FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Friday 12th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

(1) If $(A - C) \subseteq B$, then $A \subseteq B$

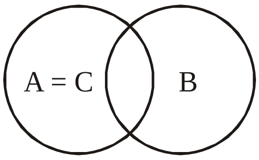
(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A - B) \subseteq C$, then $A \subseteq C$

(4) $B \cap C \neq \phi$

Official Ans. by NTA (1)

Sol.



for $A = C$, $A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option 1 is **NOT** true

Let $x \in (C\ x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C \text{ or } x \in A)$ and $(x \in C \text{ or } x \in B)$

$\Rightarrow x \in C \text{ or } x \in (A \cap B)$

$\Rightarrow x \in C \text{ or } x \in C$ (as $A \cup B \subseteq C$)

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$ (1)

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B)$ (2)

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option 2 is true

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

as $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

$\Rightarrow$ Option 4 is true.

2. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to:

(1) (420, 18) (2) (380, 19)

(3) (380, 18) (4) (420, 19)

Official Ans. by NTA (1)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

Diff. w.r.t. x

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + {}^nC_2\,(2x) + ...+ {}^nC_n\,n(x)^{n-1}$

Multiply by x both side

$\Rightarrow nx(1+x)^{n-1} = {}^nC_1\,x + {}^nC_2\,(2x^2) +....+ {}^nC_n(n\,x^n)$

Diff w.r.t. x

$\Rightarrow n\,[(1+x)^{n-1} + (n-1)x\,(1+x)^{n-2}]$

$= {}^nC_1 + {}^nC_2\,2^2x + ...\ {}^nC_n\,(n^2)x^{n-1}$

Put x = 1 and n = 20

$\Rightarrow {}^{20}C_1 + 2^2\,{}^{20}C_2 + 3^2\,{}^{20}C_3 + ... + (20)^2\,{}^{20}C_{20}$

$= 20 \times 2^{18}\,[2 + 19] = 420\,(2^{18}) = A(2^\beta)$

3. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(1) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

(2) $(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

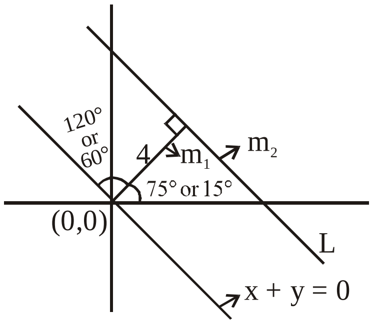
(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

Official Ans. by NTA (1)

ALLEN Ans. (1) or (2)

Sol.



$m_1 = \tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

or $m = \tan 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$m_2 = \dfrac{-1}{m_1} = \dfrac{-\left(\sqrt{3}-1\right)}{\sqrt{3}+1}$

or $m_2 = \dfrac{-1}{m_1} = \dfrac{-\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$

$\Rightarrow y = m_2x + C \Rightarrow y = \dfrac{-\left(\sqrt{3}-1\right)x}{\sqrt{3}+1} + C \Rightarrow L$

or $y = \dfrac{-\left(\sqrt{3}+1\right)x}{\sqrt{3}-1} + C \Rightarrow L$

Distance from origin = 4

$\therefore \left|\dfrac{C}{\sqrt{1+\dfrac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)^2}}}\right| = 4$ or $\left|\dfrac{C}{\sqrt{1+\dfrac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2}}}\right| = 4$

$\Rightarrow C = \dfrac{8\sqrt{2}}{\left(\sqrt{3}+1\right)}$ or $C = \dfrac{8\sqrt{2}}{\left(\sqrt{3}-1\right)}$

$\Rightarrow \left(\sqrt{3}-1\right)y + \left(\sqrt{3}+1\right)x = 8\sqrt{2}$

or $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

4. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0, \text{ is :}$$

(1) $\dfrac{7\pi}{24}$ (2) $\dfrac{\pi}{18}$ (3) $\dfrac{\pi}{9}$ (4) $\dfrac{7\pi}{36}$

Official Ans. by NTA (3)

Sol. $R_1 \to R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$\Rightarrow (1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$

$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$6\theta = \dfrac{2\pi}{3} \Rightarrow \boxed{\theta = \dfrac{\pi}{9}}$

5. If [x] denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$

(1) have infinitely many solutions if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

(3) has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \dfrac{7\pi}{6}\right)$

(4) has a unique solution if $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \cup \left(\pi, \dfrac{7\pi}{6}\right)$

Official Ans. by NTA (2)

Sol. $[\sin\theta]x + [-\cos\theta]y = 0$ and $[\cos\theta]x + y = 0$ for infinite many solution

$$\begin{vmatrix} [\sin\theta] & [-\cos\theta] \\ [\cos\theta] & 1 \end{vmatrix} = 0$$

ie $[\sin\theta] = -[\cos\theta][\cot\theta]$ (1)

when $\theta \in \left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \dfrac{1}{2}\right)$

$-\cos\theta \in \left(0, \dfrac{1}{2}\right)$

$\cot\theta \in \left(-\dfrac{1}{\sqrt{3}}, 0\right)$

when $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$$-\cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$$

$$\cot\theta \in \left(\sqrt{3}, \infty\right)$$

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ then equation (i) satisfied there fore infinite many solution.

when $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ then equation (i) not satisfied there fore infinite unique solution.

