FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Tuesday 09th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(3) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (4) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

Official Ans. by NTA (3)

Sol. $\vec{\alpha} = 3\hat{i} + \hat{j}$

$\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\vec{\beta}_1 = \lambda\left(3\hat{i} + \hat{j}\right), \vec{\beta}_2 = \lambda\left(3\hat{i} + \hat{j}\right) - 2\hat{i} + j - 3\hat{k}$

$\vec{\beta}_2 . \vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

Aliter :

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \Rightarrow \vec{\beta}.\hat{\alpha} = \vec{\beta}_1.\hat{\alpha} = \left|\vec{\beta}_1\right|$

$\Rightarrow \vec{\beta}_1 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha}$

$\Rightarrow \vec{\beta}_2 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} - \vec{\beta}$

$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = -\left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} \times \vec{\beta}$

$= \frac{-5}{10}\left(3\hat{i} + \hat{j}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

2. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is

(1) $p \wedge q$ (2) $p \leftrightarrow q$

(3) $\sim p \vee \sim q$ (4) $\sim p \wedge \sim q$

Official Ans. by NTA (4)

Sol. $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q)$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

3. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is

(1) $\frac{\pi - 2}{4}$ (2) $\frac{\pi - 2}{8}$ (3) $\frac{\pi - 1}{4}$ (4) $\frac{\pi - 1}{2}$

Official Ans. by NTA (3)

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$

$= \int_0^{\pi/4} \left(1 - \sin x \cos x\right) dx$

$$=\left(x-\frac{\sin^2 x}{2}\right)_0^{\pi/4}$$

$$=\frac{\pi}{4}-\frac{1}{4}$$

$$=\frac{\pi-1}{4}$$

4. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set S = {x ∈ R : $f(x) = f(0)$}

Contains exactly :

(1) four irrational numbers.

(2) two irrational and one rational number.

(3) four rational numbers.

(4) two irrational and two rational numbes.

Official Ans. by NTA (2)

Sol. $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$$\Rightarrow f(x)=\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu$$

Now $f(x) = f(0)$

$$\Rightarrow \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu=\mu$$

$\Rightarrow x = 0, 0, \pm\sqrt{2}$

Two irrational and one rational number

5. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

Official Ans. by NTA (2)

Sol. $$S.D=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\bar{x}=\frac{\Sigma x}{4}=\frac{-1+0+1+k}{4}=\frac{k}{4}$$

Now $$\sqrt{5}=\sqrt{\frac{\left(-1-\frac{k}{4}\right)^2+\left(0-\frac{k}{4}\right)^2+\left(1-\frac{k}{4}\right)^2+\left(k-\frac{k}{4}\right)^2}{4}}$$

$$\Rightarrow 5\times 4=2\left(1+\frac{k}{16}\right)^2+\frac{5k^2}{8}$$

$$\Rightarrow 18=\frac{3k^2}{4}$$

$\Rightarrow k^2 = 24$

$\Rightarrow k=2\sqrt{6}$

6. All the points in the set $S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}\left(i=\sqrt{-1}\right)$ lie on a

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is –1

(4) circle whose radius is $\sqrt{2}$.

Official Ans. by NTA (1)

Sol. Let $\frac{\alpha+i}{\alpha-i}=z$

$$\Rightarrow \frac{|\alpha+i|}{|\alpha-i|}=|z|$$

$\Rightarrow 1 = |z|$

$\Rightarrow$ circle of radius 1

7. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, $f(1)$) and (–1, $f(-1)$), then S is equal to :

(1) $\left\{-\frac{1}{3},-1\right\}$ (2) $\left\{\frac{1}{3},-1\right\}$

(3) $\left\{-\frac{1}{3},1\right\}$ (4) $\left\{\frac{1}{3},1\right\}$

Official Ans. by NTA (3)

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$$m=\frac{f(1)-f(-1)}{1+1}=\frac{-2-0}{2}=-1$$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$\Rightarrow x = 1, -\frac{1}{3}$

8. Let $f(x) = 15 - |x - 10|; x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) {5,10,15,20} (2) {10,15}

(3) {5,10,15} (4) {10}

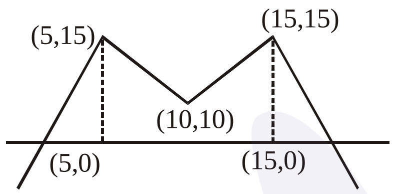
Official Ans. by NTA (3)

