NCERT Solutions for Class 12

Physics

Chapter 7 – Alternating Current

1. A 100Ω resistor is connected to a $220V$, $50Hz$ ac supply.

a) What is the rms value of current in the circuit?

Ans: It is given that,

Resistance, $R = 100\Omega$

Voltage, $V = 220V$

Frequency, $f = 50Hz$

It is known that,

$$I_{rms} = \frac{V_{rms}}{R}$$

$$\Rightarrow I_{rms} = \frac{220}{100} = 2.2A$$

Therefore, the rms value of current in the circuit is $I_{rms} = 2.2A$.

b) What is the net power consumed over a full cycle?

Ans: It is known that,

$Power = VI$

$\therefore Power = 220 \times 2.2$

$\therefore Power = 484W$

Therefore, the net power consumed over a full cycle is $484W$.

2.

a) The peak voltage of an ac supply is $300V$. What is the rms voltage?

Ans: It is given that,

Peak voltage of the ac supply, $V_0 = 300V$ It is known that,

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = \frac{300}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = 212.1\text{V}$$

Therefore, the rms voltage is 212.1V.

b) The rms value of current in an ac circuit is 10A. What is the peak current?

Ans: It is given that,

Rms value of current in an ac circuit, $I_{rms} = 10\text{A}$ It is known that,

$$I_0 = \sqrt{2}\, I_{rms}$$

$$\Rightarrow I_0 = 1.414 \times 10$$

$$\Rightarrow I_0 = 14.14\text{A}$$

Therefore, the peak current is 14.14A.

3. A 44mH inductor is connected to 220V ,50Hz ac supply. Determine the rms value of the current in the circuit.

Ans: It is known that,

Inductance, $L = 44\text{mH} = 44 \times 10^{-3}\text{H}$

Voltage, $V = 220\text{V}$

Frequency, $f = 50\text{Hz}$

Angular frequency, $\omega = 2\pi f$

It is known that,

Inductive reactance, $X_L = \omega L = 2\pi f L$

$$\Rightarrow X_L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3}\,\Omega$$

$$\Rightarrow X_L = 13.8\,\Omega$$

$$I_{rms} = \frac{V}{\ldots}$$

X_L

$$\therefore I_{rms} = \frac{220}{13.82}$$

$$\therefore I_{rms} = 15.92A$$

Therefore, the rms value of the current in the circuit is 15.92A.

4. A 60μF capacitor is connected to a 110V,60Hz ac supply. Determine the rms value of the current in the circuit.

Ans: It is given that,

Capacitance, $C = 60\mu F = 60\times 10^{-6}F$

Voltage, $V = 110V$

Frequency, $f_C = 60Hz$

It is known that,

$$I_{rms} = \frac{V}{X_C}$$

$$X_C = \frac{1}{\omega_C C} = \frac{1}{2\pi f_C C}$$

$$\therefore X_C = \frac{1}{2\times 3.14\times 60\times 60\times 10^{-6}}$$

$$\therefore X_C = 44.248\Omega$$

$$\therefore I_{rms} = \frac{110}{44.28}$$

$$\therefore I_{rms} = 2.488A$$

Therefore, the rms value of the current in the circuit is 2.488A.

5. In exercises 4 and 5 What is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Ans: From the inductive circuit,

Rms value of current, $I_{rms} = 15.92A$

Rms value of voltage, $V_{rms} = 220V$

It is known that,

Net power absorbed, $P = V_{rms} I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure inductive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$\therefore P = 220 \times 15.92 \times \cos 90^0 = 0$

Therefore, net power absorbed is zero in a pure inductive circuit. In a capacitive circuit,

Rms value of current, $I_{rms} = 2.49A$

Rms value of voltage, $V_{rms} = 110V$

It is known that,

Net power absorbed, $P = V_{rms} I_{rms} \cos\phi$

Where,

ϕ is the phase difference between voltage and current

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90^0 i.e., $\phi = 90^0$

$\therefore P = 110 \times 2.49 \cos 90^0 = 0$

Therefore, net power absorbed is zero in a pure capacitive circuit.

6. Obtain the resonant frequency ω_r of a series LCR circuit with $L = 2.0H$, $C = 32\mu F$ and $R = 10\Omega$. What is the Q-value of this current?

Ans: It is given that, Inductance,

$L = 2H$

Capacitance, $C = 32\mu F = 32 \times 10^{-6} F$

$R = 10\Omega$

It is known that,

Resonant frequency, $\omega_r = \frac{1}{\sqrt{\ }}$

LC

$\Rightarrow \omega_r = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}}$

$\Rightarrow \omega_r = \frac{1}{8\times10^{-3}}$

$\Rightarrow \omega_r = 125 \text{ rad/s}$

Q-value $= \frac{\omega_r L}{R}$

$\Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\Rightarrow Q = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}}$

$\Rightarrow Q = \frac{1}{10\times4\times10^{-3}}$

$\Rightarrow Q = 25$

Therefore, the resonant frequency is 125 rad / s and Q-value is 25.

7. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit? Ans: It is given that,

Capacitance, $C = 30\ \mu F = 30 \times 10^{-6} F$

Inductance, $L = 27 mH = 27 \times 10^{-3} H$

It is known that,

Angular frequency of free oscillations, $\omega_r = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega_r = \frac{1}{\sqrt{27\times10^{-3} \times 30\times10^{-6}}}$

$\Rightarrow \omega_r = \frac{1}{9\times10^{-4}}$

$\Rightarrow \omega_r = 1.11 \times 10^{3} \text{ rad / s}$

Therefore, the angular frequency of free oscillations of the circuit is

1.11×10^{3} rad / s.

8. Suppose the initial charge on the capacitor in exercise 7 is 6mC . What is the total energy stored in the circuit initially? What is the total energy at a later time?

Ans: It is known that,

Capacitance of the capacitor, $C = 30\,\mu F = 30 \times 10^{-6} F$

Inductance of the capacitor, $L = 27 mH = 27 \times 10^{-3} H$

Charge on the capacitor, $Q = 6 mC = 6 \times 10^{-3} C$

It is known that,

$$\text{Energy}, E = \frac{1}{2}\frac{Q^2}{C}$$

$$\Rightarrow E = \frac{1}{2}\frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}}$$

$$\Rightarrow E = \frac{6}{10} = 0.6 J$$

Therefore, the energy stored in the circuit initially is $E = 0.6J$.

Total energy at later time will remain same as the initially stored i.e., 0.6J because energy is shared between the capacitor and the inductor.

9. A series LCR circuit with $R = 20\Omega$, $L = 1.5H$ and $C = 35\,\mu F$ is connected to a variable frequency 200V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Ans: It is known that,

Resistance, $R = 20\Omega$

Inductance, $L = 1.5H$

Capacitance, $C = 35\,\mu F = 35 \times 10^{-6} F$

Voltage, $V = 200V$

It is known that,

$$\text{Impedance}, Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C$

$\therefore Z = R = 20\Omega$

$I = \frac{V}{Z} = \frac{200}{20}$

$\therefore I = 10A$

Average power, $P = I^2 R$

$\therefore P = 10^2 \times 20$

$\therefore P = 2000W$

Therefore, the average power transferred is 2000W.

10. A radio can tune over the frequency range of a portion of MW broadcast band: (800kHz to 1200kHz). If its LC circuit has an effective inductance of 200μH, what must be the range of its variable capacitor?
[Hint: For tuning, the natural frequency i.e., the frequency of free oscillations of the LC circuit should be equal to the frequency of the radio wave.]

Ans: It is given that,
The range of frequency(f) of a radio is 800kHz to 1200kHz.

Effective inductance of the circuit, $L = 200\mu H = 200 \times 10^{-6} H$ It is known that,

Capacitance of variable capacitor for f_1 is $C_1 = \frac{1}{\omega_1^2 L}$

Where,

ω_1 is the angular frequency for capacitor for $f_1 = 2\pi f_1$

$\therefore \omega_1 = 2 \times 3.14 \times 800 \times 10^3 \text{ rad / s}$

$\therefore C_1 = \frac{1}{(2 \times 3.14 \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$

$\therefore C_1 = 1.9809 \times 10^{-10} F$

□ □C_1 198.1pF 1

C_2 □ ____

□$_{2}$$_{2}$L

1

□ □C^2 ____________________________(2□$^{3\,2}$ 200□$10^{□6}$

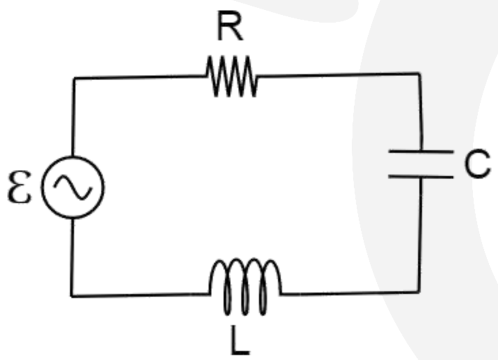
3.14□1200□10) □

□C_2□ 0.8804 10□ $^{□10}$F

□ □C_2 88.04pF

Therefore, the range of the variable capacitor is from 88.04pF to 198.1pF.

11. Figure shows a series LCR circuit connected to a variable frequency 230V source. L □ 5.0H, C□ □80 F, R □ 40□ .



a) Determine the source frequency which drives the circuit in resonance.

