$

**73.** Let $\vec{\alpha} = 4\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{\beta} = \hat{i}+2\hat{j}-4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2\cdot(\hat{i}+\hat{j}+\hat{k})$ is

(1) 6

(2) 11

(3) 7

(4) 9

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** Let $\vec{\beta}_1 = \lambda\vec{\alpha}$

Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$

$= (1-4\lambda)\hat{i} + (2-3\lambda)\hat{j} - (5\lambda+4)\hat{k}$

$\vec{\beta}_2.\vec{\alpha} = 0$

$\Rightarrow 4(1-4\lambda) + 3(2-3\lambda) - 5(5\lambda+4) = 0$

$\Rightarrow 4 - 16\alpha + 6 - 9\lambda - 25\lambda - 20 = 0$

$\Rightarrow 50\lambda = -10$

$\Rightarrow \boxed{\lambda = \frac{-1}{5}}$

$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1+4)\hat{k}$

$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$

$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

$5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

**74.** The locus of the mid points of the chords of the circle $C_1$: $(x-4)^2 + (y-5)^2 = 4$ which subtend an angle $\theta_i$ at the centre of the circle $C_1$, is a circle of radius $r_i$. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then $\theta_2$ is equal to

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

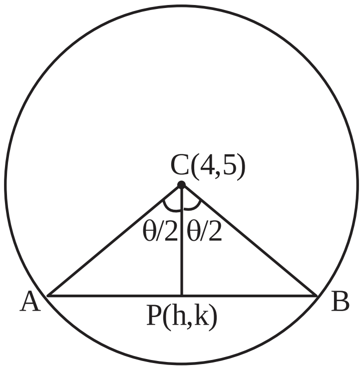
(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** **In ΔCPB**




$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h-4)^2 + (k-5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x-4)^2 + (y-5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2}}$

**75.** If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes x + 2y + z = 0 and 3y – z = 3 is (α, β, γ), then α + β + γ is equal to

(1) –1

(2) 3

(3) 1

(4) 5

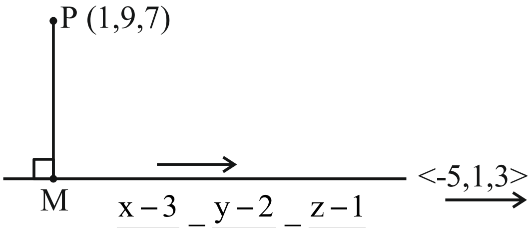
**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$




$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3}$

$M(-5\lambda+3, \lambda+2, 3\lambda+1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda+2) + (\lambda-7) + 3(3\lambda-6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

**76.** Let y = y(x) be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy\, dy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $3e^2$

(2) $e^2$

(3) $2e^2$

(4) $\frac{3e^2}{2}$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** $(x^2 - 3y^2)\, dx + 3xy\, dy = 0$

$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y}$ (1)

Put y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$

$\Rightarrow vdv = \frac{-1}{3x}$

Integrating both side

$\frac{v^2}{2} = \frac{-1}{3} \ln x + c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + c$

y (1) = 1

$\Rightarrow \boxed{\frac{1}{2} = c}$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + \frac{1}{2}$

$\Rightarrow y^2 = -\frac{2}{3} x^2 \ln x + x^2$

$y^2(e) = -\frac{2}{3} e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow \boxed{6y^2(e) = 2e^2}$

**77.** Let p and q be two statements.

Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge (\sim q))$

(2) $p \vee ((\sim p) \wedge q)$

(3) $(\sim p) \vee q$

(4) $p \vee (p \wedge q)$

**Official Ans. by NTA (3)**

**Ans. (3)**

**Sol.** $\sim (p \wedge (p \rightarrow \sim q))$

$\equiv \sim p \vee \sim(\sim p \vee \sim q)$

$\equiv \sim p \vee (p \wedge q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$

$\equiv t \wedge (\sim p \vee q)$

$\equiv \sim p \vee q$

**78.** The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

(3) 150

(4) 125

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** $\begin{bmatrix} - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - & - & - & - & - \end{bmatrix}$

