FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2 (3) 3.0 (4) 2.5

Official Ans. by NTA (1)

Sol. $\sum f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$$

2. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

(2) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 - \Delta_2 = -2x^3$

(4) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

Official Ans. by NTA (2)

Sol. $\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$

and $\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$

So $\Delta_1 + \Delta_2 = -2x^3$

3. If $\lim\limits_{x\to 1} \frac{x^4-1}{x-1} = \lim\limits_{x\to k} \frac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 1} \frac{x^4-1}{x-1} = \lim\limits_{x\to k} \frac{x^3-k^3}{x^2-k^2}$

$\Rightarrow \lim\limits_{x\to 1} (x+1)(x^2+1) = \frac{k^2+k^2+k^2}{2k}$

$\Rightarrow k = 8/3$

4. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 12 (2) 10 (3) 9 (4) 7

Official Ans. by NTA (2)

Sol. $x + 3y + \lambda z - \mu = p(x + y + z - 5) + q(x + 2y + 2z - 6)$

on comparing the coefficient;

$p + q = 1$ and $p + 2q = 3$

$\Rightarrow (p,q) = (-1,2)$

Hence $x + 3y + \lambda z - \mu = x + 3y + 3z - 7$

$\Rightarrow \lambda = 3, \mu = 7$

5. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(1) exactly two values of K

(2) exactly one value of K

(3) no value of K.

(4) infinitely many values of K

Official Ans. by NTA (3)

Sol. Equation of common chord

$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$ ....(1)

and given line is $4x + 5y - k = 0$ .....(2)

On comparing (1) & (2), we get

$$k = \frac{1}{10} = \frac{k+\frac{1}{2}}{-k}$$

$\Rightarrow$ No real value of k exist

6. Le $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

(1) $f(g(S)) \neq f(S)$ (2) $f(g(S)) = S$

(3) $g(f(S)) = g(S)$ (4) $g(f(S)) \neq S$

Official Ans. by NTA (3)

Sol. $g(S) = [-2, 2]$

So, $f(g(S)) = [0, 4] = S$

And $f(S) = [0, 16] \Rightarrow f(g(S) \neq f(S)$

Also, $g(f(S)) = [-4, 4] \neq g(S)$

So, $g(f(S) \neq S$

7. Let $f(x) = e^x - x$ and $g(x)\ x^2 - x$, $\forall x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$

(3) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$ (4) $[0,\infty)$

Official Ans. by NTA (2)

Sol. $h(x) = f(g(x))$

$\Rightarrow h'(x) = f'(g(x)).\ g'(x)$ and $f'(x) = e^x - 1$

$\Rightarrow h'(x) = (e^{g(x)} - 1)\ g'(x)$

$\Rightarrow h'(x) = \left(e^{x^2-x} - 1\right)(2x - 1) \geq 0$

Case-I $e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1,\infty)$ ......(1)

Case-II $e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow x \in \left[0,\frac{1}{2}\right]$.....(2)

Hence, $x \in \left[0,\frac{1}{2}\right] \cup [1,\infty)$

8. Which one of the following Boolean expressions is a tautology ?

(1) $(P \vee q) \wedge (\sim p \vee \sim q)$ (2) $(P \wedge q) \vee (p \wedge \sim q)$

(3) $(P \vee q) \wedge (p \vee \sim q)$ (4) $(P \vee q) \vee (p \vee \sim q)$

Official Ans. by NTA (4)

Sol. (1) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim (p \wedge q) \rightarrow$ Not tautology (Take both p and q as T)

(2) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$

(3) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$

(4) $(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$

9. All the pairs (x, y) that satisfy the inequality

$2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the

eauation.

(1) $\sin x = |\sin y|$ (2) $\sin x = 2 \sin y$

(3) $2|\sin x| = 3\sin y$ (4) $2\sin x = \sin y$

Official Ans. by NTA (1)

Sol. $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot 4^{-\sin^2 y} \leq 1$

$\Rightarrow 2^{\sqrt{(\sin x - 1)^2 + 4}} \leq 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \leq 2\sin^2 y$

$\Rightarrow \sin x = 1$ and $|\sin y| = 1$

10. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(1) 36 (2) 60 (3) 48 (4) 72

Official Ans. by NTA (2)

Sol. Sum of given digits 0, 1, 2, 5, 7, 9 is 24.

