FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Friday 12th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS — TEST PAPER WITH ANSWER & SOLUTION

1. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4, 3\sqrt{2})$ (2) $(4, 3\sqrt{3})$

(3) $(3, 3\sqrt{3})$ (4) $(5, 3\sqrt{6})$

Official Ans. by NTA (2)

Sol. $f(x) = x\sqrt{kx - x^2}$

$f'(x) = \dfrac{3kx - 4x^2}{2\sqrt{kx - x^2}}$

For ↑ $f'(x) \geq 0$ | $3kx - 4x^2 \geq 0$

$kx - x^2 \geq 0$ | $4x^2 - 3kx \leq 0$

$x^2 - kx \leq 0$ | $4x\left(x - \dfrac{3k}{4}\right) \leq 0$

$x(x-k) \leq 0$ so $x \in [0, 3]$ | $3 - \dfrac{3k}{4} \leq 0$

+ve $\boxed{x \geq 3}$ | $\boxed{k \geq 4}$

minimum value of k is $\boxed{m = 4}$

$f(x) = x\sqrt{kx - x^2}$

$= 3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3}, M = 3\sqrt{3}$

2. The equation $|z-i| = |z-1|$, $i = \sqrt{-1}$, represents:

(1) the line through the origin with slope –1.

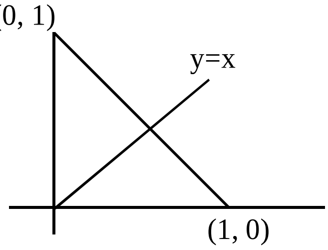
(2) a circle of radius 1.

(3) a circle of radius $\dfrac{1}{2}$.

(4) the line through the origin with slope 1.

Official Ans. by NTA (4)

Sol.



$|z - i| = |z - 1|$

$y = x$

3. For $x \in \left(0, \dfrac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \dfrac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\dfrac{\pi}{3}\right)$ is equal to :

(1) $\tan\dfrac{\pi}{12}$ (2) $\tan\dfrac{7\pi}{12}$

(3) $\tan\dfrac{11\pi}{12}$ (4) $\tan\dfrac{5\pi}{12}$

Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$fog(x) = \sqrt{\tan x}$

$hofog(x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\dfrac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\dfrac{\pi}{4} - x\right)$

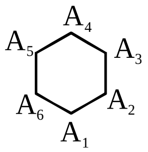
$\phi\left(\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{3}\right) = \tan\left(-\dfrac{\pi}{12}\right) = -\tan\dfrac{\pi}{12}$

$= \tan\left(\pi - \dfrac{\pi}{12}\right) = \tan\dfrac{11\pi}{12}$

4. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\dfrac{3}{10}$ (2) $\dfrac{1}{10}$ (3) $\dfrac{3}{20}$ (4) $\dfrac{1}{5}$

Official Ans. by NTA (2)

Sol. 

Only two equilateral tringles are possible A_1 A_3 A_5 and $A_2A_5A_6$

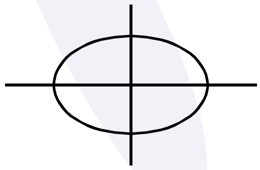
$$\frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

5. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{221}}{2}$ (2) $\frac{\sqrt{157}}{2}$ (3) $\frac{\sqrt{61}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

Official Ans. by NTA (4)

Sol. $3x^2 + 4y^2 = 12$



$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$x = 2\cos\theta$, $y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3}\sin\theta)$

Equation of normal is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$$2x\sin\theta - \sqrt{3}\cos\theta y = \sin\theta\cos\theta$$

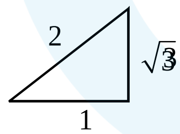
Slope $\frac{2}{\sqrt{3}}\tan\theta = -2 \quad \therefore \tan\theta = -\sqrt{3}$

Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + 2\sqrt{3}\sin\theta\, y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$



$\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ $\therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$$P\left(-1, \frac{3}{2}\right), \ Q\ (4, 4)$$

$$PQ = \frac{5\sqrt{5}}{2}$$

6. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

(1) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

(3) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

Official Ans. by NTA (1)

Sol. $e^y = xy = e$

differentiate w.r.t. x

$$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(x + e^y) = -y, \quad \left.\frac{dy}{dx}\right|_{(0,1)} = -\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot e^y\cdot\frac{dy}{dx} + x\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$(x + e^y)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\cdot e^y + 2\frac{dy}{dx} = 0$$

$$e\frac{d^2y}{dx^2} + \frac{1}{e^2}e + 2\left(-\frac{1}{e}\right) = 0$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

7. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the palne $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to :

(1) $2\sqrt{14}$ (2) $\sqrt{14}$ (3) $2\sqrt{7}$ (4) 14

Official Ans. by NTA (1)

Sol. $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

$x = 3\lambda + 2$, $y = 2\lambda - 1$, $z = -\lambda + 1$

Intersection with plane $2x + 3y - z + 13 = 0$

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0$ $\boxed{\lambda = -1}$

$\therefore$ P(–1, –3, 2)

Intersection with plane

$3x + y + 4z = 16$

$3(3\lambda+2) + (2\lambda-1) + 4(-\lambda+1) = 16$

$\lambda = 1$

Q(5, 1, 0)

$PQ = \sqrt{6^2+4^2+2^2} = \sqrt{56} = 2\sqrt{14}$

8. The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to:

(1) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$ (2) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

(3) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (4) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$

Official Ans. by NTA (3)

Sol. $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$

9. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r+\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{7}{116}$

Official Ans. by NTA (3)

Sol. $375x^2 - 25x - 2 = 0$

$\alpha+\beta=\frac{25}{375}$, $\alpha\beta=\frac{-2}{375}$

$\Rightarrow$ ($\alpha + \alpha^2 + \ldots$ upto infinite terms) + ($\beta + \beta^2 + \ldots$ upto infinite terms) = $=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}=\frac{1}{12}$

10. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx = m(\pi + n)$, then m . n is equal to :

(1) –1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Official Ans. by NTA (1)

Sol. $\int_0^{\pi/2}\frac{\cot x\,dx}{\cot x+\operatorname{cosec} x}$

$\int_0^{\pi/2}\frac{\cos x}{\cos x+1}=\int\frac{2\cos^2\frac{x}{2}-1}{2\cos^2\frac{x}{2}}$

$\int_0^{\pi/2}\left(1-\frac{1}{2}\sec^2\frac{x}{2}\right)dx$

$\left[x-\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi-2]$ $m=\frac{1}{2}$, $n=-2$

$mn = -1$

11. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) 2^{20} (2) $2^{20} - 1$

(3) $2^{20} + 1$ (4) 2^{21}

Official Ans. by NTA (1)

Sol.

10 Identical Object	21Distinct	1 0
0	10	${}^{21}C_{10} \times 1$
1	9	${}^{21}C_{9} \times 1$
⋮	⋮	⋮
10	0	${}^{21}C_{0} \times 1$

${}^{21}C_0 + \ldots + {}^{21}C_{10} + {}^{21}C_1 + \ldots + {}^{21}C_0 = 2^{21}$

$({}^{21}C_0 + \ldots + {}^{21}C_{10}) = 2^{20}$

12. If the data $x_1, x_2, \ldots, x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) 4 (2) 2 (3) $\sqrt{2}$ (4) $2\sqrt{2}$

Official Ans. by NTA (2)

Sol. $x_1 + \ldots + x_4 = 44$

$x_5 + \ldots + x_{10} = 96$

$\bar{x} = 14$, $\Sigma x_i = 140$

Variance $=\frac{\Sigma x_i^2}{n}-\bar{x}^2 = 4$

Standard deviation = 2

13. The number of solutions of the equation $1+\sin^4 x=\cos^2 3x,\ x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$ is :

(1) 5 (2) 4 (3) 7 (4) 3

Official Ans. by NTA (1)

Sol. $1 + \sin^4x = \cos^2 3x$

$\sin x = 0$ & $\cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

14. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) –320 (2) –260 (3) –380 (4) –410

Official Ans. by NTA (1)

Sol. $2\{2a+3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$\boxed{d=-12}$

$S_{10} = 5\{44 - 9 \times 12\}$

$= -320$

15. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is :

(1) 0 (2) 2 (3) 1 (4) –1

Official Ans. by NTA (3)

Sol. $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha^2 - 2\alpha - 25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

16. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(x\le 2)=\frac{k}{2^{16}}$, then k is equal to :

(1) 17 (2) 1 (3) 121 (4) 137

Official Ans. by NTA (4)

