

FINAL JEE-MAIN EXAMINATION – APRIL, 2019

(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

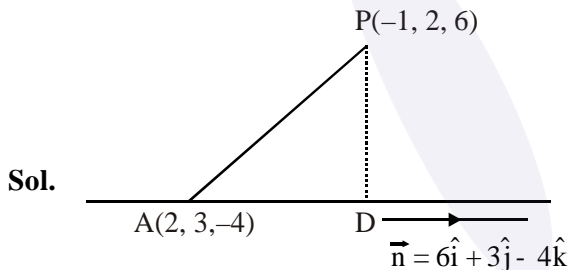
MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is :

- (1) 7 (2) $4\sqrt{3}$
 (3) $2\sqrt{13}$ (4) 6

Official Ans. by NTA (1)



$$AD = \frac{|\vec{AP} \cdot \vec{n}|}{|\vec{n}|} = \sqrt{61}$$

$$\Rightarrow PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$$

2. If both the mean and the standard deviation of 50 observations x_1, x_2, \dots, x_{50} are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, \dots, (x_{50} - 4)^2$ is :

- (1) 525 (2) 380
 (3) 480 (4) 400

Official Ans. by NTA (4)

Sol. Mean (μ) = $\frac{\sum x_i}{50} = 16$

standard deviation (σ) = $\sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$$

\Rightarrow New mean

$$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8 \sum x_i}{50}$$

$$= (256) \times 2 + 16 - 8 \times 16 = 400$$

3. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

- (1) (2, 0, 1) (2) (4, 0, -1)
 (3) (-1, 0, 4) (4) (1, 0, 2)

Official Ans. by NTA (1)

Sol. Let point P on the line is $(2\lambda + 1, -\lambda - 1, \lambda)$ foot of perpendicular Q is given by

$$\frac{x - 2\lambda - 1}{1} = \frac{y + \lambda + 1}{1} = \frac{z - \lambda}{1} = \frac{-(2\lambda - 3)}{3}$$

\therefore Q lies on $x + y + z = 3$ & $x - y + z = 3$
 $\Rightarrow x + z = 3$ & $y = 0$

$$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda + 3}{3} \Rightarrow \lambda = 0$$

$$\Rightarrow Q \text{ is } (2, 0, 1)$$

4. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

- (1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{68}{15}$ (4) $\frac{34}{15}$

Official Ans. by NTA (3)

Sol. $3x^2 + 5y^2 = 32$

$$\left. \frac{dy}{dx} \right|_{(2,2)} = -\frac{3}{5}$$

Tangent : $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area is = $\frac{1}{2}(QR) \times 2 = QR = \frac{68}{15}$.

Final JEE-Main Exam April, 2019/10-04-2019/Evening Session

5. Let λ be a real number for which the system of linear equations
 $x + y + z = 6$
 $4x + \lambda y - \lambda z = \lambda - 2$
 $3x + 2y - 4z = -5$
 has infinitely many solutions. Then λ is a root of the quadratic equation.

- (1) $\lambda^2 - 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$
 (3) $\lambda^2 + 3\lambda - 4 = 0$ (4) $\lambda^2 + \lambda - 6 = 0$

Official Ans. by NTA (2)

Sol. $D = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

6. The smallest natural number n , such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$

is ${}^n C_{23}$, is :

- (1) 35 (2) 38
 (3) 23 (4) 58

Official Ans. by NTA (2)

Sol. $T_r = \sum_{r=0}^n {}^n C_r x^{2n-2r} \cdot x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$
 for $r = 15$, $n = 38$
 smallest value of n is 38.

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm³/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

- (1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

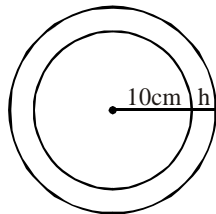
Official Ans. by NTA (3)

Sol. $V = \frac{4}{3}\pi((10+h)^3 - 10^3)$

$$\frac{dV}{dt} = 4\pi(10+h)^2 \frac{dh}{dt}$$

$$-50 = 4\pi(10+5)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{1}{18} \text{ cm/min}$$



8. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

- (1) $\left(-\frac{5}{3}, 0\right)$ (2) (5, 0)

- (3) (-5, 0) (4) $\left(\frac{5}{3}, 0\right)$

Official Ans. by NTA (3)

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a = 3, b = 4$ & $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

corresponding focus will be $(-ae, 0)$ i.e., $(-5, 0)$.