Let the six digit number be abcdef and to be divisible by 11

so $|(a + c + e) - (b + d + f)|$ is multiple of 11.

Hence only possibility is $a + c + e = 12 = b + d + f$

Case-I $\{a, c, e\} = \{9, 2, 1\}$ & $\{b, d, f\} = \{7, 5, 0\}$

So, Number of numbers $= 3! \times 3! = 36$

Case-II $\{a,c,e\} = \{7,5,0\}$ and $\{b,d,f\} = \{9,2,1\}$

So, Number of numbers $2 \times 2! \times 3! = 24$

Total = 60

11. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$ (3) $\frac{1}{10}$ (4) $\frac{1}{12}$

Official Ans. by NTA (1)

Sol. P(B) = P(G) = 1/2

Required Proballity =

$$\frac{\text{all 4 girls}}{(\text{all 4girls})+(\text{exactly 3 girls}+1\text{boy})+(\text{exactly 2girls}+2\text{boys})}$$

$$=\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4+{}^4C_3\left(\frac{1}{2}\right)^4+{}^4C_2\left(\frac{1}{2}\right)^4}=\frac{1}{11}$$

12. The sum

$$\frac{3\times1^3}{1^2}+\frac{5\times\left(1^3+2^3\right)}{1^2+2^2}+\frac{7\times\left(1^3+2^3+3^3\right)}{1^2+2^2+3^2}+.......$$

(1) 660 (2) 620 (3) 680 (4) 600

Official Ans. by NTA (1)

Sol. $T_n=\frac{(3+(n-1)\times2)\left(1^3+2^3+...+n^3\right)}{\left(1^2+2^2+...+n^2\right)}$

$=\frac{3}{2}n(n+1) \quad =\frac{n(n+1)(n+2)-(n-1)n(n+1)}{2}$

$\Rightarrow S_n=\frac{n(n+1)(n+2)}{2}$

$\Rightarrow S_{10}=660$

13. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4,-2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 35 = 0$

(2) $4e^4 + 8e^2 - 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

Official Ans. by NTA (1)

Sol. Hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{a}{e}=\frac{4}{\sqrt{5}}$ and $\frac{16}{a^2}-\frac{12}{b^2}=1$

$a^2=\frac{16}{5}e^2$ ....(1) and $\frac{16}{a^2}-\frac{12}{a^2(e^2-1)}=1$ ....(2)

From (1) & (2)

$16\left(\frac{5}{16e^2}\right)-\frac{12}{(e^2-1)}\left(\frac{5}{16e^2}\right)=1$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

14. If $f(x)$ = $\begin{cases}\frac{\sin(p+1)+\sin x}{x}, & x<0\\ q, & x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$

is continuous at x = 0, then the ordered pair (p,q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$

(3) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

Official Ans. by NTA (4)

Sol. $\text{RHL}=\lim_{x\to0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}}=\lim_{x\to0^+}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$

$\text{LHL}=\lim_{x\to0}\frac{\sin(p+1)x+\sin x}{x}=(p+1)+1=p+2$

for continuity LHL = RHL = f(0)

$\Rightarrow (p,q)=\left(\frac{-3}{2},\frac{1}{2}\right)$

15. If y = y (x) is the solution of the differential equation

$\frac{dy}{dx}$ = (tanx –y) sec² x, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that

y(0) = 0, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $2+\frac{1}{e}$ (2) $\frac{1}{2}-e$ (3) e – 2 (4) $\frac{1}{2}-e$

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx}$ = (tan x –y) sec²x

Now, put tanx = t $\Rightarrow \frac{dt}{dx} = \sec^2 x$

So $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2$

16. If the line x – 2y = 12 is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

(1) 9 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 5

Official Ans. by NTA (1)

Sol. Tangent at $\left(3, -\frac{9}{2}\right)$

$$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$$

Comparing this with x – 2y = 12

$$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$$

we get a = 6 and b = $3\sqrt{3}$

L(LR) = $\frac{2b^2}{a} = 9$

17. The value of $\int_0^{2\pi} [\sin 2x(1+\cos 3x)]\,dx$, where [t] denotes the greatest integer function, is :

(1) –2π (2) π (3) –π (4) 2π

Official Ans. by NTA (3)