6. $\lim\limits_{x\to 0} \frac{x + 2\sin x}{\sqrt{x^2 - 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is :

(1) 3 (2) 2 (3) 6 (4) 1

Official Ans. by NTA (2)

Sol. Rationalize

$$\lim_{x\to 0} \frac{(x + 2\sin x)\left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right)}{x^2 + 2\sin x + 1 - \sin^2 x + x - 1}$$

$$\lim_{x\to 0} \frac{(x + 2\sin x)(2)}{x^2 + 2\sin x - \sin^2 x + x}$$

$\frac{0}{0}$ form using L' hospital

$$\Rightarrow \lim_{x\to 0} \frac{(1 + 2\cos x)\times 2}{2x + 2\cos x - 2\sin x\cos x + 1}$$

$$\Rightarrow \frac{2\times 3}{(2+1)} = 2$$

7. If a_1, a_2, a_3,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 200 (2) 280

(3) 120 (4) 150

Official Ans. by NTA (1)

Sol. $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40 \qquad \Rightarrow \boxed{a + 7d = \frac{40}{3}}$

$$S_{15} = \frac{15}{2}(2a + 14d) = 15(a + 7d)$$

$$S_{15} = 15\times\frac{40}{3} \Rightarrow 200 \quad \boxed{S_{15} = 200}$$

8. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is :

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{3}$ (4) 3

Official Ans. by NTA (1)

Sol. perpendicular vector to the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Eq. of plane

$-3(x - 1) + 3(y - 1) + 3z = 0$

$\Rightarrow x - y - z = 0$

$$d_{(2,1,4)} = \frac{|2 - 1 - 4|}{\sqrt{1^2 + 1^2 + 1^2}} = \sqrt{3}$$

9. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) $\beta\gamma$ (2) 0 (3) $\alpha\gamma$ (4) $\alpha\beta$

Official Ans. by NTA (1)

Sol. $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t$, $\gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0 \qquad (1)$

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t \,.\, \alpha t^2 = \alpha^2 t^3 = a^2(t^2 + t)$

(from equation 1)

10. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) 36 (2) – 108 (3) – 72 (4) – 36

Official Ans. by NTA (4)

Sol. $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6-\frac{1}{81}\cdot x^8\left(2x^2-\frac{3}{x^2}\right)^6$

its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r}-\frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

for term independent of x, r for I[st] expression is 3 and r for second expression is 5

∴ term independent of x = – 36

11. Let α ∈ R and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set S = {α : $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar}

(1) is singleton

(2) Contains exactly two numbers only one of which is positive

(3) Contains exactly two positive numbers

(4) is empty

Official Ans. by NTA (4)

Sol. $\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -4\\ \alpha & -2 & 3\end{vmatrix}=0$

$\Rightarrow 3\alpha^2+18=0$

$\Rightarrow \alpha\in\phi$

12. A value of α such that $\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$ is :

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) – 2

Official Ans. by NTA (4)

Sol. $\int_{\alpha}^{\alpha+1}\frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)}dx=\left(\ell n|x+\alpha|-\ell n|x+\alpha+1|\right)_{\alpha}^{\alpha+1}$

$=\ell n\left|\frac{2\alpha+1}{2\alpha+2}\times\frac{2\alpha+1}{2\alpha}\right|=\ell n\frac{9}{8}$

$\Rightarrow \alpha=-2,1$

13. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3},1\right)$ (2) $\left(\frac{1}{3},2\right)$

(3) $\left(1,\frac{7}{3}\right)$ (4) $\left(\frac{1}{3},\frac{5}{3}\right)$

Official Ans. by NTA (2)

Sol. Let B(α,β) and C(γ,δ)

$\frac{\alpha+1}{2}=-1\Rightarrow\alpha=-3$

$\frac{\beta+2}{2}=1\Rightarrow\beta=0$

$\Rightarrow B(-3,0)$

Now $\frac{\gamma+1}{2}=2\Rightarrow\gamma=3$

$\frac{\delta+2}{2}=3\Rightarrow\delta=4$

⇒ C(3,4)

⇒ centroid of triangle is $G\left(\frac{1}{3},2\right)$

14. Let α ∈ (0, π/2) be fixed. If the integral $\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx=$ A(x) cos2α + B(x) sin2α + C, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) x – α and $\log_e|\cos(x-\alpha)|$