Ans: It is given that,

Voltage, V □ 230V

Inductance, L □5.0H

Capacitance, C□ □ □ □80 F80 $10^{□6}$F

Resistance, R □ □40

It is known that,

Source frequency at resonance□ $\dfrac{1}{\sqrt{LC}}$

$$\omega = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\ rad / s$$

Therefore, the source frequency of the circuit in resonance is 50rad / s.

b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

Ans: It is known that,

At resonance, Impedance, $Z =$ Resistance, R

$$\therefore Z = R = 40\Omega$$

$$I = \frac{V}{Z}$$

$$\therefore I = \frac{230}{40} = 5.75A$$

Amplitude, $I_0 = 1.414 \times I$

$$\therefore I_0 = 1.414 \times 5.75$$

$$\therefore I_0 = 8.13A$$

Therefore, the impedance of the circuit is 40Ω and the amplitude of current at resonating frequency is 8.13A.

c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Ans: It is known that,

Potential drop, $V = IR$

Across resistor, $V_R = IR$

$$\therefore V_R = 5.75 \times 40 = 230V$$

Across capacitor, $V_C = IX_C = \frac{I}{\omega C}$

$$\therefore V_C = 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}}$$

$\therefore V_C = 1437.5V$

Across Inductor, $V_L = IX_L = \omega I L$

$\therefore V_L = 5.75 \times 50 \times 5$

$\therefore V_L = 1437.5V$

Across LC combination, $V_{LC} = I(X_L - X_C)$

At resonance, $X_L = X_C$

$\therefore V_{LC} = 0$

Therefore, the rms potential drop across Resistor is 230V, Capacitor is 1437.5V , Inductor is 1437.5V and the potential drop across LC combination is zero at resonating frequency.

12. An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.

a) What is the total energy stored initially? Is it conserved during LC oscillations?

Ans: It is given that,

Inductance of the inductor, $L = 20mH = 20 \times 10^{-3} H$

Capacitance of the capacitor, $C = 50\mu F = 50 \times 10^{-6} F$

Initial charge on the capacitor, $Q = 10mC = 10 \times 10^{-3} C$

It is known that,

Total energy stored initially in the circuit, $E = \frac{1}{2}\frac{Q^2}{C}$

$$\therefore E = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1J$$

Therefore, the total energy stored in the LC circuit will be conserved because there is no resistor (R = 0) connected in the circuit.

b) What is the natural frequency of the circuit?

Ans: It is known that,

Natural frequency of the circuit, $\nu_r = \frac{1}{2\pi\sqrt{LC}}$

$\therefore \nu_r = \frac{1}{2\pi\sqrt{20\times10^{-3}\times50\times10^{-6}}}$

$\therefore \nu_r = \frac{10^3}{2\pi} = 159.24\text{Hz}$

Natural angular frequency, $\omega_r = \frac{1}{\sqrt{LC}}$

$\therefore \omega_r = \frac{1}{\sqrt{20\times10^{-3}\times50\times10^{-6}}}$

$\therefore \omega_r = \frac{1}{\sqrt{10^{-6}}} = 10^3\text{rad/s}$

Therefore, the natural frequency is 159.24Hz and the natural angular frequency is 10^3rad / s.

c) At what time is the energy stored (i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)? Ans:

(i) Completely electrical

It is known that,

Time period for LC oscillations, $T = \frac{1}{\nu}$

$\therefore T = \frac{1}{159.24} = 6.28\text{ms}$

Total charge on the capacitor at time t, $Q' = Q\cos\left(\frac{2\pi}{T}t\right)$

If energy stored is electrical, $Q' = \pm Q$

Therefore, it can be inferred that the energy stored in the capacitor is completely electrical at time, $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots\ldots\ldots$ where, $T = 6.3\text{ms}$.

(ii) Completely magnetic

Magnetic energy is maximum, when electrical energy Q' is equal to 0.

Therefore, it can be inferred that the energy stored is completely magnetic at time,

$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots\ldots\ldots$ where, $T = 6.3\text{ms}$.

d) At what times is the total energy shared equally between the inductor and the capacitor?

Ans: Consider, Q' be the charge on capacitor when total energy is equally shared between the capacitor and the inductor at time t.