9. The sum $1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + \dots$

$$+ \frac{1^3 + 2^3 + 3^3 + \dots + 15^3}{1+2+3+\dots+15} - \frac{1}{2}(1+2+3+\dots+15)]$$

- (1) 1240 (2) 1860
 (3) 660 (4) 620

Official Ans. by NTA (4)

Sol. Sum = $\sum_{n=1}^{15} \frac{1^3 + 2^3 + \dots + n^3}{1+2+\dots+n} - \frac{1}{2} \cdot \frac{15 \cdot 16}{2}$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2 - (n-1))}{6} - 60$$

$$= \frac{15 \cdot 16 \cdot 17}{6} - 60 = 620$$

10. If the line $ax + y = c$, touches both the curves

$x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to :

- (1) $1/2$ (2) 2
 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Official Ans. by NTA (3)

Sol. Tangent to $y^2 = 4\sqrt{2}x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$$\Rightarrow \left| \frac{\sqrt{2}/m}{\sqrt{1+m^2}} \right| = 1 \Rightarrow m = \pm 1$$

\Rightarrow Tangent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$
 compare with $y = -ax + C$

$$\Rightarrow a = \pm 1 \text{ \& } C = \pm\sqrt{2}$$

11. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \leq x \leq 1, -2 \leq y \leq 2, x \leq \frac{y}{2}$,

then for all $x, y, 4x^2 - 4xy \cos \alpha + y^2$ is equal to

- (1) $4 \sin^2 \alpha - 2x^2y^2$ (2) $4 \cos^2 \alpha + 2x^2y^2$
 (3) $4 \sin^2 \alpha$ (4) $2 \sin^2 \alpha$

Official Ans. by NTA (3)

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos \alpha$$

$$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2} \sqrt{1-\frac{y^2}{4}} = \cos \alpha$$

$$\Rightarrow \left(\cos \alpha - \frac{xy}{2} \right)^2 = (1-x^2) \left(1 - \frac{y^2}{4} \right)$$

$$x^2 + \frac{y^2}{4} - xy \cos \alpha = 1 - \cos^2 \alpha = \sin^2 \alpha$$

12. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to :

- (1) $-\frac{5}{2}$ (2) 1
 (3) $-\frac{1}{2}$ (4) -1

Official Ans. by NTA (1)

Sol. Let $x^2 = t$ $2x dx = dt$

$$\Rightarrow \frac{1}{2} \int t^2 \cdot e^{-t} dt = \frac{1}{2} \left[-t^2 \cdot e^{-t} + \int 2t \cdot e^{-t} dt \right]$$

$$= \frac{1}{2} \left(-t^2 \cdot e^{-t} \right) + \left(-t \cdot e^{-t} + \int 1 \cdot e^{-t} dt \right)$$

$$= -\frac{t^2 e^{-t}}{2} - t e^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1 \right) e^{-t}$$

$$= \left(-\frac{x^4}{2} - x^2 - 1 \right) e^{-x^2} + C$$

$$g(x) = -1 - x^2 - \frac{x^4}{2} + k e^{x^2}$$

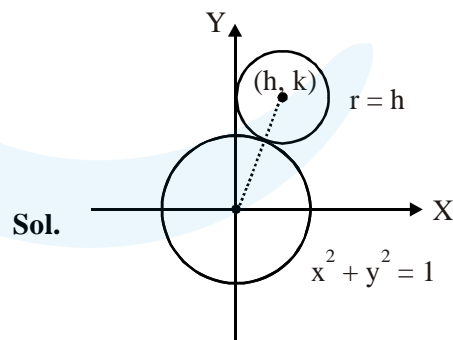
for $k = 0$

$$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

13. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

- (1) $y = \sqrt{1+4x}, x \geq 0$
 (2) $x = \sqrt{1+4y}, y \geq 0$
 (3) $x = \sqrt{1+2y}, y \geq 0$
 (4) $y = \sqrt{1+2x}, x \geq 0$

Official Ans. by NTA (4)



$$\sqrt{h^2 + k^2} = |h| + 1$$

$$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$$

$$\Rightarrow y^2 = 1 + 2x$$

$$\Rightarrow y = \sqrt{1+2x}; x \geq 0.$$

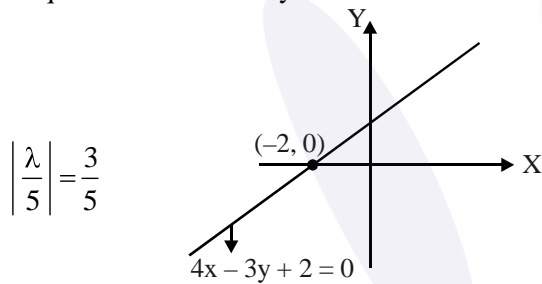
14. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin.