In each row and each column exactly one is to be placed –

$\therefore$ No. of such matrices = 5 × 4 × 3 × 2 × 1 = 120

**Alternate :**

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to 5 \text{ ways} \\ \to 4 \text{ ways} \\ \to 3 \text{ ways} \\ \to 2 \text{ ways} \\ \to 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways = 5 × 4 × 3 × 2 × 1 = 120

**79.** $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$ is equal to

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$

(4) $2\pi$

**Official Ans. by NTA (4)**

**Ans. (4)**

**Sol.** $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$

**We have** $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

**Hence** $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx = \frac{48}{2} \times \left[\sin^{-1}\frac{2x}{3}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$= 24 \times \left[\sin^{-1}\left(\frac{2}{3} \times \frac{3\sqrt{3}}{4}\right) - \sin^{-1}\left(\frac{2}{3} \times \frac{3\sqrt{2}}{4}\right)\right]$

$= 24 \times \left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]$

$= 24 \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= 24 \times \frac{\pi}{12} = 2\pi$

**80.** Let A be a 3 × 3 matrix such that $\left|\text{adj}\left(\text{adj}(\text{adj}A)\right)\right| = 12^4$. Then $|A^{-1}\text{adj } A|$ is equal to

(1) $2\sqrt{3}$

(2) $\sqrt{6}$

(3) 12

(4) 1

**Official Ans. by NTA (1)**

**Ans. (1)**

**Sol. Given |adj(adj(adj.A))| = $12^4$**

$\Rightarrow |A|^{(n-1)^3} = 12^4$

Given n = 3

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A|^2 = 12$

$|A| = 2\sqrt{3}$

We are asked

$|A^{-1}.\text{adj} A|$

$= |A^{-1}|.|\text{adj } A|$

$= \frac{1}{|A|}.|A|^{3-1}$

$= |A| = 2\sqrt{3}$

**81.** The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is

**Official Ans. by NTA (432)**

**Ans. (432)**

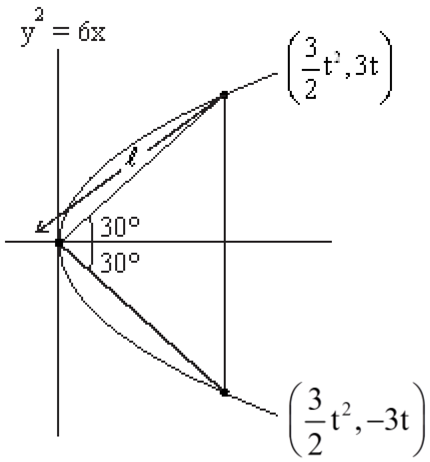
**Sol.**

| Urn A | | Urn B | | Urn C | |
|---|---|---|---|---|---|
| Red | Black | Red | Black | Red | Black |
| 4 | 6 | 5 | 5 | λ | 4 |

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right) + P(B)P\left(\frac{R}{B}\right) + P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$$\Rightarrow \lambda = 6$$




$$\tan 30° = 3t = \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$$

$$\ell^2 = 18^2 + (6\sqrt{3})^2$$

$= 324 + 108$

$= 432$

**82.** If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8A is equal to

**Official Ans. by NTA (36)**

**Ans. (36)**

**Sol.** $y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

$(y-1)^2 = -(x-1)$

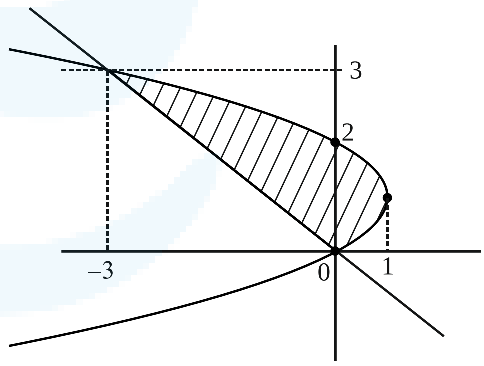
$y = -x$

Points of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$

$x = 0, -3$




$$A = \int_0^3 (-y^2 + 2y + y)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$$

$8A = 36$

**83.** If $\frac{1^3 + 2^3 + 3^3 + \ldots\ldots \text{upto n terms}}{1\cdot 3 + 2\cdot 5 + 3\cdot 7 + \ldots\ldots \text{upto n terms}} = \frac{9}{5}$, then the value of n is

