

FINAL JEE(Advanced) EXAMINATION - 2021**(Held On Sunday 03rd OCTOBER, 2021)****PAPER-2****TEST PAPER WITH SOLUTION****PART-1 : PHYSICS****SECTION-1 : (Maximum Marks : 24)**

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If unanswered;

Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then

choosing **ONLY** (A), (B) and (D) will get +4 marks;

choosing **ONLY** (A) and (B) will get +2 marks;

choosing **ONLY** (A) and (D) will get +2 marks;

choosing **ONLY** (B) and (D) will get +2 marks;

choosing **ONLY** (A) will get +1 mark;

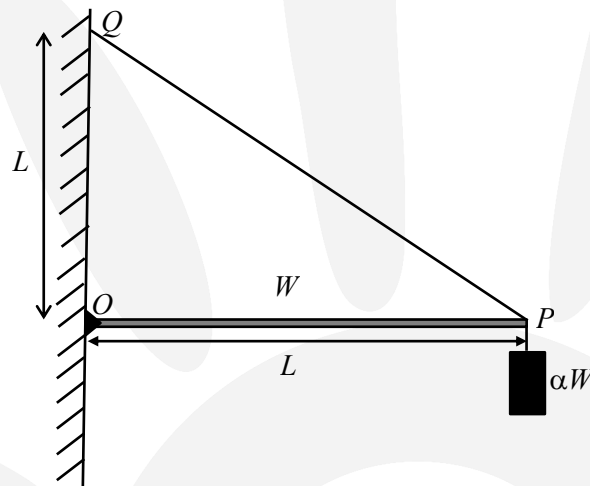
choosing **ONLY** (B) will get +1 mark;

choosing **ONLY** (D) will get +1 mark;

choosing no option(s) (i.e. the question is unanswered) will get 0 marks and

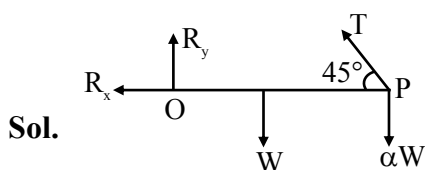
choosing any other option(s) will get -2 marks.

1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q , at a height L above the hinge at point O . A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $(2\sqrt{2})W$. Which of the following statement(s) is(are) correct ?



- (A) The vertical component of reaction force at O does **not** depend on α
 (B) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$
 (C) The tension in the rope is $2W$ for $\alpha = 0.5$
 (D) The rope breaks if $\alpha > 1.5$

Ans. (ABD)



$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \quad \dots(i)$$

$$R_x = \frac{T}{\sqrt{2}} \quad \dots(ii)$$

Taking torque about 'O'

$$W \frac{\ell}{2} + \alpha W \ell = \frac{T}{\sqrt{2}} \ell$$

$$T = \sqrt{2} \left(\frac{W}{2} + \alpha W \right) \quad \dots(iii)$$

$$R_x = \frac{T}{\sqrt{2}} = \left(\frac{W}{2} + \alpha W \right)$$

Taking torque about P

$$R_y \ell = W \frac{\ell}{2}$$

$$R_y = \frac{W}{2};$$

when $T = T_{\max}$

$$2\sqrt{2}W = \sqrt{2} \left(\frac{W}{2} + \alpha W \right)$$

$$\text{we get } \alpha = \frac{3}{2}$$

2. A source, approaching with speed u towards the open end of a stationary pipe of length L , is emitting a sound of frequency f_s . The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s , will the sound reaching the pipe lead to a resonance ?

(A) $u = 0.8 v$ and $f_s = f_0$

(B) $u = 0.8 v$ and $f_s = 2f_0$

(C) $u = 0.8 v$ and $f_s = 0.5 f_0$

(D) $u = 0.5 v$ and $f_s = 1.5 f_0$

Ans. (AD)

Sol. $f = f_s \left(\frac{v}{v-u} \right)$

(A) $f = f_0 \left(\frac{v}{v-0.8v} \right) = 5f_0$

(B) $f = 2f_0 \left(\frac{v}{v-0.8v} \right) = 10f_0$

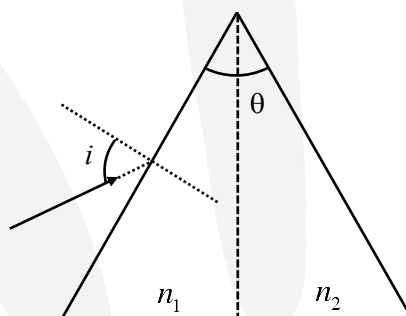
(C) $f = 0.5f_0 \left(\frac{v}{v-0.8v} \right) = 2.5f_0$