When total energy is equally shared between the inductor and capacitor, the energy stored in the capacitor $= \frac{1}{2}$(maximum energy).

$$\Rightarrow \frac{1}{2}\frac{(Q')^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$

$$\Rightarrow \frac{1}{2}\frac{(Q')^2}{C} = \frac{1}{4}\frac{Q^2}{C}$$

$$\Rightarrow Q' = \frac{Q}{\sqrt{2}}$$

It is known that, $Q' = Q\cos\frac{2\pi}{T}t$

$$\frac{Q}{\sqrt{2}} = Q\cos\frac{2\pi}{T}t$$

$\Rightarrow \cos\frac{2\pi}{T}t = \pm\frac{1}{\sqrt{2}} = \cos(2n+1)\frac{\pi}{4};\ n = 0,1,2,3.....$

$\Rightarrow t = (2n+1)\frac{T}{8}$

Therefore, total energy is equally shared between the inductor and the capacitor at time, $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$

e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Ans: If a resistor is included in the circuit, then the total initial energy gets dissipated as heat energy in the circuit. The LC oscillation gets damped due to the resistance.

13. A coil of inductance 0.5H and resistance 100Ω is connected to a 240V,50Hz ac supply.

a) What is the maximum current in the coil?

Ans: It is given that,

Inductance of the inductor, $L = 0.5H$

Resistance of the resistor, $R = 100\Omega$

Potential of the supply voltage, $V = 240V$

Frequency of the supply, $\nu = 50Hz$

It is known that,

Peak voltage, $V_0 = \sqrt{2}V$

$\Rightarrow V_0 = \sqrt{2} \times 240$

$\Rightarrow V_0 = 339.41V$

Angular frequency of the supply, $\omega = 2\pi\nu$

$\Rightarrow \omega = 2\pi \times 50 = 100\pi\ rad/s$

V_0

Maximum current in the circuit, I_0 □ $\sqrt{\ ^2 \quad ^{2\,2}L}$

R □□

339.41

□ □$I_{0\,2}$ $(\sqrt{100}$) (0.50)□ $_{2\,2}$□1.82A (100) □

Therefore, the maximum current in the coil is 1.82A.

b) What is the time lag between the voltage maximum and the current maximum?

Ans: It is known that,

Equation for voltage, V □ V $\cos_0$□t

Equation for current, I □ I cos(t_0 □ □□)

Where,

□ is the phase difference between voltage and current.

At time t □ 0, V □ V_0 (voltage is maximum) If □ □□□t

0 i.e., at t □ $\frac{□}{□}$, I □ I_0 (current is maximum)

Therefore, the time lag between maximum voltage and maximum current is $\frac{□}{□}$.

□ □□tan $\frac{□L}{R}$

□ tan□□ $\frac{2\pi\times 50\times 0.5}{100}$ □1.57

□□□ tan $(1.57)^{□1}$

□□□57.5 □$^\circ$ $\frac{57.5^{□}}{180}$ rad

Time lag, $t = \frac{\phi}{\omega}$

$\phi = 57.5^\circ$

$\therefore t = \frac{57.5\pi}{180 \times 2\pi \times 50}$

$\therefore t = 3.19 \times 10^{-3}\, s$

$\therefore t \approx 3.2\, ms$

Therefore, the time lag between the maximum voltage and maximum current is 3.2ms.

14. Obtain the answers (a) to (b) in Exercise 13 if the circuit is connected to a high frequency supply (240V,10kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Ans: It is given that,

Inductance of the inductor, $L = 0.5H$

Resistance of the resistor, $R = 100\Omega$

Potential of the supply voltage, $V = 240V$

Frequency of the supply, $\nu = 10kHz = 10^4 Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 10^4 rad/s$ Peak

Voltage, $V_0 = V\sqrt{2} = 240\sqrt{2}V$

a) Maximum current, $I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$\therefore I_0 = \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 (0.5)^2}} = 1.1 \times 10^{-2} A$

Therefore, the maximum current in the coil is 1.1×10^{-2}A.

b) The time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

For phase difference ϕ: $\tan\phi = \frac{\omega L}{R}$

$$\Rightarrow \tan\phi = \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$$

$$\Rightarrow \phi = \tan^{-1}(100\pi)$$

$$\Rightarrow \phi = 89.82^\circ = \frac{89.82\pi}{180} \text{ rad}$$

Time lag, $t = \frac{\phi}{\omega}$

$$\Rightarrow t = \frac{89.82\pi}{180 \times 2\pi \times 10^4}$$

$$\Rightarrow t = 25 \times 10^{-6} \text{ s}$$

$$\Rightarrow t = 25 \ \mu\text{s}$$

Therefore, the time lag between the maximum voltage and maximum current is $25\mu s$.

It can be observed that I_0 is very small in this case.

Thus, at high frequencies, the inductor amounts to an open circuit.

In a dc circuit, after a steady state is achieved, $\omega = 0$. Thus, inductor L behaves like a pure conducting object.

15. A 100μF capacitor in series with a 40Ω resistance is connected to a 110V,60Hz supply.

a) What is the maximum current in the circuit?