Sol. $I = \int_0^{2\pi} \left[\sin 2x\left(1+\cos 3x\right)\right] dx$

$$I = \int_0^{\pi} \left(\left[\sin 2x + \sin 2x \cos 3x\right] + \left[-\sin 2x - \sin 2x \cos 3x\right]\right) dx$$

$$= \int_0^{\pi} -dx = -\pi$$

18. The region represented by |x–y| ≤ 2 and |x+y| ≤ 2 is bounded by a :

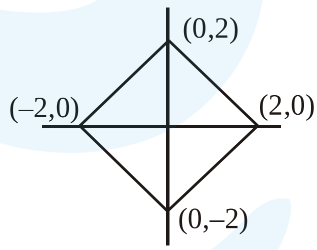
(1) square of side length $2\sqrt{2}$ units (2) rhombus of side length 2 units

(3) square of area 16 sq, units

(4) rhombus of area 8 $\sqrt{2}$ sq. units

Official Ans. by NTA (1)

Sol. |x–y| ≤ 2 and |x + y| ≤ 2



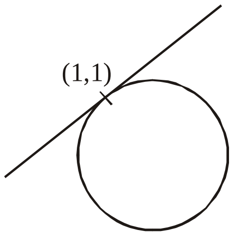
Square whose side is $2\sqrt{2}$

19. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

(1) $3\sqrt{2}$ (2) 3 (3) $2\sqrt{2}$ (4) 2

Official Ans. by NTA (1)

ALLEN Ans. (3)

Sol. 

Equation of circle can be written as $(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

It passes through (1, –3)

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

So $(x-1)^2 + (y-1)^2 - 4(x-y) = 0$

$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$

(correct key is 3)

20. Let A(3, 0, –1), B (2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos(\angle GOA)$ (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{30}}$ (2) $\frac{1}{6\sqrt{10}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{2\sqrt{15}}$

Official Ans. by NTA (3)

Sol. G is the centroid of Δ ABC

$G \equiv (2,4,2)$

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$\cos(\angle GOA) = \frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$

21. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at x = c, g is :

(1) differentiable if $f'(c) = 0$

(2) not differentiable

(3) differentiable if $f'(c) \neq 0$

(4) not differentiable if $f'(c) = 0$

Official Ans. by NTA (1)

Sol. $g'(c) = \lim\limits_{h\to 0} \frac{|f(c+h)| - |f(c)|}{h}$

$= \lim\limits_{h\to 0} \frac{|f(c+h)|}{h} = \lim\limits_{h\to 0} \frac{|f(c+h) - f(c)|}{h}$

$= \lim\limits_{h\to 0} \left|\frac{f(c+h)-f(c)}{h}\right| \frac{|h|}{h}$

$= \lim\limits_{h\to 0} |f'(c)| \frac{|h|}{h} = 0$, if $f'(c) = 0$

i.e., g(x) is differentiable at x = c, if $f'(c) = 0$

22. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$ is equal to :

(1) $\frac{2^6}{(\sin\theta + 8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta - 8)^6}$

(3) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$

Official Ans. by NTA (4)

Sol. $\frac{\alpha^{12} + \beta^{12}}{\left(\frac{1}{\alpha^{12}} + \frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$= \frac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2 - 4\alpha\beta\right]^{12}} = \left[\frac{\alpha\beta}{(\alpha+\beta)^2 - 4\alpha\beta}\right]^{12}$

$= \left(\frac{-2\sin\theta}{\sin^2\theta + 8\sin\theta}\right)^{12} = \frac{2^{12}}{(\sin\theta+8)^{12}}$

23. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) –1 (2) 2 (3) –2 (4) 1

Official Ans. by NTA (1)

Sol. One of the point on line is P(0, 1, –1) and given point is Q(β, 0, β).

