FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (4)

Sol. Probability of observing at least one head out of n tosses

$$=1-\left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$$

$\Rightarrow n \geq 4$

$\Rightarrow$ minimum number of tosses = 4

2. A student scores the following marks in five tests : 45,54,41,57,43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$ (3) $\frac{100}{3}$ (4) $\frac{10}{3}$

Official Ans. by NTA (1)

Sol. Let x be the 6^{th} observation

$\Rightarrow$ 45 + 54 + 41 + 57 + 43 + x = 48 × 6 = 288

$\Rightarrow x = 48$

$$\text{variance} = \left(\frac{\Sigma x_i^2}{6} - (\bar{x})^2\right)$$

$$\Rightarrow \text{variance} = \frac{14024}{6} - (48)^2$$

$$= \frac{100}{3}$$

$$\Rightarrow \text{standard deviation} = \frac{10}{\sqrt{3}}$$

3. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to-

(1) $2-\frac{3}{2^{17}}$ (2) $2-\frac{11}{2^{19}}$

(3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{21}{2^{20}}$

Official Ans. by NTA (2)

Sol. $$S = \sum_{k=1}^{20} \frac{1}{2^k}$$

$$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + \ldots + \frac{20}{2^{20}}$$

$$S \times \frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{19}{2^{20}} + \frac{20}{2^{21}}$$

$$\Rightarrow \left(1-\frac{1}{2}\right)S = \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{20}} - \frac{20}{2^{21}}$$

$$\Rightarrow S = 2 - \frac{11}{2^{19}}$$

4. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}| = r$ is possible if :

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (2) $0 < r \leq \sqrt{\frac{3}{2}}$

(3) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$ (4) $r \geq 5\sqrt{\frac{3}{2}}$

Official Ans. by NTA (4)

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$$

$$= (2+x)\hat{i} + (x-3)\hat{j} - 5k$$

$$|\vec{a}\times\vec{b}| = \sqrt{4+x^2+4x+x^2+9-6x+25}$$

$$= \sqrt{2x^2-2x+38}$$

$$\Rightarrow |\vec{a}\times\vec{b}| \geq \sqrt{\frac{75}{2}}$$

$$\Rightarrow |\vec{a}\times\vec{b}| \geq 5\sqrt{\frac{3}{2}}$$

5. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x,y,z), $z \neq 0$, then (x,y) lies on the straight line whose equation is :

(1) $3x - 4y - 1 = 0$ (2) $3x - 4y - 4 = 0$

(3) $4x - 3y - 4 = 0$ (4) $4x - 3y - 1 = 0$

Official Ans. by NTA (3)

Sol. $x - 2y + kz = 1$ ...(1)

$2x + y + z = 2$ ...(2)

$3x - y - kz = 3$ ...(3)

(1) +(3)

$\Rightarrow 4x - 3y = 4$

6. If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is-

(1) $2x - y - 2 = 0$ (2) $3x - 2y = 0$

(3) $2x - 3y + 10 = 0$ (4) $x - 2y + 8 = 0$

Official Ans. by NTA (1)

Sol. Let us Suppose equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through (4,6) $\Rightarrow a^2 = 4,\ b^2 = 12$

$\Rightarrow$ equaiton of tangent

$$x - \frac{y}{2} = 1$$

$$\Rightarrow 2x - y - 2 = 0$$

7. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

(1) 5 : 9 : 13 (2) 5 : 6 : 7

(3) 4 : 5 : 6 (4) 3 : 4 : 5

Official Ans. by NTA (3)

Sol. $a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$$\Rightarrow \sin C = \sin 2A$$

$$\Rightarrow \sin C = 2\sin A.\cos A$$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A$$

$$\Rightarrow \frac{c}{a} = 2\frac{b^2 + c^2 - a^2}{2bc}$$

put $a = b - \lambda$, $c = b + \lambda$, $\lambda > 0$

$$\Rightarrow \lambda = \frac{b}{5}$$

$$\Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

8. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x)f_2(y)$

(3) $2f_1(x + y)f_2(x - y)$

(4) $2f_1(x + y)f_1(x - y)$

Official Ans. by NTA (1)

Sol. $f(x) = a^x$, $a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$= 2f_1(x)\ f_1(y)$

9. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is :

(1) 10^3 (2) 100 (3) 10^4 (4) 10

Official Ans. by NTA (4)