Ans: It is given that,

Capacitance of the capacitor, $C = 100\ \mu F = 100 \times 10^{-6} F$

Resistance of the resistor, $R = 40\ \Omega$

Supply voltage, $V = 110 V$

Frequency oscillations, $\nu = 60 Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60 \text{ rad / s}$

It is known that,

For a RC circuit, Impedance: $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Peak Voltage, $V_0 = V\sqrt{2} = 110\sqrt{2} V$

Maximum current; $I_0 = \frac{V_0}{Z}$

$$\Rightarrow I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 (10^{-4})^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{1600 + \frac{1}{(120\pi)^2 (10^{-4})^2}}} = 3.24 A$$

Therefore, the maximum current in the circuit is 3.24A.

b) What is the time lag between the current maximum and the voltage maximum?

Ans: It is known that,

In a capacitor circuit, the voltage lags behind the current by a phase angle of ϕ.

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$\therefore \tan\phi = \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635$$

$$\phi = \tan^{-1}(0.6635)$$

$$\phi = 33.56^{\circ} = \frac{33.56\pi}{180}\,\text{rad}$$

It is known that,

$$\text{Time lag, } t = \frac{\phi}{\omega}$$

$$\therefore t = \frac{33.56\pi}{180 \times 120\pi}$$

$$\therefore t = 1.55 \times 10^{-3}\,\text{s}$$

$$\therefore t = 1.55\,\text{ms}$$

Therefore, the time lag between maximum current and maximum voltage is 1.55ms.

16. Obtain the answers to (a) and (b) in Exercise 15 if the circuit is connected to a 110V,12kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Ans: It is given that,

Capacitance of the capacitor, $C = 100\,\mu F = 100 \times 10^{-6}\,F$

Resistance of the resistor, $R = 40\,\Omega$

Supply voltage, $V = 110\,V$

Frequency oscillations, $\nu = 12\,kHz = 12 \times 10^{3}\,Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 12 \times 10^{3}\,rad/s = 24\pi \times 10^{3}\,rad/s$

Peak Voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}\,V$

a) It is known that,

For a RC circuit, Impedance: $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Maximum current; $I_0 = \frac{V_0}{Z}$

$$\therefore I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10)^2 (10)^{-4\cdot 2}}}}$$

$$\Rightarrow I_0 = \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9\,A$$

Therefore, the maximum current in the circuit is 3.9A. b)

It is known that,

In a capacitor circuit, the voltage lags behind the current by a phase angle of ϕ.

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$\therefore \tan\phi = \frac{1}{24\pi \times 10^{3} \times 10^{-4} \times 40} = \frac{1}{96\pi}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{96\pi}\right)$$

$$= 0.2^\circ$$

$$\Rightarrow \phi = 0.2 \times \frac{\pi}{180}\ \text{rad}$$

It is known that,

Time lag, $t = \frac{\phi}{\omega}$

□

0.2□

□ □t180______________□ 3

24□□10

□ □t 0.04 10□ □6s

□ □t 0.04□s

Therefore, the time lag between maximum current and maximum voltage is 0.04□s.

It can be concluded that □ tends to become zero at high frequencies. At a high frequency, capacitor C acts as a conductor.

In a dc circuit, after the steady state is achieved, □□ 0. Therefore, capacitor C amounts to an open circuit.

17. Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if three elements, L,C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 11 for this frequency. Ans: It is given that,

An inductor (L), a capacitor (C) and a resistor (R) is connected in parallel with each other in a circuit where,

Inductance, L □5.0H

Capacitance, C 80 F 80 10 F□ □ □ □ □6

Resistance, R □ □40

Potential of the voltage source, V □ 230V

It is known that,

Impedance (Z) of the given LCR circuit is given as:

Z1 □ √(R1 □□□□1 □□C□□2

2□ L □)

Where,

ω is the angular frequency

At resonance: $\frac{1}{\omega L} - \omega C = 0$

$\therefore \omega = \frac{1}{\sqrt{LC}}$

$\therefore \omega = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{rad / s}$

Therefore, the magnitude of Z is maximum at 50rad / s and the total current is minimum.

Rms current flowing through inductor L: $I_L = \frac{V}{\omega L}$

$\therefore I_L = \frac{230}{50 \times 5} = 0.92A$

Rms current flowing through capacitor C: $I_C = \frac{V}{\frac{1}{\omega C}} = \omega CV$

$\therefore I_C = 50 \times 80 \times 10^{-6} \times 230 = 0.92A$

Rms current flowing through resistor R : $I_R = \frac{V}{R}$

$\therefore I_R = \frac{230}{40} = 5.75A$

Current rms value in inductor is 0.92A, in capacitor is 0.92A and in resistor is 5.75A.