Sol. $np = 8$

$npq = 4$

$q=\frac{1}{2} \Rightarrow p=\frac{1}{2}$

$n = 16$

$p(x=r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p(x\le 2) = \frac{{}^{16}C_0+{}^{16}C_1+{}^{16}C_2}{2^{16}}$

$=\frac{137}{2^{16}}$

17. If the truth value of the statement $P\to(\sim p\vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T (2) T, F, F

(3) T, T, F (4) T, F, T

Official Ans. by NTA (3)

Sol. $P\to(\sim q\vee r)$

$\sim p\vee(\sim q\vee r)$

$$\left.\begin{matrix}\sim p\to F\\ \sim q\to F\\ r\to F\end{matrix}\right\}\Rightarrow\left.\begin{matrix}p\to T\\ q\to T\\ r\to F\end{matrix}\right\}$$

18. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, the the value of x for which y = 2, is :

(1) $\frac{1}{2}+\frac{1}{\sqrt{e}}$ (2) $\frac{3}{2}-\sqrt{e}$

(3) $\frac{5}{2}+\frac{1}{\sqrt{e}}$ (4) $\frac{3}{2}-\frac{1}{\sqrt{e}}$

Official Ans. by NTA (4)

Sol. $y^2dx + xdy = \frac{dy}{y}$

$$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$$

$$IF = e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$$

$$e^{-\frac{1}{y}}\cdot x=\int e^{-\frac{1}{y}}\cdot\frac{1}{y^3}dy+C$$

$$xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}+\frac{e^{-\frac{1}{y}}}{y}+C$$

$C = -\frac{1}{e}$

$x=\frac{3}{2}-\frac{1}{\sqrt{e}}$ when y = 2

19. Let $f : R \to R$ be a continuously differentiable function wuch that $f(2) = 6$ and $f(2) = \frac{1}{48}$.

If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to :

(1) 24 (2) 36 (3) 12 (4) 18

Official Ans. by NTA (4)

Sol. $\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x-2}$

$= \lim_{x\to 2} \frac{4.f^3(x)\cdot f'(x)}{1}$

$= 4f^3(2)\ f'(2) = 18$

20. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1 + x + x^2)^9$ is :

(1) –84 (2) 84 (3) 126 (4) –126

Official Ans. by NTA (2)

Sol. $(1 + x)\ (1 - x)^{10}\ (1 + x + x^2)^9$

$(1 - x^2)\ (1 - x^3)^9$

${}^9C_6 = 84$

21. For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is

(1) –153 (2) –133 (3) –131 (4) –135

Official Ans. by NTA (2)

Sol. $\underbrace{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+...+\left[-\frac{1}{3}-\frac{66}{100}\right]}_{(-1)67}$

$+\underbrace{\left[-\frac{1}{3}-\frac{67}{100}\right]+...+\left[-\frac{1}{3}-\frac{99}{100}\right]}_{-2(33)} = -133$

22. The equation $y = \sin x \sin(x + 2) - \sin^2(x+1)$ represents a straight line lying in :

(1) second and third quadrants only
(2) third and fourth quadrants only
(3) first, third and fourth quadrants
(4) first, second and fourth quadrants

Official Ans. by NTA (2)

Sol. $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1-\cos(2x+2))$

$= \cos 2 - 1$

$2y = -2\sin^2 \frac{1}{2}$

$y = -\sin^2 \frac{1}{2} \le 0$

23. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:

(1) 5:4 (2) 14:13 (3) 2:1 (4) 13:11

Official Ans. by NTA (1)

Sol. Equation of tangents

$y^2 = 12x \quad \Rightarrow \quad y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$

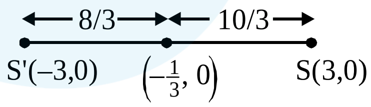
Since they are common tangent

$\therefore \frac{3}{m} = \pm\sqrt{m^2-8}$ | $\frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^4 - 8m^2 - 9 = 0$ | $e = 3$

$m = \pm 3$ | $ae = 3$

$\therefore$ $y = 3x + 1$, $y = -3x - 1$ $\Big\rangle$ $P\left(-\frac{1}{3}, 0\right)$, $S = (3, 0)$, $S' = (-3, 0)$



24. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to :

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

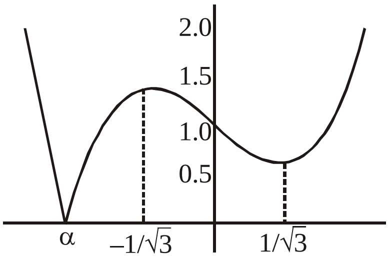
Official Ans. by NTA (3)