Sol. $200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{\frac{3}{2}} \times x^{\frac{1}{4}}$

$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$

$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$

where t = $\log_{10}x$

$\Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow$ x = 10 (As x > 1)

10. Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

Official Ans. by NTA (2)

Sol. $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$A(\alpha) = 2\int_0^{\alpha}\sqrt{x}dx = 2\alpha^{\frac{3}{2}}$

$A(4) = 2 \times 4^{3/2} = 16$

$A(\lambda) = 2 \times \lambda^{3/2}$

$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4.\left(\frac{4}{25}\right)^{1/3}$

11. Given that the slope of the tangent to a curve y = y(x) at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is :

(1) $x \log_e|y| = 2(x - 1)$
(2) $x \log_e|y| = x - 1$
(3) $x^2 \log_e|y| = -2(x - 1)$
(4) $x \log_e|y| = -2(x - 1)$

Official Ans. by NTA (1)

Sol. given $\frac{dy}{dx} = \frac{2y}{x^2}$

$\Rightarrow \int\frac{dy}{2y} = \int\frac{dx}{x^2}$

$\Rightarrow \frac{1}{2}\ell ny = -\frac{1}{x} + c$

passes through centre (1,1)

$\Rightarrow c = 1$

$\Rightarrow x\ell ny = 2(x-1)$

12. The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y+ 4z = 5 which is perpendicular to the plane x – y + z = 0 is :

(1) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

(2) $\vec{r}.(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}.(\hat{i}-\hat{k})+2=0$

(4) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

Official Ans. by NTA (3)

Sol. Let the plane be

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$

$\Rightarrow (2\lambda+1)x + (3\lambda+1)y + (4\lambda+1)z - (5\lambda+1) = 0$

$\perp$ to the plane x – y + z = 0

$\Rightarrow \lambda = -\frac{1}{3}$

$\Rightarrow$ the required plane is x – z + 2 = 0

13. Which one of the following statements is not a tautology ?

(1) $(p \wedge q) \to p$

(2) $(p \wedge q) \to (\sim p) \vee q$

(3) $p \to (p \vee q)$

(4) $(p \vee q) \to (p \vee (\sim q))$

Official Ans. by NTA (4)

Sol. Tautology

(1)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

(2)

p	q	$p \wedge q$	$\sim p \vee q$	$(p \wedge q) \to (\sim p) \vee q$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Tautology

(3)

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology

(4)

p	q	$p \vee q$	$\sim p$	$p \vee \sim q$	$p \vee q \to p \wedge (\sim q)$
T	T	T	F	F	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	F	T

14. Let f : R $\to$ R be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim\limits_{x \to 0} \left(\dfrac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)} \right)^{\frac{1}{x}}$ is equal to

(1) e^2 (2) e

(3) e^{-1} (4) 1

Official Ans. by NTA (4)

Sol. $\lim\limits_{x \to 0} \left(\dfrac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)} \right)^{\frac{1}{x}}$ (1^∞ form)

$\Rightarrow e^{\lim\limits_{x \to 0} \frac{f(3+x) - f(2-x) - f(3) + f(2)}{x(1 + f(2-x) - f(2))}}$

using L'Hopital

$\Rightarrow e^{\lim\limits_{x \to 0} \frac{f'(3+x) + f'(2-x)}{-x f'(2-x) + (1 + f(2-x) - f(2))}}$

$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}} = 1$

15. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

(1) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (2) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

(3) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (4) $\left(\frac{1}{4}, \frac{3}{4}\right)$

Official Ans. by NTA (3)

Sol. Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

by (1) and (2)

$\Rightarrow x = 1 \text{ and } y = 2$

equation of tangent at (1,2) to $y^2 = 4x$

is $y = x + 1$

16. Let the number 2,b,c be in an A.P. and

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2,16]$, then c

lies in the interval :

(1) [2,3)

(2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$

(4) [4,6]

Official Ans. by NTA (4)

Sol. put $b = \dfrac{2+c}{2}$ in determinant of A

$|A| = \dfrac{c^3 - 6c^2 + 12c - 8}{4} \in [2,16]$

$\Rightarrow (c-2)^3 \in [8, 64]$

$\Rightarrow c \in [4, 6]$

17. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d,e,f are in A.P.

(2) $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in G.P.

(3) $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

(4) d,e,f are in G.P.