18. A circuit containing an 80mH inductor and a 60μF capacitor in series is connected to a 230V,50Hz supply. The resistance of the circuit is negligible.

a) Obtain the current amplitude and rms values.

Ans: It is given that,

Inductance, $L = 80\,mH = 80 \times 10^{-3}\,H$

Capacitance, $C = 60\,\mu F = 60 \times 10^{-6}\,F$

Supply voltage, $V = 230\,V$

Frequency, $\nu = 50\,Hz$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$ rad / s

Peak voltage, $V_0 = V\sqrt{2} = 230\sqrt{2}\,V$

It is known that,

Maximum current: $I_0 = \dfrac{V_0}{\left(\omega L - \dfrac{1}{\omega C}\right)}$

$\Rightarrow I_0 = \dfrac{230\sqrt{2}}{\left(100\pi \times 80 \times 10^{-3} - \dfrac{1}{100\pi \times 60 \times 10^{-6}}\right)}$

$\Rightarrow I_0 = \dfrac{230\sqrt{2}}{\left(8\pi - \dfrac{1000}{6\pi}\right)} = -11.63\,A$

The negative sign is because $\omega L < \dfrac{1}{\omega C}$

Amplitude of maximum current, $|I_0| = 11.63\,A$

$\therefore I = \dfrac{I_0}{\sqrt{2}} = \dfrac{11.63}{\sqrt{2}}$

$\therefore I = 8.22\,A$, which is the rms value of current.

b) Obtain the rms values of potential drops across each element.

Ans: It is known that,

Potential difference across the inductor, $V_L = I \times \omega L$

$\therefore V_L = 8.22 \times 100\pi \times 80 \times 10^{-3}$

$\therefore V_L = 206.61V$

Potential difference across the capacitor, $V_C = I \times \dfrac{1}{\omega C}$

$\therefore V_C = 8.22 \times \dfrac{1}{100\pi \times 60 \times 10^{-6}}$

$\therefore V_C = 436.3V$, which is the rms value of potential drop.

c) What is the average power transferred to the inductor?

Ans: Average power transferred to the inductor is zero as actual voltage leads the current by $\dfrac{\pi}{2}$.

d) What is the average power transferred to the capacitor?

Ans: Average power transferred to the capacitor is zero as actual voltage lags the current by $\dfrac{\pi}{2}$.

e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Ans: The total average power absorbed (averaged over one cycle) is zero.

19. Suppose the circuit in Exercise 18 has a resistance of 15Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Ans: It is given that,

Average power transferred to the resistor = 788.44W

Average power transferred to the capacitor = 0W

Total power absorbed by the circuit = 788.44W

Inductance of inductor, $L = 80\text{mH} = 80 \times 10^{-3}\text{H}$ Capacitance of capacitor, $C = 60\ \mu F = 60 \times 10^{-6}F$

Resistance of resistor, $R = 15\ \Omega$

Potential of voltage supply, $V = 230V$

Frequency of signal, $\nu = 50Hz$

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi\ \text{rad/s}$

It is known that,

Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\therefore Z = \sqrt{(15)^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)^2}$

$\therefore Z = \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728\ \Omega$

Now,

$I = \frac{V}{Z}$

$\therefore I = \frac{230}{31.728} = 7.25A$

The elements are connected in series to each other. Therefore, impedance of the circuit is given as current flowing in the circuit,

Average power transferred to resistance is given as: $P_R = I^2 R$

$\therefore P_R = (7.25)^2 \times 15 = 788.44W$

Average power transferred to capacitor, $P_C =$ Average power transferred to inductor, $P_L = 0$

Total power absorbed by the circuit: $P_T = P_R + P_C + P_L$

$P_T = 788.44 + 0 + 0 = 788.44W$

Therefore, the total power absorbed by the circuit is 788.44W.

20. A series LCR circuit with $L = 0.12H, C = 480nF, R = 23\ \Omega$ is connected to a 230V variable frequency supply.

a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

Ans: It is given that,

Inductance, $L = 0.12H$

Capacitance, $C = 480nF = 480 \times 10^{-9}F$

Resistance, $R = 23\ \Omega$

Supply voltage, $V = 230V$

Peak voltage, $V_0 = 230 \times \sqrt{2} = 325.22V$

It is known that,

Current flowing in the circuit, $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

Where,

I_0 is maximum at resonance.