(D) $f = 1.5f_0 \left(\frac{v}{v-0.5v} \right) = 3f_0$

_____ close

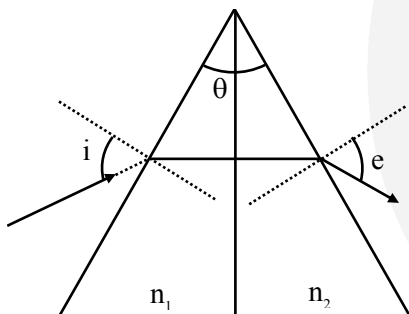
All odd harmonics are available in closed pipe therefore
correct Ans (A,D)

3. For a prism of prism angle $\theta = 60^\circ$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n \ll n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is (are) correct ?



- (A) The value of Δe (in radians) is greater than that of Δn
- (B) Δe is proportional to Δn
- (C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
- (D) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

Ans. (BC)



Sol.

$$1 \times \sin i = \mu \sin\left(\frac{A}{2}\right)$$

$$\sin i = \frac{3}{4}$$

$$n_1 \sin 30^\circ = n_2 \sin(e)$$

on differentiating both sides

$$dn \sin 30^\circ = de \cos(e)$$

$$de = \frac{dn}{2 \cos(e)}$$

$$= \frac{dn}{2 \times \frac{\sqrt{7}}{4}}$$

$$de = \frac{2}{\sqrt{7}} dn \Rightarrow de < dn$$

$$de = \frac{2.8 \times 10^{-3} \times 2}{\sqrt{7}} = 2.11 \text{ mrad}$$

4. A physical quantity \vec{S} is defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where \vec{E} is electric field, \vec{B} is magnetic field and μ_0 is the permeability of free space. The dimensions of \vec{S} are the same as the dimensions of which of the following quantity (ies) ?

- (A) $\frac{\text{Energy}}{\text{charge} \times \text{current}}$ (B) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$ (C) $\frac{\text{Energy}}{\text{Volume}}$ (D) $\frac{\text{Power}}{\text{Area}}$

Ans. (BD)

Sol. $\vec{S} = [\vec{E} \times \vec{B}] \frac{1}{\mu_0}$

\vec{S} is pointing vector denotes flow of energy per unit area per unit time

$$\vec{S} = \frac{\text{watt}}{\text{m}^2}$$

Hence B, D are correct

5. A heavy nucleus N , at rest, undergoes fission $N \rightarrow P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P , M_Q and M_N are the masses of P , Q and N , respectively. E_P and E_Q are the kinetic energies of P and Q , respectively. The speed of P and Q are v_P and v_Q , respectively. If c is the speed of light, which of the following statement(s) is(are) correct ?

(A) $E_P + E_Q = c^2 \delta$

(B) $E_P = \left(\frac{M_P}{M_P + M_Q} \right) c^2 \delta$

(C) $\frac{v_P}{v_Q} = \frac{M_Q}{M_P}$

(D) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \frac{M_P M_Q}{(M_P + M_Q)}$

Ans. (ACD)

Sol. $N \longrightarrow P + Q$

$$\text{Energy released} = (m_N - m_P - m_Q)c^2 = \delta c^2$$

This will be distributed kinetic energy of P and Q

$$\Rightarrow E_P + E_Q = \delta c^2 \quad \dots(i)$$

By conservation of momentum

$$v_p = \frac{p}{m_p} \leftarrow \textcircled{P} \quad \textcircled{Q} \rightarrow \frac{p}{m_q} = v_q$$

So $\frac{v_p}{v_q} = \frac{M_q}{M_p}$ (ii)

Kinetic energy be written as $KE = \frac{p^2}{2m}$

Hence divided in inverse ratio of masses.