So, $\overrightarrow{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $\beta^2 + 1 + (\beta+1)^2 - \frac{(\beta-\beta-1)^2}{2} = \frac{3}{2}$

$\Rightarrow 2\beta^2 + 2\beta = 0$

$\Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

24. If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration, then :

(1) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

Official Ans. by NTA (4)

Sol. $\int \frac{dx}{\left((x-1)^2+9\right)^2} = \frac{1}{27}\int \cos^2\theta\, d\theta$ (Put x–1= 3tanθ)

$= \frac{1}{54}\int (1+\cos 2\theta)d\theta = \frac{1}{54}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$

$= \frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right)+C$

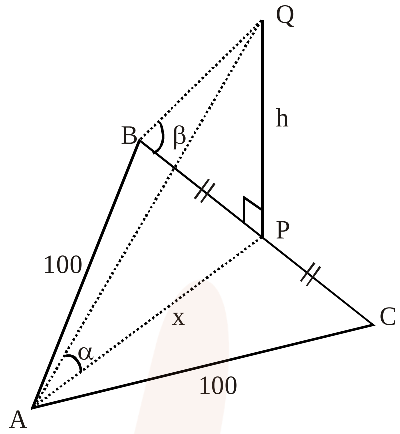
25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is :

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$ (3) 20 (4) 25

Official Ans. by NTA (3)

Sol. $\cot\alpha = 3\sqrt{2}$

& $\text{cosec}\,\beta = 2\sqrt{2}$



So, $\frac{x}{h} = 3\sqrt{2}$ ...(i)

And $\frac{h}{\sqrt{10^4-x^2}} = \frac{1}{\sqrt{7}}$ ...(ii)

So, from (i) & (ii)

$\Rightarrow \frac{h}{\sqrt{10^4-18h^2}} = \frac{1}{\sqrt{7}}$

$\Rightarrow 25h^2 = 100 \times 100$

$\Rightarrow h = 20.$

26. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98 (3) 76 (4) 64

Official Ans. by NTA (3)

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$

$\Rightarrow a_1 + a_{16} = 38$

So, $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 = 76$

27. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+\ldots\ldots+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$

(2) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$

(4) $\frac{4}{3}(2)^{3/4}$

Official Ans. by NTA (3)

Sol. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3}$

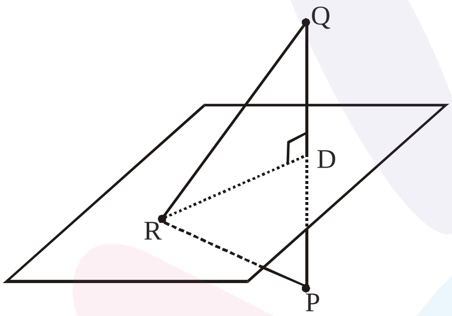
$= \int_0^1 (1+x)^{1/3}dx = \frac{3}{4}\left(2^{4/3}-1\right)$

28. If Q(0, –1, –3) is the image of the point P in the plane 3x –y + 4z = 2 and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

Official Ans. by NTA (4)

Sol. R lies on the plane.



DQ = $\frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

$\Rightarrow$ PQ = $\sqrt{26}$

Now, RQ = $\sqrt{9+1}=\sqrt{10}$

$\Rightarrow$ RD = $\sqrt{10-\frac{13}{2}} = \sqrt{\frac{7}{2}}$

Hence, ar(ΔPQR) = $\frac{1}{2}\times\sqrt{26}\times\sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$.

29. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 – 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (28, 315) (2) (–54, 315)
(3) (–21, 714) (4) (24, 861)

Official Ans. by NTA (1)

Sol. Coefiicient of x^2 = ${}^{15}C_2\times 9-3a\left({}^{15}C_1\right)+b=0$

$\Rightarrow -45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)

Also, $-27\times{}^{15}C_3 + 9a\times{}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24a + 672 = 0$

$\Rightarrow a = 28$

So, b = 315

30. If a > 0 and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{3}{5}-\frac{1}{5}i$ (2) $-\frac{1}{5}+\frac{3}{5}i$
(3) $-\frac{1}{5}-\frac{3}{5}i$ (4) $\frac{1}{5}-\frac{3}{5}i$

Official Ans. by NTA (3)

Sol. Given a > 0

$z = \frac{(1+i)^2}{a-i}=\frac{2i(a+i)}{a^2+1}$

Also $|z| = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{a^2+1}}=\sqrt{\frac{2}{5}}\Rightarrow a=3$

So $\bar{z}=\frac{-2i(3-i)}{10}=\frac{-1-3i}{5}$