(2) x + α and $\log_e|\sin(x-\alpha)|$

(3) x – α and $\log_e|\sin(x-\alpha)|$

(4) x + α and $\log_e|\sin(x+\alpha)|$

Official Ans. by NTA (3)

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let x – α = t

$\Rightarrow \int \frac{\sin(t+2\alpha)}{\sin t}dt = \int \cos 2\alpha dt + \int \cot(t)\sin 2\alpha dt$

$= t.\cos 2\alpha + \ell n|\sin t|.\sin 2\alpha + C$

= (x – α) cos2α + ln|sin(x – α)|.sin2α + C

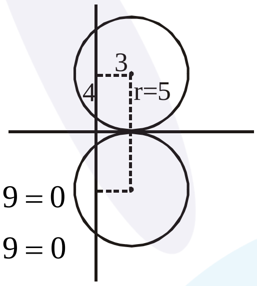
15. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (3, 10) (2) (2,3) (3) (1,5) (4) (3,5)

Official Ans. by NTA (1)

Sol. Equaiton of circles are

$$\begin{cases}(x-3)^2+(y-5)^2=25\\(x-3)^2+(y+5)^2=25\end{cases}$$



$$\Rightarrow \begin{cases}x^2+y^2-6x-10y+9=0\\x^2+y^2-6x+10y+9=0\end{cases}$$

16. For and initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (4) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

Official Ans. by NTA (2)

Sol. Let X be random varibale which denotes number of problems that candidate is unbale to solve

$\because\ p = \frac{1}{5}$ and X < 2

$\Rightarrow P(X<2) = P(X=0) + P(X=1)$

$= \left(\frac{4}{5}\right)^{50} + {}^{50}C_1.\left(\frac{1}{5}\right).\left(\frac{4}{5}\right)^{49}$

17. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 1 (4) 2

Official Ans. by NTA (4)

Sol. $f(x) = \tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$

$= \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right) = \tan^{-1}\left(\tan\left(x - \frac{\pi}{4}\right)\right)$

$\because\ x - \frac{\pi}{4} \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\therefore\ f(x) = x - \frac{\pi}{4}$

$\Rightarrow$ its derivative w.r.t. $\frac{x}{2}$ is $\frac{1}{1/2} = 2$

18. Let S be the set of all α ∈ R such that the equation, cos2x + αsinx = 2α – 7 has a solution. Then S is equal to :

(1) [2, 6] (2) [3,7] (3) R (4) [1,4]

Official Ans. by NTA (1)

Sol. cos2x + αsinx = 2α – 7

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\sin^2 x - \frac{\alpha}{2}\sin x + \alpha - 4 = 0$

$\Rightarrow \sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\Rightarrow \left|\frac{\alpha - 4}{2}\right| \le 1$

$\Rightarrow \alpha \in [2, 6]$

19. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

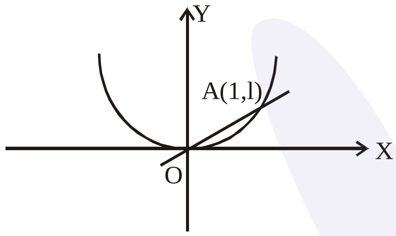
(1) $\left(-\frac{5}{2}, -1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$

(3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(\frac{5}{2}, 1\right)$

Official Ans. by NTA (3)

Sol. Put $x - 2 = X$ & $y + 1 = Y$

$\therefore$ given curve becomes $Y = X^2$ and $Y = X$



tangent at origin is X-axis

and tangent at A(1,1) is $Y + 1 = 2X$

$\therefore$ there intersection is $\left(\frac{1}{2}, 0\right)$

$\therefore \; x - 2 = \frac{1}{2}$ & $y + 1 = 0$

therefore $x = \frac{5}{2}, y = -1$

20. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 25 (2) 28 (3) 27 (4) 24

Official Ans. by NTA (1)

Sol. ${}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$

$n^2 + 3n = 700$

$\therefore \; n = 25$

21. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is :

(1) $x + y + 4 = 0$ (2) $x - 2y + 16 = 0$

(3) $2x - y + 2 = 0$ (4) $x - y + 4 = 0$

Official Ans. by NTA (4)

Sol. tangent to the parabola $y^2 = 16x$ is $y = mx + \frac{4}{m}$

solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

condition of common tangent is $D = 0$

$\therefore \; m^3 = 1$

$\Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$

22. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z - n}{2z + n} = 2i - 1$ for some natural number n. Then :

(1) $n = 20$ and $\text{Re}(z) = -10$

(2) $n = 20$ and $\text{Re}(z) = 10$

(3) $n = 40$ and $\text{Re}(z) = -10$

(4) $n = 40$ and $\text{Re}(z) = 10$

Official Ans. by NTA (3)