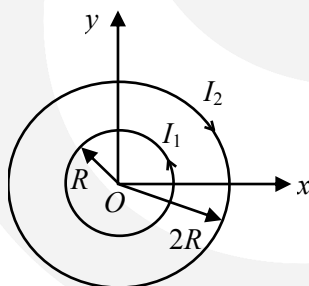
$$E_p = \frac{M_q}{M_p + M_q} c^2 \delta \quad \dots\text{(iii)}$$

By equation (i) $\Rightarrow \frac{p^2}{2M_p} + \frac{p^2}{2M_q} = \delta c^2$

$$\Rightarrow \frac{p^2}{2\mu} = \delta c^2 \Rightarrow p = c\sqrt{2\mu\delta}$$

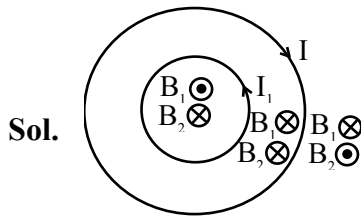
Ans. (A,C,D)

6. Two concentric circular loops, one of radius R and the other of radius $2R$, lie in the xy -plane with the origin as their common center, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy -plane. Which of the following statement(s) is(are) correct?



- (A) $\vec{B}(x, y)$ is perpendicular to the xy -plane at any point in the plane
 (B) $|\vec{B}(x, y)|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$
 (C) $|\vec{B}(x, y)|$ is non-zero at all points for $r < R$
 (D) $\vec{B}(x, y)$ points normally outward from the xy -plane for all the points between the two loops

Ans. (AB)



$$(A) \quad d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \vec{r}}{4\pi r^3}$$

$d\vec{\ell}$ is in xy plane & \vec{r} is also in xy plane

so $d\vec{B}$ is perpendicular to xy plane

(B) Due to symmetry it depends only on $r = \sqrt{x^2 + y^2}$

(C) At centre $B_1 = \frac{\mu_0 I_1}{2R}$; $B_2 = \frac{\mu_0 I_2}{4R} \Rightarrow B_2 > B_1$

but as we approach towards first loop B_1 increases to infinity hence B_1 dominates.

So it would be zero at some point between inner loops and centre.

Ans. (A,B)

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

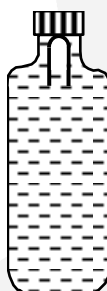
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

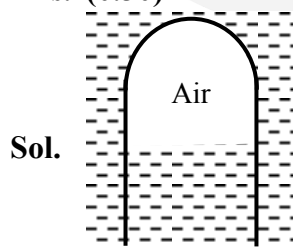
A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $P_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.

Let $\Delta v = X$ cc and $\Delta p = Y \times 10^3$ Pa.



7. The value of X is _____.

Ans. (0.30)



When it starts sinking

$$F_B = mg$$

$$\rho_0 (V_{\text{glass}} + V_{\text{gas}}) = m$$

$$1(2 + V_{\text{gas}}) = 5 \Rightarrow V_{\text{gas}} = 3 \text{ cc}$$

Hence $\Delta V = 0.3$ cc.

8. The value of Y is _____.

Ans. (10.00)

Sol. Isothermal process for air

$$P_1 V_1 = P_2 V_2$$

$$10^5 (3.3) = P_2 (3)$$

$$P_2 = 1.1 \times 10^5$$

$$\Delta P = P_2 - P_1 = 1.1 \times 10^5 - 10^5$$

$$= 0.1 \times 10^5$$

$$= 10 \times 10^3 \text{ Pascal}$$

$$= Y \times 10^3 \text{ Pascal}$$

$$\text{So } Y = 10$$

Question Stem for Question Nos. 9 and 10

Question Stem

A pendulum consists of a bob of mass $m = 0.1$ kg and a massless inextensible string of length $L = 1.0$ m. It is suspended from a fixed point at height $H = 0.9$ m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse $P = 0.2$ kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m²/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

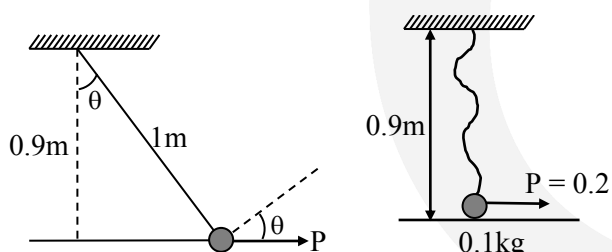
9. The value of J is ____.