Official Ans. by NTA (3)

Sol. a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

18. The number of integral values of m for which the equation

$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

has no real root is :

(1) infinitely many (2) 2

(3) 3 (4) 1

Official Ans. by NTA (1)

Sol. $D < 0$

$4(1 + 3m)^2 - 4(1 + m^2)(1 + 8m) < 0$

$\Rightarrow m(2m - 1)^2 > 0 \Rightarrow m > 0$

19. If a point R(4,y,z) lies on the line segment joining the points P(2,–3,4) and Q(8,0,10), then the distance of R from the origin is :

(1) $2\sqrt{14}$ (2) 6

(3) $\sqrt{53}$ (4) $2\sqrt{21}$

Official Ans. by NTA (1)

Sol. $\frac{4}{2} = \frac{-y}{y+3} = \frac{10-z}{z-4}$

$\Rightarrow z = 6$ & $y = -2$

$\Rightarrow R(4, -2, 6)$

dist. from origin $= \sqrt{16+4+36} = 2\sqrt{14}$

20. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$,

then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to

(1) –1 (2) 1

(3) 0 (4) $(-1+2i)^9$

Official Ans. by NTA (1)

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$$\Rightarrow z^5 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}+i}{2}$$

and $z^8 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\left(\frac{1+i\sqrt{3}}{2}\right)$

$$\Rightarrow \left(1+iz+z^5+iz^8\right)^9 = \left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{i}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$$

$$= \left(\frac{1+i\sqrt{3}}{2}\right)^9 = \cos 3\pi + i\sin 3\pi = -1$$

21. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$ equals-

(1) $\int_{x+5}^{5} g(t)dt$ (2) $5\int_{x+5}^{5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $2\int_{5}^{x+5} g(t)dt$

Official Ans. by NTA (1)

Sol. $f(x) = \int_0^x g(t)dt$

$f(-x) = \int_0^{-x} g(t)dt$

put $t = -u$

$= -\int_0^x g(-u)du$

$= -\int_0^x g(u)d(u) = -f(x)$

$\Rightarrow f(-x) = -f(x)$

$\Rightarrow f(x)$ is an odd function

Also $f(5 + x) = g(x)$
$f(5 - x) = g(-x) = g(x) = f(5 + x)$
$\Rightarrow f(5 - x) = f(5 + x)$
Now

$$I = \int_0^x f(t)dt$$

$t = u + 5$

$$I = \int_{-5}^{x-5} f(u+5)du$$

$$= \int_{-5}^{x-5} g(u)du$$

$$= \int_{-5}^{x-5} f'(u)du$$

$= f(x - 5) - f(-5)$
$= -f(5 - x) + f(5)$
$= f(5) - f(5+x)$

$$= \int_{5+x}^{5} f'(t)dt = \int_{5+x}^{5} g(t)dt$$

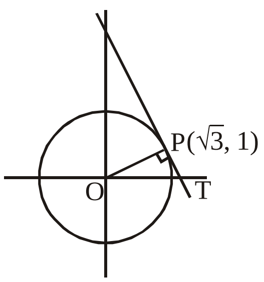
22. The tangent and the normal lines at the point $(\sqrt{3},1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{1}{3}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

Official Ans. by NTA (4)

Sol.



Given $x^2 + y^2 = 4$
equation of tangent

$$\Rightarrow \sqrt{3}x + y = 4 \qquad ...(1)$$

Equation of normal

$$x - \sqrt{3}y = 0 \qquad ...(2)$$

Coordinate of $T\left(\frac{4}{\sqrt{3}},0\right)$

$\therefore$ Area of triangle = $= \frac{2}{\sqrt{3}}$

23. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0,5\sqrt{3})$, then the length of its latus rectum is:

(1) 10 (2) 8 (3) 5 (4) 6

Official Ans. by NTA (3)

Sol. Let equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$2a - 2b = 10 \qquad ...(1)$

$ae = 5\sqrt{3} \qquad ...(2)$

$\frac{2b^2}{a} = ?$

$b^2 = a^2(1 - e^2)$
$b^2 = a^2 - a^2e^2$
$b^2 = a^2 - 25 \times 3$
$\Rightarrow b = 5$ and $a = 10$

$\therefore$ length of L.R. $= \frac{2(25)}{10} = 5$

24. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is :

(1) 12 (2) 33 (3) 9 (4) 15

Official Ans. by NTA (2)