At resonance: $\omega_R L - \dfrac{1}{\omega_R C} = 0$

Where,

ω_R is the resonance angular frequency

$\therefore \omega_R = \dfrac{1}{\sqrt{LC}}$

$\therefore \omega_R = \dfrac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}}$

$\therefore \omega_R = 4166.67 \text{ rad/s}$ Resonant

frequency, $\nu_R = \dfrac{\omega_R}{2\pi}$

$\therefore \nu_R = \dfrac{4166.67}{2\times 3.14} = 663.48\text{Hz}$

Maximum current, $(I_0)_{Max} = \dfrac{V_0}{R}$

$\therefore (I_0)_{Max} = \dfrac{325.22}{23} = 14.14\text{A}$

b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power. Ans:

It is known that,

Maximum average power absorbed by the circuit; $(P_{V})_{Max} = \dfrac{1}{2}(I_0)^2_{Max} R$

$\therefore (P_{V})_{Max} = \dfrac{1}{2}(14.14)^2 \times 23$

$\therefore (P_{V})_{Max} = 2299.3\text{W}$

Therefore, the resonant frequency, $\nu_R = 663.48\text{Hz}$

c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

Ans: It is known that,

The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half, $\omega = \omega_R \pm \Delta\omega = 2\pi(\nu_R \pm \Delta\nu)$

Where,

$\Delta\omega = \dfrac{R}{2L}$

$$\Delta\omega = \frac{R}{2L} = \frac{23}{2\times 0.12} = 95.83\,\text{rad/s}$$

Therefore, the change in frequency, $\Delta\nu = \frac{1}{2\pi}\Delta\omega$

$$\Delta\nu = \frac{95.83}{2\pi} = 15.26\,\text{Hz}$$

$\therefore \nu_1 = \nu_R + \Delta\nu = 663.48 + 15.26 = 678.74\,\text{Hz}$

$\therefore \nu_2 = \nu_R - \Delta\nu = 663.48 - 15.26 = 648.22\,\text{Hz}$

Therefore, at 648.22Hz and 678.74Hz frequencies, the power transferred is half.

At these frequencies, current amplitude: $I' = \frac{1}{\sqrt{2}} \times (I_0)_{Max}$

$$\therefore I' = \frac{14.14}{\sqrt{2}} = 10\,\text{A}$$

Therefore, the current amplitude is 10A.

d) What is the Q-factor of the given circuit? Ans:

It is known that,

Q-factor of the given circuit, $Q = \frac{\omega_r L}{R}$

$$\therefore Q = \frac{4166.67\times 0.12}{23} = 21.74$$

Therefore, the Q-factor of the given circuit is 21.74.

21. Obtain the resonant frequency and Q-factor of a series LCR circuit with $L = 3.0\,\text{H}$, $C = 27\,\mu\text{F}$ and $R = 7.4\,\Omega$. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Ans: It is given that,

Inductance, $L = 3.0\,\text{H}$

Capacitance, $C = 27\ \mu F = 27 \times 10^{-6} F$

Resistance, $R = 7.4\ \Omega$

It is known that,

At resonance, angular frequency of the source for the given LCR series circuit is

$$\therefore \omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \omega_r = \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}}$$

$$\therefore \omega_r = \frac{10^3}{9} = 111.11 \text{ rad/s}$$

Therefore, the resonant frequency is 111.11 rad / s.

Q-factor of the series, $Q = \frac{\omega_r L}{R}$

$$\therefore Q = \frac{111.11 \times 3}{7.4} = 45.0446$$

Therefore, the Q-factor is 45.0446.

To improve the sharpness of the resonance by reducing 'full width at half maximum' by a factor of 2 without changing ω_r, reduce the resistance to half.

$$\therefore R = \frac{7.4}{2} = 3.7$$

Therefore, required resistance is $3.7\ \Omega$.

22. Answer the following questions:

a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

Ans: Yes, in any ac circuit, the applied instantaneous voltage is equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit.

The same is not true for rms voltage because voltages across different elements may not be in phase.

b) **A capacitor is used in the primary circuit of an induction coil. Ans:** Yes, a capacitor is used in the primary circuit of an induction coil. This is because, when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

c) **An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.**

Ans: The dc signal will appear across capacitor C because for dc signals, the impedance of an inductor L is negligible while the impedance of a capacitor C is very high (almost infinite).

Therefore, a dc signal appears acrossC.

For an ac signal of high frequency, the impedance of Lis high and that of C is very low.

Thus, an ac signal of high frequency appears across L.

d) **A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.**

Ans: When an iron core is inserted in the choke coil (which is in series with a lamp connected to an ac line), the lamp will glow dimly.

This is because the choke coil and the iron core increase the impedance of the circuit.

e) **Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?**

Ans: As the choke coil reduces the voltage across the tube without wasting much power, it is used in the fluorescent tubes with ac mains. An ordinary resistor cannot be used instead of choke coil because it wastes power in the form of heat.

23. A power transmission line feeds input power at 2300V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230V ?

Ans: It is given that,

Input voltage, V_1 □ 2300V

Number of turns in primary coil, n_1 □ 4000

Output voltage, V_2 □ 230V

Number of turns in secondary coil ,n_2 □ ?