ALLEN Ans. Bonus

Sol. Volume of parallelopiped = $\left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graph as follows



where $\alpha \approx -1.32$

$\because$ Question is asking minimum value of volume of parallelopiped & corresponding value of λ; the minimum value is zero, $\because$ cubic always has atleast one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

In JEE (Screening) 2003 same Question was asked and answer was given to be none of these, where the options were :

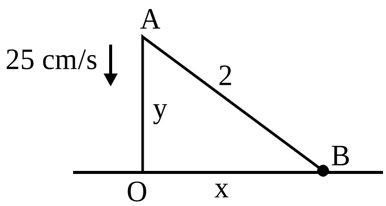
(A) –3 (B) 3 (C) $\frac{1}{\sqrt{3}}$ (D) none of these

25. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/ sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

(1) $25\sqrt{3}$ (2) 25 (3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{3}$

Official Ans. by NTA (3)

Sol.



$x^2 + y^2 = 4 \quad \left(\frac{dy}{dt} = -25\right)$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$\sqrt{3}\frac{dx}{dt} - 1(25) = 0$

$\frac{dx}{dt} = \frac{25}{\sqrt{3}}$ cm / sec

26. If a is A symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

Official Ans. by NTA (3)

Sol. A = A', B = –B'

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ ...(1)

$A' + B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$ ...(2)

After adding Eq. (1) & (2)

$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

27. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i} + 2\hat{j} - \hat{k})$ (2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$

(3) $4(2\hat{i} - 2\hat{j} - \hat{k})$ (4) $4(2\hat{i} + 2\hat{j} + \hat{k})$

Official Ans. by NTA (3)

Sol. $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$= 2(\vec{b} \times \vec{a})$

$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$

$= 2(8\hat{i} - 8\hat{j} + 4\hat{k})$

Required vector $= \pm 12\dfrac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$

$= \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

28. The integral $\int \dfrac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\dfrac{x^3+1}{x}\right|+C$

(2) $\dfrac{1}{2}\log_e\dfrac{(x^3+1)^2}{|x^3|}+C$

(3) $\dfrac{1}{2}\log_e\dfrac{|x^3+1|}{x^2}+C$

(4) $\log_e\dfrac{|x^3+1|}{x^2}+C$

Official Ans. by NTA (1)

Sol. $\int \dfrac{2x^3-1}{x^4+x}dx$

$\int \dfrac{2x-\dfrac{1}{x^2}}{x^2+\dfrac{1}{x}}dx$

$x^2+\dfrac{1}{x}=t$

$\left(2x-\dfrac{1}{x^2}\right)dx=dt$

$\int \dfrac{dt}{t}=\ell n(t)+C$

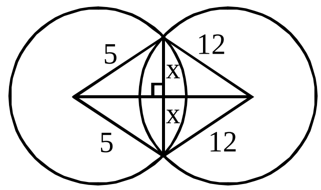
$=\ell n\left(x^2+\dfrac{1}{x}\right)+C$

29. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\dfrac{60}{13}$ (2) $\dfrac{120}{13}$ (3) $\dfrac{13}{2}$ (4) $\dfrac{13}{5}$

Official Ans. by NTA (2)

Sol.



Let length of common chord = 2x

$\sqrt{25-x^2}+\sqrt{144-x^2}=13$

after solving

$x=\dfrac{12\times 5}{13}$

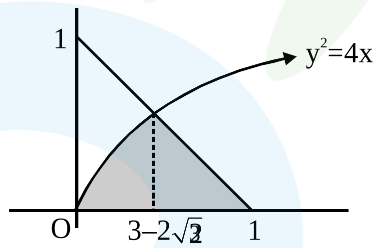
$2x=\dfrac{120}{13}$

30. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2}+b$, then a – b is equal to :

(1) $\dfrac{8}{3}$ (2) $\dfrac{10}{3}$ (3) 6 (4) $-\dfrac{2}{3}$

Official Ans. by NTA (3)

Sol. $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$



$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx+\dfrac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-(3-2\sqrt{2})\right)$

$=\dfrac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2}+\dfrac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$

$=\dfrac{8\sqrt{2}}{3}+\left(-\dfrac{10}{3}\right)$

$a=\dfrac{8}{3}$, $b=-\dfrac{10}{3}$

a – b = 6