Sol. $y = f(f(f(x))) + (f(x))^2$

$\frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$
$= f'(1)f'(1)f'(1) + 2f(1)f'(1)$
$= 3 \times 5 \times 3 + 2 \times 1 \times 3$
$= 27 + 6$
$= 33$

25. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$

where C is a constant of integration, then the function $f(x)$ is equal to-

(1) $-\frac{1}{6x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{2x^3}$

Official Ans. by NTA (4)

Sol. $\displaystyle\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$

$$\int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$$

Let $t = \frac{1}{x^6} + 1$

$dt = \frac{-6}{x^7}dx$

$$-\frac{1}{6}\int \frac{dt}{t^{2/3}} = -\frac{1}{2}t^{1/3}$$

$$= -\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} = -\frac{1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^2}$$

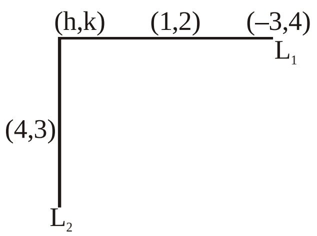
$\therefore\ f(x) = -\frac{1}{2x^3}$

26. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1. If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1, then $\frac{k}{h}$ equals :

(1) 3 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 0

Official Ans. by NTA (3)

Sol.



equation of L_1 is

$y = -\frac{1}{2}x + \frac{5}{2}$ ...(1)

equation of L_2 is

$y = 2x - 5$ ...(2)

by (1) and (2)

$x = 3$

$y = 1 \Rightarrow h = 3,\ k = 1$

$\frac{k}{h} = \frac{1}{3}$

27. Let $f : [-1,3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x] & , & -1 \le x < 1 \\ x + |x| & , & 1 \le x < 2 \\ x + [x] & , & 2 \le x \le 3 \end{cases},$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

(1) four or more points

(2) only one point

(3) only two points

(4) only three points

Official Ans. by NTA (4)

Sol.
$$f(x) = \begin{cases} -(x+1) & , & -1 \le x < 0 \\ x & , & 0 \le x < 1 \\ 2x & , & 1 \le x < 2 \\ x+2 & , & 2 \le x < 3 \\ x+3 & , & x = 3 \end{cases}$$

function discontinuous at x = 0,1,3

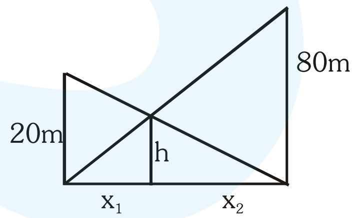
28. Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :

(1) 12 (2) 15

(3) 16 (4) 18

Official Ans. by NTA (3)

Sol.



by similar triangle

$\frac{h}{x_1} = \frac{80}{x_1+x_2}$ ...(1)

by $\frac{h}{x_2} = \frac{20}{x_1+x_2}$ ..(2)

by (1) and (2)

$\frac{x_2}{x_1} = 4$ or $x_2 = 4x_1$

$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$

or h = 16m

29. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is :

(1) 288 (2) 306 (3) 360 (4) 310

Official Ans. by NTA (4)

Sol. (1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$

(2) Starting with 44 and 55 is equal to $36 \times 2 = 72$

(3) Starting with 433,434 and 435 is equal to $6 \times 3 = 18$

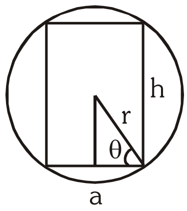
(3) Remaining numbers are 4322,4323,4324,4325is equal to 4 so total numbers are $216 + 72 + 18 + 4 = 310$

30. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $2\sqrt{3}$ (2) $\sqrt{3}$ (3) $\sqrt{6}$ (4) $\frac{2}{3}\sqrt{3}$

Official Ans. by NTA (1)

Sol.



$h = 2r\sin\theta$

$a = 2r\cos\theta$

$v = \pi(r\cos\theta)^2(2r\sin\theta)$

$v = 2\pi r^3\cos^2\theta\sin\theta$

$$\frac{dv}{d\theta} = \pi r^3\left(-2\cos\theta\sin^2\theta + \cos^3\theta\right) = 0$$

or $\tan\theta = \frac{1}{\sqrt{2}}$

$\because\ h = 2 \times 3 \times \frac{1}{\sqrt{3}}$

$= 2\sqrt{3}$