Ans. (0.18)

10. The value of K is ____.

Ans. (0.16)

Sol.



$$L = P \times 0.9 = 0.18 \text{ kgm}^2/\text{s}$$

Just after string becomes taut; there will be no velocity along the string.

$$\therefore V_{\perp} = \frac{P \cos \theta}{m} = \frac{0.2 \times 0.9}{1 \times 0.1} = 1.8 \text{ m/s}$$

$$\therefore K = \frac{1}{2} m v_{\perp}^2 = \frac{1}{2} \times 0.1 \times 1.8^2$$

$$= 0.162 \text{ J}$$

Question Stem for Question Nos. 11 and 12
Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance $C \mu F$ across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take *rms* values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ .

Assume, $\pi\sqrt{3} \approx 5$.

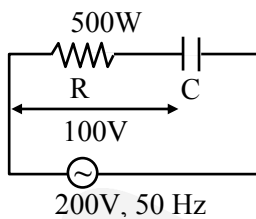
11. The value of C is ____.

Ans. (100.00)

12. The value of ϕ is ____.

Ans. (60.00)

Sol.



$$\sqrt{V_C^2 + V_R^2} = \varepsilon_{rms}$$

$$\Rightarrow V_C^2 + 100^2 = 200^2$$

$$V_C = 100\sqrt{3}V \quad \dots(i)$$

$$\tan \phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100}$$

$$\therefore \phi = 60^\circ \quad \dots(ii)$$

$$P = I_{rms} \varepsilon_{rms} \cos \phi$$

$$= \frac{\varepsilon_{rms}^2}{Z} \frac{1}{2}$$

$$500 = \frac{200^2}{Z} \frac{1}{2}$$

$$\therefore Z = 40 \Omega \quad \dots(iii)$$

$$\cos \phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{40}$$

$$\therefore R = 20$$

$$\& x_C = \sqrt{Z^2 - R^2} = \sqrt{40^2 - 20^2} = 20\sqrt{3} \Omega$$

$$\Rightarrow \frac{1}{\omega C} = 20\sqrt{3} \quad \therefore C = 100$$

SECTION-3 : (Maximum Marks : 12)

- This section contains **TWO (02) paragraphs**. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If ONLY the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1	In all other cases.

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a , with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r ($\gg a$) from the center of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m , at a point on its axis at distance r , is $\frac{\mu_0}{2\pi} \frac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2 , separated by a distance r on the common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



13. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to
- (A) $\frac{m}{r^3}$ (B) $\frac{m^2}{r^2}$ (C) $\frac{m}{r^2}$ (D) $\frac{m^2}{r}$

Ans. (A)

14. The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to
- (A) $\frac{m}{r^5}$ (B) $\frac{m^2}{r^5}$ (C) $\frac{m^2}{r^6}$ (D) $\frac{m^2}{r^7}$

Ans. (C)

Sol. $\phi = Li = \frac{\mu_0 m}{2\pi r^3} \times \pi a^2$

$$\Rightarrow i = \frac{\mu_0 m \pi a^2}{2\pi r^3 L}$$

$$\Rightarrow i \propto \frac{m}{r^3}$$

$$m' = \pi a^2 i = \frac{\mu_0 m \pi^2 a^4}{2\pi r^3 L}$$

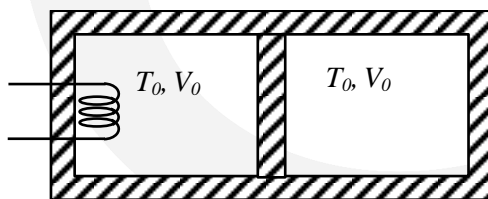
$$F = \frac{k m^2 \pi^2 a^4}{2\pi r^7 L}$$

$$W = \int F dr \propto \int \frac{m^2 dr}{r^7}$$

$$W \propto \frac{m^2}{r^6}$$

Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_v = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0 . The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R , respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.