Sol. $f(x) = 15 - |x - 10|, x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



x = 5, 10, 15 are points of non differentiability

Aliter :

At x = 10 $f(x)$ is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow x = 5, 15$

$\therefore$ non differentiability points are {5, 10, 15}

9. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $q^2 + 4p + 14 = 0$ (2) $p^2 - 4q - 12 = 0$

(3) $q^2 - 4p - 16 = 0$ (4) $p^2 - 4q + 12 = 0$

Official Ans. by NTA (2)

ALLEN Ans. (2) or (Bonus)

Sol. In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$$x^2 - (\alpha + 2 - \sqrt{3})x + \alpha.\left(2-\sqrt{3}\right) = 0$$

Now, we can have none or any of the options can be correct depending upon 'α'

Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2+\sqrt{3}$

$$\Rightarrow p = -\left(2+\sqrt{3}-2-\sqrt{3}\right) = -4$$

and $q = \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (2) is correct

10. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is

(1) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

(3) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

Official Ans. by NTA (3)

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$

$\Rightarrow 1 + \sin 2\theta = \frac{1}{4}$

$\Rightarrow \sin 2\theta = -\frac{3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$\Rightarrow m = \frac{-4\pm\sqrt{7}}{1-7}$

$$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$$

11. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m = n = 78 (2) n = m – 8

(3) m + n = 68 (4) m = n = 68

Official Ans. by NTA (1)

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13\times 12}{2} = 78$$

12. If the fourth term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8 (2) 8^2 (3) 8^{-2} (4) 8^3

Official Ans. by NTA (2)

Sol. $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 .\left(x^{\log_8 x}\right)^3$

$20\times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$

$8^6 = x^{\log_2 x} - 3$

$2^{18} = x^{\log_2 x - 3}$

$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow (t-6)(t+3) = 0$

$\Rightarrow t = 6, -3$

$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$

$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

13. The solution of the differential equation $x\frac{dy}{dx}+2y = x^2$ (x ≠ 0) with y(1) = 1, is

(1) $y = \frac{x^3}{5}+\frac{1}{5x^2}$ (2) $y = \frac{4}{5}x^3+\frac{1}{5x^2}$

(3) $y = \frac{3}{4}x^2+\frac{1}{4x^2}$ (4) $y = \frac{x^2}{4}+\frac{3}{4x^2}$

Official Ans. by NTA (4)

Sol. $x\frac{dy}{dx}+2y = x^2 : y(1) = 1$

$\frac{dy}{dx}+\left(\frac{2}{x}\right)y = x$ (LDE in y)

IF $= e^{\int \frac{2}{x}dx} = e^{2\ell nx} = x^2$

$y.(x^2) = \int x.x^2 dx = \frac{x^4}{4}+C$

y(1) =1

$1 = \frac{1}{4}+C \Rightarrow C = 1-\frac{1}{4} = \frac{3}{4}$

$yx^2 = \frac{x^4}{4}+\frac{3}{4}$

$y = \frac{x^2}{4}+\frac{3}{4x^2}$

14. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point

(1) $(-\sqrt{2},1,-4)$ (2) $(\sqrt{2},1,4)$

(3) $(\sqrt{2},-1,4)$ (4) $(-\sqrt{2},-1,-4)$

Official Ans. by NTA (2)

Sol. Let ax + by + cz = 1 be the equation of the plane

$\Rightarrow 0 - b + 0 = 1$

$\Rightarrow b = -1$

0 + 0 + c = 1

$\Rightarrow c = 1$

$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$

$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$

$\Rightarrow a^2 + 2 = 4$

$\Rightarrow a = \pm\sqrt{2}$

$\Rightarrow \pm\sqrt{2}x - y + z = 1$

Now for –sign

$-\sqrt{2}.\sqrt{2} - 1 + 4 = 1$

option (2)

15. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to (Hence C is a constant of integration)

(1) $3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$

(3) $-3\cot^{-1/3}x + C$ (4) $-3\tan^{-1/3}x + C$

Official Ans. by NTA (4)

Sol. $I=\int \frac{dx}{(\sin x)^{4/3}.(\cos x)^{2/3}}$

$I=\int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$

$\Rightarrow I=\int \frac{\sec^2 x}{(\tan x)^{4/3}}dx$

put tanx = t $\Rightarrow$ $\sec^2 x dx$ = dt

$\therefore I=\int \frac{dt}{t^{4/3}} \Rightarrow I=\frac{-3}{t^{1/3}}+c$

$\Rightarrow I=\frac{-3}{(\tan x)^{1/3}}+c$

16. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be $50n+\frac{n(n-7)}{2}A$, wherre A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