**Official Ans. by NTA (5)**

**Ans. (5)**

**Sol.** $1^3+2^3+3^3.....+n^3=\left(\frac{n(n+1)}{2}\right)^2$

$1\cdot 3+2\cdot 5+3\cdot 7+.......+n$ terms $=$

$\sum_{r=1}^{n} r(2r+1)=\sum_{r=1}^{n}\left(2r^2+r\right)$

$=\frac{2\cdot n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$

$=\frac{n(n+1)}{6}\left(2(2n+1)+3\right)$

$=\frac{n(n+1)}{2}\times\frac{(4n+5)}{3}$

$=\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}\times\frac{(4n+5)}{3}}=\frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2}=\frac{9(4n+5)}{3}$

$\Rightarrow 15n(n+1)=18(4n+5)$

$\Rightarrow 15n^2+15n=72n+90$

$\Rightarrow 15n^2-57n-90=0\Rightarrow 5n^2-19n-30=0$

$\Rightarrow (n-5)(5n+6)=0$

$\Rightarrow n=\frac{-6}{5}$ or 5

$\Rightarrow n=5.$

**84.** Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that $f(x)>0$ and

$f(x)+\int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\,dt=e,\ \forall x\in\left[0,\frac{\pi}{2}\right].$

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _______.

**Official Ans. by NTA (27)**

**Ans. (27)**

**Sol.** $f(x)+\int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\,dt=e$

$\Rightarrow f(0)=e$

$f'(x)+f(x)\sqrt{1-\left(\ln f(x)\right)^2}=0$

$f(x)=y$

$\frac{dy}{dx}=-y\sqrt{1-(\ln y)^2}$

$\int\frac{dy}{y\sqrt{1-(\ln y)^2}}=-\int dx$

Put $\ln y=t$

$\int\frac{dt}{\sqrt{1-t^2}}=-x+C$

$\sin^{-1}t=-x+C\Rightarrow\sin^{-1}(\ln y)=-x+C$

$\sin^{-1}\left(\ln f(x)\right)=-x+C$

$f(0)=e$

$\Rightarrow\frac{\pi}{2}=C$

$\Rightarrow\sin^{-1}\left(\ln f(x)\right)=-x+\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{-\pi}{6}+\frac{\pi}{2}$

$\Rightarrow\sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right)=\frac{\pi}{3}$

$\Rightarrow\ln f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$, we need $\left(6\times\frac{\sqrt{3}}{2}\right)^2=27.$

**85.** The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is __________.

**Official Ans. by NTA (13)**

**Ans. (13)**

**Sol.** Given R = {(a, b), (b, c), (b, d)}

In order to make it equivalence relation as per given set, R must be

{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}

There already given so 13 more to be added.

**86.** Let $a = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $b = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a}\cdot\vec{c} = 7$, $2\vec{b}\cdot\vec{c} + 43 = 0$, $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

**Official Ans. by NTA (8)**

**Ans. (8)**

**Sol.** $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k},\ \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k},\ \vec{a}\cdot\vec{c} = 7$

$\vec{a}\times\vec{c} - \vec{b}\times\vec{c} = \vec{0}$,

$(\vec{a}-\vec{b})\times\vec{c} = 0 \Rightarrow (\vec{a}-\vec{b})$ is paralleled to $\vec{c}$

$\vec{a}-\vec{b} = \mu\vec{c}$, where $\mu$ is a scalar

$-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu\cdot\vec{c}$

Now $\vec{a}\cdot\vec{c} = 7$ gives $2\lambda^2 + 12 = 7\mu$

And $\vec{b}\cdot\vec{c} = -\dfrac{43}{2}$ gives $4\lambda^2 + 82 = 43\mu$

$\mu = 2$ and $\lambda^2 = 1$

$|\vec{a}\cdot\vec{b}| = 8$

**87.** Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \dfrac{3}{x^2}\right)^n$, $x \neq 0$, $n \in N$, be 376. Then the coefficient of $x^4$ is ________.