Sol. Put $z = x + 10i$

$\therefore \; 2(x + 10i) - n = (2i - 1) \cdot [2(x+10i) + n]$

compare real and imginary coefficients

$x = -10, \; n = 40$

23. The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0 \; (x \neq 0)$ is :

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$

(2) $y^2 + 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$

(4) $y^2 - 2x^2 + cx^3 = 0$

Official Ans. by NTA (1)

Sol. $xy\frac{dy}{dx} - y^2 + x^3 = 0$

put $y^2 = k \Rightarrow y\frac{dy}{dx} = \frac{1}{2}\frac{dk}{dx}$

$\therefore$ given differential equation becomes

$$\frac{dk}{dx} + k\left(-\frac{2}{x}\right) = -2x^2$$

I.F. = $e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

$\therefore$ solution is $k \cdot \frac{1}{x^2} = \int -2x^2 \cdot \frac{1}{x^2} dx + \lambda$

$y^2 + 2x^3 = \lambda x^2$

take $\lambda = -c$ (integration constant)

24. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) 2 gain (2) $\frac{1}{2}$ loss (3) $\frac{1}{4}$ loss (4) $\frac{1}{2}$ gain

Official Ans. by NTA (2)

Sol. win Rs.15 $\to$ number of cases = 6

win Rs.12 $\to$ number of cases = 4

loss Rs.6 $\to$ number of cases = 26

p(expected gain/loss) = $15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36} = -\frac{1}{2}$

25. Let f(x) = 5– |x–2| and g(x) = |x + 1|, x $\in$ R. If f(x) attains maximum value at α and g(x) attains minimum value at β, then $\lim\limits_{x\to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to :

(1) 1/2 (2) –3/2 (3) 3/2 (4) –1/2

Official Ans. by NTA (1)

Sol. Maxima of f(x) occured at x = 2 i.e. α = 2

Minima of g(x) occured at x = –1 i.e. β = –1

$\therefore \lim\limits_{x\to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \frac{1}{2}$

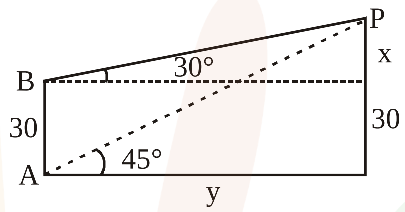
26. The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:

(1) $15(3-\sqrt{3})$ (2) $15(3+\sqrt{3})$

(3) $15(1+\sqrt{3})$ (4) $15(5-\sqrt{3})$

Official Ans. by NTA (2)

Sol.



$\tan 45° = 1 = \frac{x+30}{y} \Rightarrow x + 30 = y$ (i)

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{x}{y} \Rightarrow x = \frac{y}{\sqrt{3}}$ (ii)

from (i) and (ii) $y = 15(3+\sqrt{3})$

27. The Boolean expression ~(p$\Rightarrow$ (~q)) is equivalent to :

(1) (~p)$\Rightarrow$ q (2) $p \vee q$

(3) q$\Rightarrow$ ~p (4) p ^ q

Official Ans. by NTA (4)

Sol. $\sim(p \to (\sim q)) = \sim(\sim p \vee \sim q)$

$= p \wedge q$

28. A plane which bisects the angle between the two given planes 2x – y + 2z – 4 = 0 and x + 2y + 2z – 2 = 0, passes through the point:

(1) (2,4,1) (2) (2, – 4, 1)

(3) (1, 4, – 1) (4) (1, – 4, 1)

Official Ans. by NTA (2)

Sol. equation of bisector of angle

$\frac{2x-y+2z-4}{3} = \pm \frac{x+2y+2z-2}{3}$

(+) gives x – 3y = 2

(–) gives 3x + y + 4z = 6

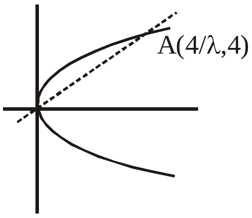
therefore option (ii) satisfy

29. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 24 (2) 48 (3) $4\sqrt{3}$ (4) $2\sqrt{6}$

Official Ans. by NTA (1)

Sol.



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}} \left(\sqrt{4\lambda x} - \lambda x\right) dx$$

$$\Rightarrow \lambda = 24$$

30. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ?

(1) $(1, 2\sqrt{2})$

(2) $(2, \sqrt{2})$

(3) $(2, 2\sqrt{2})$

(4) $(\sqrt{2}, 2)$

Official Ans. by NTA (4)

Sol. given that be = 2 and a = 2
(here a < b)
$\because a^2 = b^2(1 - e^2)$
$\therefore b^2 = 8$

$\therefore$ equation of ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$