(1) (A, 50+46A) (2) (A, 50+45A)

(3) (50, 50+46A) (4) (50, 50+45A)

Official Ans. by NTA (1)

Sol. $S_n=50n+\frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$=50n+\frac{n(n-7)}{2}A-50(n-1)-\frac{(n-1)(n-8)}{2}A$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

= 50 + A(n – 4)

$d = T_n - T_{n-1}$

= 50 + A(n – 4) – 50 – A(n – 5)

= A

T_{50} = 50 + 46A

(d, A_{50}) = (A, 50+46A)

17. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$ (3) $\frac{31}{6}$ (4) $\frac{13}{6}$

Official Ans. by NTA (2)

Sol. $x^2 \le y \le x + 2$

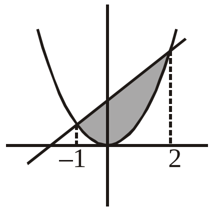
$x^2 = y$; $y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

x = 2, –1



Area = $\int_{-1}^{2} (x+2)-x^2\, dx=\frac{9}{2}$

18. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is

(1) $\frac{9}{2}$ (2) $2\sqrt{5}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{7}{2}$

Official Ans. by NTA (1)

Sol. Any point on the given line can be

$(1 + 2\lambda , -1 + 3\lambda, 2 + 4\lambda)$; $\lambda \in R$

Put in plane

$1 + 2\lambda + (-2 + 6\lambda) + (6 + 12\lambda) = 15$

$20\lambda + 5 = 15$

$20\lambda = 10$

$\lambda = \frac{1}{2}$

$\therefore$ Point $\left(2,\frac{1}{2},4\right)$

Distance from origin

$=\sqrt{4+\frac{1}{4}+16}=\frac{\sqrt{16+1+64}}{2}=\frac{\sqrt{81}}{2}$

$=\frac{9}{2}$

19. Let $\sum_{k=1}^{10} f(a+k)=16\left(2^{10}-1\right)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1)$ = 2. then the natural number 'a' is

(1) 4 (2) 3 (3) 16 (4) 2

Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall\ x \in N$

$2^{a+1} + 2^{a+2}+....+2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2+....+2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.(2^{10}-1)}{1}=16(2^{10}-1)$$

$2^{a+1} = 16 = 2^4$

a = 3

20. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(1) y^3 (2) $y^3 - 1$

(3) $y(y^2 - 1)$ (4) $y(y^2 - 3)$

Official Ans. by NTA (1)

Sol. Roots of the equation $x^2 + x + 1 = 0$ are $\alpha = \omega$ and $\beta = \omega^2$

where ω, ω^2 are complex cube roots of unity

$$\therefore \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta = y.y^2 \Rightarrow D = y^3$

21. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?

(1) (–2, 2) (2) (2, –2)

(3) (2, –1) (4) (–2, 1)

Official Ans. by NTA (2)

Sol. $y = x^3 + ax - b$

(1, –5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'_{(1,-5)} = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

By (i) and (ii) : a = –4, b = 2

$\therefore y = x^3 - 4x - 2$.

(2,–2) lies on this curve.

22. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. if all hit at the target independently, then the probability that the target would be hit, is

(1) $\frac{25}{192}$ (2) $\frac{1}{192}$ (3) $\frac{25}{32}$ (4) $\frac{7}{32}$

Official Ans. by NTA (3)

Sol. Let persons be A,B,C,D

P(Hit) = 1 – P(none of them hits)

$$=1-P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D})$$

$$=1-P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$$

$$=1-\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$$

$$=\frac{25}{32}$$

23. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{15}}{2}$

Official Ans. by NTA (3)

Sol. $\frac{x^2}{24}-\frac{y^2}{18}=1 \Rightarrow a=\sqrt{24}; b=\sqrt{18}$

Parametric normal :

$$\sqrt{24}\cos\theta.x+\sqrt{18}.y\cot\theta=42$$

At x = 0 : $y=\frac{42}{\sqrt{18}}\tan\theta=7\sqrt{3}$ (from given equation)

$$\Rightarrow \tan\theta=\sqrt{\frac{3}{2}} \Rightarrow \sin\theta=\pm\sqrt{\frac{3}{5}}$$

slope of parametric normal $=\frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta}=m$

$$\Rightarrow m=-\sqrt{\frac{4}{3}}\sin\theta=-\frac{2}{\sqrt{5}} \text{ or } \frac{2}{\sqrt{5}}$$