Then which one of the following points lies on any of these lines ?

- (1) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$
 (3) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$ (4) $\left(\frac{1}{4}, -\frac{1}{3}\right)$

Official Ans. by NTA (1)

Sol. Required line is $4x - 3y + \lambda = 0$



$$\Rightarrow \lambda = \pm 3.$$

So, required equation of line is $4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$

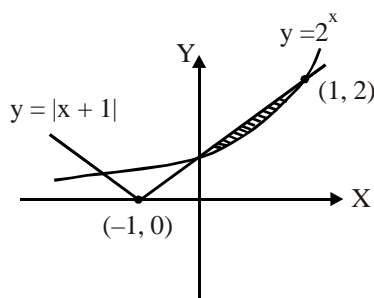
15. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

- (1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$
 (3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

Sol. Required Area

$$\int_0^1 ((x+1) - 2^x) dx$$



$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2} \right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2} \right) - \left(0 + 0 - \frac{1}{\ln 2} \right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

16. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$

and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

- (1) 15 (2) 5
 (3) 13 (4) 9

Official Ans. by NTA (3)

Sol. $4x - 2y + 4z + 6 = 0$

$$\frac{|\lambda - 6|}{\sqrt{16 + 4 + 16}} = \frac{|\lambda - 6|}{6} = \frac{1}{3}$$

$$|\lambda - 6| = 2$$

$$\lambda = 8, 4$$

$$\frac{|\mu - 3|}{\sqrt{4 + 4 + 1}} = \frac{2}{3}$$

$$|\mu - 3| = 2$$

$$\mu = 5, 1$$

\therefore Maximum value of $(\mu + \lambda) = 13$.

17. If z and w are two complex numbers such that

$$|zw| = 1 \text{ and } \arg(z) - \arg(w) = \frac{\pi}{2}, \text{ then :}$$

- (1) $\bar{z}w = i$ (2) $\bar{z}w = -i$
 (3) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (4) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

Official Ans. by NTA (2)

Sol. $|z| \cdot |w| = 1$ $z = re^{i(\theta + \pi/2)}$ and $w = \frac{1}{r} e^{i\theta}$

$$\bar{z} \cdot w = e^{-i(\theta + \pi/2)} \cdot e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$z \cdot \bar{w} = e^{i(\theta + \pi/2)} \cdot e^{-i\theta} = e^{i(\pi/2)} = i$$

18. Let a, b and c be in G. P. with common ratio r, where

$a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

Official Ans. by NTA (2)

Sol. $b = ar$

$c = ar^2$

3a, 7b and 15c are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r-1)(5r-3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$.

Only acceptable value is $r = \frac{1}{3}$, because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore c, d = 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

\therefore 4th term = $15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

19. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ equal to :

(1) $3^{7/6} - 3^{5/6}$

(2) $3^{5/3} - 3^{1/3}$

(3) $3^{4/3} - 3^{1/3}$

(4) $3^{5/6} - 3^{2/3}$

Official Ans. by NTA (1)

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x \cdot \sin x} dx$

$= \int \frac{\tan^{2/3} x \cdot \sec^2 x \cdot dx}{\tan^2 x}$

$= \int \frac{\sec^2 x}{\tan^{4/3} x} \cdot dx \quad \{\tan x = t, \sec^2 x dx = dt\}$

$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3(t^{-1/3})$

$\Rightarrow I = -3 \tan(x)^{-1/3}$

$\Rightarrow I = \frac{3}{(\tan x)^{1/3}} \Big|_{\pi/6}^{\pi/3} = -3 \left[\frac{1}{(\sqrt{3})^{1/3}} - (\sqrt{3})^{1/3} \right]$

$= 3 \left(3^{1/3} - \frac{1}{3^{1/6}} \right) = 3^{7/6} - 3^{5/6}$

20. Let $y = y(x)$ be the solution of the differential

equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$,

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$. Then :

(1) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

I.F = $e^{\int \tan x dx} = e^{\ln \sec x} = \sec x$

$\therefore y \cdot \sec x = \int (2x + x^2 \tan x) \sec x dx$

$= \int 2x \sec x dx + \int x^2 (\sec x \tan x) dx$

$y \sec x = x^2 \sec x + \lambda$

$\Rightarrow y = x^2 + \lambda \cos x$

$y(0) = 0 + \lambda = 1 \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

21. Let a_1, a_2, a_3, \dots be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1 a_4 a_5$, is :

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Official Ans. by NTA (2)