15. The value of $\frac{T_R}{T_0}$ is

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 2

(D) 3

Ans. (A)

16. The value of $\frac{Q}{RT_0}$ is

(A) $4(2\sqrt{2} + 1)$

(B) $4(2\sqrt{2} - 1)$

(C) $(5\sqrt{2} + 1)$

(D) $(5\sqrt{2} - 1)$

Ans. (B)

Sol. Finally $V_L = \frac{3V_0}{2}, V_R = \frac{V_0}{2}$

$$C_V = \frac{R}{\gamma - 1} = 2R \Rightarrow \gamma - 1 = \frac{1}{2}$$

$$\gamma = \frac{3}{2}$$

$$T_0 V_0^{\gamma-1} = T_R \left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

$$\rho \left(\frac{V_0}{2}\right)^\gamma = P_0 V_0^\gamma \Rightarrow P = P_0 \times 2^{\frac{3}{2}}$$

$$\frac{PV}{T_L} = \frac{P_0 V_0}{T_0} \Rightarrow T_L = 2^{\frac{3}{2}} \times \frac{3}{2} T_0 = 3\sqrt{2} T_0$$

$$Q = nC_V \Delta T_1 + nC_V \Delta T_2$$

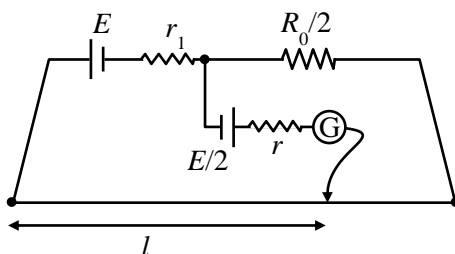
$$= 1 \times 2R \times (3\sqrt{2} - 1)T_0 + 1 \times 2R \times (\sqrt{2} - 1)T_0$$

$$\frac{Q}{RT_0} = 2(3\sqrt{2} - 1) + 2(\sqrt{2} - 1) = 8\sqrt{2} - 4$$

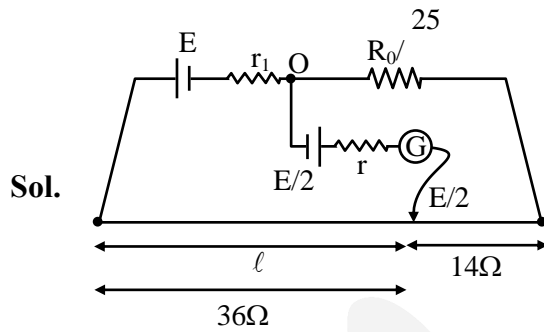
SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.

17. In order to measure the internal resistance r_1 of a cell of emf E , a meter bridge of wire resistance $R_0 = 50 \Omega$, a resistance $R_0/2$, another cell of emf $E/2$ (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at $l = 72$ cm, then the value of $r_1 = \underline{\hspace{2cm}} \Omega$.



Ans. (3)



$$i \left(\frac{R_0}{2} + 0.28R_0 \right) = \frac{E_0}{2}$$

$$i \times 0.78R_0 = \frac{E_0}{2}$$

$$i = \frac{E_0}{2 \times 0.78R_0} = \frac{E_0}{r_1 + \frac{3}{2}R_0}$$

$$r_1 + 1.5 R_0 = 1.56 R_0$$

$$r_1 = 0.06 R_0$$

$$= 0.06 \times 50 = 3 \Omega$$

18. The distance between two stars of masses $3M_S$ and $6M_S$ is $9R$. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT , where T is the period of Earth's revolution around the Sun. The value of n is ____.

Ans. (9)

Sol. Circular orbits

$$T = 2\pi \sqrt{\frac{R^3}{GM_S}}$$

Binary stars

$$nT = 2\pi \sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$n \times 2\pi \sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi \sqrt{\frac{R^3}{GM_S}}$$

$$n = 9$$

19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P , Q and R are E_P , E_Q and E_R , respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R . The work functions for metals P , Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R , in eV, is ____.

Ans. (6)

Sol. For P & Q

$$E_1 - 4 = E_P$$

$$E_1 - 4.5 = E_Q$$

$$E_P = 2 E_Q$$

$$E_1 - 4 = 2 (E_1 - 4.5)$$

$$E_1 = 5 \text{ eV}$$

$$E_P = 1 \text{ eV}, E_Q = E_R = 0.5 \text{ eV}$$

$$\text{For } E_2 - 5.5 = 0.5$$

$$E_2 = 6 \text{ eV}$$