It is known that,

Voltage is related to number of terms: $\frac{V^1}{V_2}$ □ $\frac{n^1}{n_2}$

□ $\frac{2300}{230}$ □ $\frac{4000}{n_2}$

□ n_2 □ $\frac{4000 \times 230}{2300}$ □ 400

Therefore, the number of turns in the second winding is 400.

24. At a hydroelectric power plant, the water pressure head is at a height of 300m and the water flow available is $100m / s^3$. If the turbine generator efficiency is 60% , estimate the electric power available from the plant (g □ $9.8m / s^2$).

Ans: It is known that,

Height of water pressure head, h □300m

Volume of water flow per second, V □$100m / s^3$

Efficiency of turbine generator, n □60% □0.6

Acceleration due to gravity, g □$9.8m / s^2$

Density of water, ρ □$10^3 kg / m^3$

It is known that,

Electric power available from the plant $= \eta h \rho g V$

$\Rightarrow P = 0.6 \times 300 \times 10^3 \times 9.8 \times 100$

$\Rightarrow P = 176.4 \times 10^6 W$

$\Rightarrow P = 176.4 MW$

Therefore, the estimated electric power available from the plant is 176.4MW.

25. A small town with a demand of 800kW of electric power at 220V is situated 15km away from an electric plant generating power at 440V . The resistance of the two-wire line carrying power is 0.5 Ω perkm. The town gets power from the line through a 4000-220V step-down transformer at a sub-station in the town.

a) Estimate the line power loss in the form of heat.

Ans: It is given that,

Total electric power required, $P = 800kW = 800 \times 10^3 W$

Supply voltage, $V = 220V$

Voltage at which electric plant is generating power, $V' = 440V$

Distance between the town and power generating station, $d = 15km$

Resistance of the two wire lines carrying power $= 0.5 \Omega / km$

Total resistance of the wires, $R = (15 + 15) \times 0.5 = 15 \Omega$

A step-down transformer of rating 4000-220V is used in the sub-station.

Input voltage, $V_1 = 4000V$ Output voltage, $V_2 = 220V$

It is known that,

Rms current in the wire lines: $I = \dfrac{P}{V_1}$

$\Rightarrow I = \dfrac{800 \times 10^3}{4000} = 200A$

Line power loss $= I^2 R$

$= (200)^2 \times 15$

$= 600 \times 10^3 W = 600kW$

Therefore, the line power loss is 600kW.

b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

Ans: Assuming that there is negligible power loss due to leakage of the current: Total power supplied by the plant□800kW□600kW □1400kW Therefore, the plant must supply 1400kW of power.

c) Characterise the step up transformer at the plant.

Ans: It is known that,

Voltage drop in the power line□ IR

□ □V 200 15□ □3000V

Total voltage transmitted from the plant□3000□ 4000 □7000V The power generated is 440V.

Therefore, the rating of the step-up transformer situated at the power plant is 440V□7000V.

26. Do the same exercise as above with the replacement of the earlier transformer by a 40,000□220V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Ans: It is given that,

Total electric power required, P □800kW □800 10□ ^{3}W

Supply voltage, V □ 220V

Voltage at which electric plant is generating power, V' □ 440V

Distance between the town and power generating station, d □15km

Resistance of the two wire lines carrying power□0.5□/ km

Total resistance of the wires, R □□15□15 0.5□ □15 Ω

The rating of a step-down transformer is 40000V□220V.

Input voltage, V_1 □ 40000V Output voltage, V_2 □ 220V

a) It is known that,

Rms current in the wire lines: $I = \frac{P}{V_1}$

$\therefore I = \frac{800 \times 10^3}{40000} = 20A$

Line power loss $= I^2 R$

$= (20)^2 \times 15$

$= 6 \times 10^3 W = 6kW$

Therefore, the line power loss is 6kW.

b) Assume that there is negligible power loss due to leakage of the current: Total power supplied by the plant $= 800kW + 6kW = 806kW$ Therefore, the plant must supply 806kW of power.

c) It is known that,

Voltage drop in the power line $= IR$

$\therefore V = 20 \times 15 = 300V$

Total voltage transmitted from the plant $= 300 + 40000 = 40300V$ The power generated in the plant is generated at 440V.

Therefore, the rating of the step-up transformer situated at the power plant is $440V \rightarrow 40300V$.

Power loss during transmission $= \frac{600}{1400} \times 100 = 42.8\%$

In previous exercise the power loss due to the same reason is

$= \frac{6}{806} \times 100 = 0.744\%$

As the power loss is less for a high voltage transmission, High voltage transmissions are preferred for this purpose.