Sol. Let a is first term and d is common difference then, $a + 5d = 2$ (given) ... (1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

$f'(d) = 0 \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0$ at $d = 8/5$

$\Rightarrow d = \frac{8}{5}$

22. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq. cm) of this triangle is :

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

Official Ans. by NTA (3)

Sol. $\angle B = \frac{\pi}{3}$, by sine Rule

$\sin A = \frac{1}{2}$

$\Rightarrow A = 30^\circ, a = 2, b = 2\sqrt{3}, c = 4$

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$ sq. cm

23. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

Official Ans. by NTA (3)

Sol. $1 - \left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100}$

$\Rightarrow n = 7.$

24. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :

(1) 210 (2) 190

(3) 170 (4) 180

Official Ans. by NTA (3)

Sol. Total cases = number of diagonals
 $= {}^{20}C_2 - 20 = 170$

25. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0, \text{ is equal to :}$$

- (1) 6 (2) 1
(3) 0 (4) -4

Official Ans. by NTA (3)

Sol. By expansion, we get

$$\begin{aligned} & -5x^3 + 30x - 30 + 5x = 0 \\ \Rightarrow & -5x^3 + 35x - 30 = 0 \\ \Rightarrow & x^3 - 7x + 6 = 0, \text{ All roots are real} \end{aligned}$$

So, sum of roots = 0

26. Let $f(x) = \log_e(\sin x)$, ($0 < x < \pi$) and $g(x) = \sin^{-1}(e^{-x})$, ($x \geq 0$). If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

- (1) $a\alpha^2 - b\alpha - a = 0$
(2) $a\alpha^2 + b\alpha - a = -2\alpha^2$
(3) $a\alpha^2 + b\alpha + a = 0$
(4) $a\alpha^2 - b\alpha - a = 1$

Official Ans. by NTA (4)

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$
 $(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

27. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in \mathbb{R}$,

($x \neq \pm\sqrt{3}$), at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

- (1) $|6\alpha + 2\beta| = 19$
(2) $|2\alpha + 6\beta| = 11$
(3) $|6\alpha + 2\beta| = 9$
(4) $|2\alpha + 6\beta| = 19$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx}\bigg|_{(\alpha, \beta)} = \frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2}$

Given that :

$$\frac{-\alpha^2 - 3}{(\alpha^2 - 3)^2} = -\frac{1}{3}$$

$$\Rightarrow \alpha = 0, \pm 3 \quad (\alpha \neq 0)$$

$$\Rightarrow \beta = \pm \frac{1}{2}. \quad (\beta \neq 0)$$

$$|6\alpha + 2\beta| = 19$$

28. The number of real roots of the equation

$$5 + |2^x - 1| = 2^x(2^x - 2) \text{ is :}$$

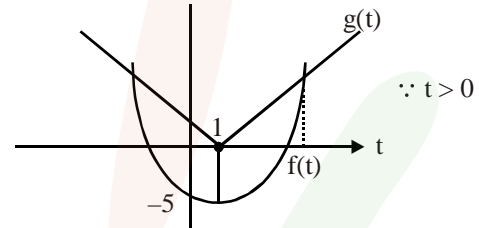
- (1) 2 (2) 3
(3) 4 (4) 1

Official Ans. by NTA (4)

Sol. Let $2^x = t$

$$\begin{aligned} 5 + |t - 1| &= t^2 - 2t \\ \Rightarrow |t - 1| &= (t^2 - 2t - 5) \end{aligned}$$

From the graph



So, number of real root is 1.

29. If $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to :-

- (1) -7 (2) -4
(3) 5 (4) 1

Official Ans. by NTA (1)

Sol. $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$

$$1 - a + b = 0 \quad \dots(i)$$

$$2 - a = 5 \quad \dots(ii)$$

$$\Rightarrow a + b = -7.$$

30. The negation of the boolean expression

$\sim s \vee (\sim r \wedge s)$ is equivalent to :

- (1) r (2) $s \wedge r$
(3) $s \vee r$ (4) $\sim s \wedge \sim r$

Official Ans. by NTA (2)

Sol. $\sim(\sim s \vee (\sim r \wedge s))$

$$s \wedge (r \vee \sim s)$$

$$(s \wedge r) \vee (s \wedge \sim s)$$

$$(s \wedge r) \vee (c)$$

$$(s \wedge r)$$