**Official Ans. by NTA (405)**

**Ans. (405)**

**Sol.** Given Binomial $\left(x - \dfrac{3}{x^2}\right)^n, x \neq 0, n \in N$,

Sum of coefficients of first three terms

${}^nC_0 - {}^nC_1\cdot 3 + {}^nC_2 3^2 = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n-10)(3n+25) = 0$

$\Rightarrow n = 10$

Now general term ${}^{10}C_r x^{10-r}\left(\dfrac{-3}{x^2}\right)^r$

$= {}^{10}C_r x^{10-r}(-3)^r\cdot x^{-2r}$

$= {}^{10}C_r(-3)^r\cdot x^{10-3r}$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$\Rightarrow r = 2$

${}^{10}C_2(-3)^2 = 405$

**88.** If the shortest between the lines

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4} \text{ and}$$

$$\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$$ is 6, then the square of sum of all possible values of $\lambda$ is

**Official Ans. by NTA (384)**

**Ans. (384)**

**Sol.** Shortest distance between the lines

$$\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$$

$$\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{2+2\sqrt{6}}{5}$$ is 6

Vector along line of shortest distance

$$= \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}, \Rightarrow -\hat{i} + 2\hat{j} - k \text{ (its magnitude is } \sqrt{6}\text{)}$$

$$\text{Now } \frac{1}{\sqrt{6}}\begin{vmatrix} \sqrt{6}+\lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \pm 6$$

$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$

So, square of sum of these values is 384.

**89.** Let S = $\{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta\in S} \sin^2\left(\theta + \dfrac{\pi}{4}\right)$ is equal to

**Official Ans. by NTA (2)**

**Ans. (2)**

**Sol.** $\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta$

$\sin\theta + \cos\theta = n$ where $n \in I$

possible values are n = 0, 1 and –1 because

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$

Now it gives $\theta \in \left\{0, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, \dfrac{3\pi}{2}, \pi\right\}$

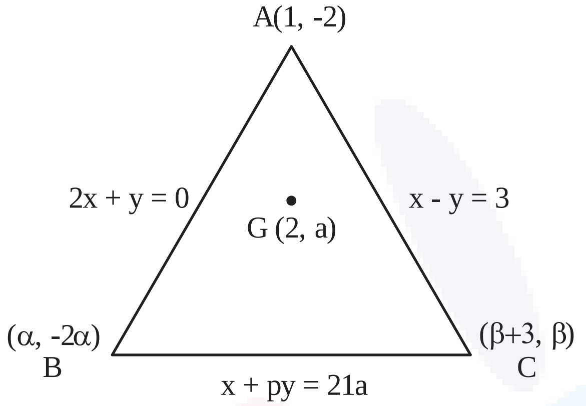
So $\sum_{\theta\in S}\sin^2\left(\theta + \dfrac{\pi}{4}\right) = 2(0) + 4\left(\dfrac{1}{2}\right) = 2$

**90.** The equations of the sides AB, BC and CA of a triangle ABC are: $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P (2, a) be the centroid of $\Delta ABC$. Then $(BC)^2$ is equal to

**Official Ans. by NTA (122)**

**Ans. (122)**

**Sol.**




Assume B(α, -2α) and C(β + 3, β)

$\frac{\alpha+\beta+3+1}{3} = 2$ also $\frac{-2\alpha-2+\beta}{3} = a$

$\Rightarrow \alpha + \beta = 2$ $\quad -2\alpha - 2 + \beta = 3a$

$\Rightarrow \beta = 2 - \alpha$ $\quad -2\alpha - \cancel{2} + \cancel{2} - \alpha = 3a \Rightarrow \alpha = -a$

Now both B and C lies as given line

$\alpha - p \cdot 2\alpha = 21a$

$\alpha(1-2p) = 21\,a$ ….. (1)

$-\alpha(1-2p) = 21\,a \Rightarrow p = 11$

$\beta + 3 + p\beta = 21\,a$

$\beta + 3 + 11\beta = 21\,a$

$21\alpha + 12\beta + 3 = 0$

Also $\beta = 2 - \alpha$

$21\alpha + 12(2-\alpha) + 3 = 0$

$21\alpha + 24 - 12\alpha + 3 = 0$

$9\alpha + 27 = 0$

$\alpha = -3,\ \beta = 5$

So $BC = \sqrt{122}$ and $(BC)^2 = 122$