24. Let S = {$\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0$}. Then the sum of the elements of S is

(1) $\frac{13\pi}{6}$ (2) π (3) 2π (4) $\frac{5\pi}{3}$

Official Ans. by NTA (3)

Sol. $2\left(1-\sin^2\theta\right)+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2};\sin\theta=2(\text{reject})$

roots : $\pi+\frac{\pi}{6}, 2\pi-\frac{\pi}{6}, -\frac{\pi}{6}, -\pi+\frac{\pi}{6}$

$\Rightarrow$ sum of values $= 2\pi$

25. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

(1) $\frac{3}{2}(1+\cos 20°)$ (2) $\frac{3}{4}$

(3) $\frac{3}{4}+\cos 20°$ (4) $\frac{3}{2}$

Official Ans. by NTA (2)

Sol. $\frac{1}{2}\left(2\cos^2 10° - 2\cos 10° \cos 50° + 2\cos^2 50°\right)$

$\Rightarrow \frac{1}{2}\left(1+\cos 20° - (\cos 60° + \cos 40°) + 1 + \cos 100°\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20° + 2\sin 70° \sin(-30°)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2}+\cos 20° - \sin 70°\right)$

$\Rightarrow \frac{3}{4}$ Ans. (2)

26. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

(1) $x^2 + y^2 - 2xy = 0$

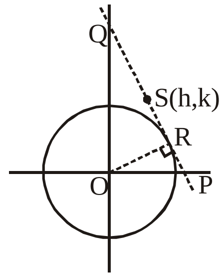
(2) $x^2 + y^2 - 16x^2y^2 = 0$

(3) $x^2 + y^2 - 4x^2y^2 = 0$

(4) $x^2 + y^2 - 2x^2y^2 = 0$

Official Ans. by NTA (3)

Sol.



Let the mid point be S(h,k)

$\therefore$ P(2h,0) and Q(0,2k)

equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

$\because$ PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\Rightarrow \frac{1}{4h^2}+\frac{1}{4k^2}=1$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

Aliter :

tangent to circle

$x\cos\theta + y\sin\theta = 1$

P : $(\sec\theta, 0)$

Q : $(0, \operatorname{cosec}\theta)$

$2h = \sec\theta \Rightarrow \cos\theta=\frac{1}{2h}$ & $\sin\theta=\frac{1}{2k}$

$\frac{1}{(2x)^2}+\frac{1}{(2y)^2}=1$

27. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$$ is continuous,

then k is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) 2

Official Ans. by NTA (1)

Sol. $\therefore$ function should be continuous at $x = \frac{\pi}{4}$

$$\therefore \lim_{x \to \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$$

$$\Rightarrow \lim_{x \to \frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = k \quad \text{(Using L'H}\hat{o}\text{pital rule)}$$

$$\lim_{x \to \frac{\pi}{4}} \sqrt{2}\sin^3 x = k$$

$$\Rightarrow k = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2}$$

28. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots\ldots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$, then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is

(1) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

Official Ans. by NTA (1)

Sol. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots\ldots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots\ldots+n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{n(n-2)}{2} = 78 \Rightarrow n = 13, -12\text{(reject)}$$

$\therefore$ We have to find inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

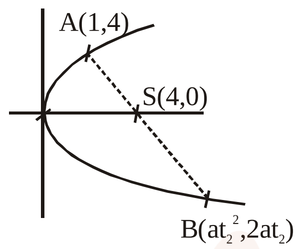
$$\therefore \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$$

29. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is

(1) 25 (2) 24 (3) 20 (4) 22

Official Ans. by NTA (1)

Sol.



$$y^2 = 4ax = 16x \Rightarrow a = 4$$

$$A(1,4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$$

$$\therefore \text{ length of focal chord } = a\left(t + \frac{1}{t}\right)^2$$

$$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$$

30. If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to

(1) R – [–1, 0) (2) R – (–1, 0)

(3) R – {–1} (4) [0, ∞)

Official Ans. by NTA (1)

Sol. $y = \frac{x^2}{1-x^2}$

Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.