FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

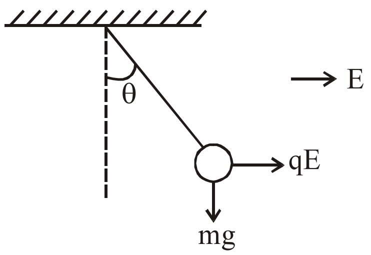
1. The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : (take g = 10 m/s^2)

(1) $\tan^{-1}(5.0)$ (2) $\tan^{-1}(2.0)$

(3) $\tan^{-1}(0.5)$ (4) $\tan^{-1}(0.2)$

Official Ans. by NTA (3)

Sol.



$$\tan\theta = \frac{qE}{mg} = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}$$

$$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}(0.5)$$

2. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas)

(1) 10^6 (2) 10^3

(3) 10^4 (4) 10^2

Official Ans. by NTA (3)

Sol. Reynolds Number = $\frac{\rho v d}{\eta}$

Volume flow rate = $v \times \pi r^2$

$$v = \frac{100\times10^{-3}}{60}\times\frac{1}{\pi\times25\times10^{-4}}$$

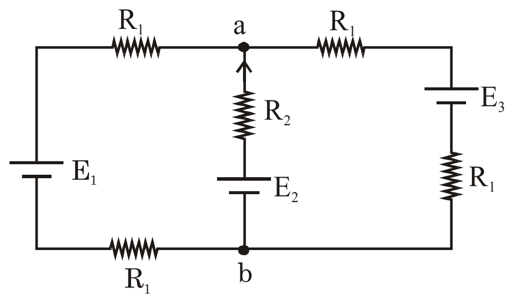
$$v = \frac{2}{3\pi} m/s$$

$$\text{Reynolds Number} = \frac{10^3\times2\times10\times10^{-2}}{10^{-3}\times3\pi}$$

$$\simeq 2\times10^4$$

order 10^4

3. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\ \Omega$, E_1 = 2 V and $E_2 = E_3$ = 4 V, the potential difference between the points 'a' and 'b' is approximately (in V) :



(1) 2.7 (2) 3.3

(3) 2.3 (4) 3.7

Official Ans. by NTA (2)

Sol.
$$E_{eq} = \frac{\frac{E_1}{2R_1}+\frac{E_2}{R_2}+\frac{E_3}{2R_1}}{\frac{1}{2R_1}+\frac{1}{R_2}+\frac{1}{2R_1}}$$

$$= \frac{\frac{2}{2}+\frac{4}{2}+\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}$$

$$= \frac{5}{\frac{3}{2}} = \frac{10}{3} = 3.3$$

4. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be :

(1) 100 Ω (2) 400 Ω

(3) 500 Ω (4) 300 Ω

Official Ans. by NTA (3)

Sol. When red is replaced with green 1st digit changes to 5 so new resistance will be 500Ω

5. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02kg on it and stretches the cord by 20 cm by applying a constant force. When released, the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to:

(1) 10^4 Nm^{-2} (2) 10^8 Nm^{-2}

(3) 10^6 Nm^{-2} (4) 10^3 Nm^{-2}

Official Ans. by NTA (3)

Sol. Energy of catapult $=\frac{1}{2}\times\left(\frac{\Delta\ell}{\ell}\right)^2\times Y\times A\times\ell$

= Kinetic energy of the ball $=\frac{1}{2}mv^2$

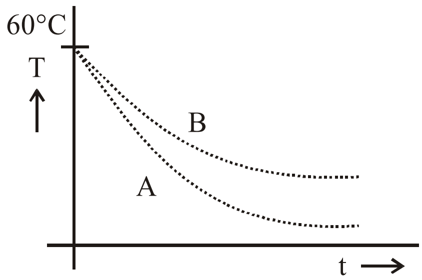
therefore,

$$\frac{1}{2}\times\left(\frac{20}{42}\right)^2\times Y\times\pi\times3^2\times10^{-6}\times42\times10^{-2}=\frac{1}{2}\times2\times10^{-2}\times(20)^2$$

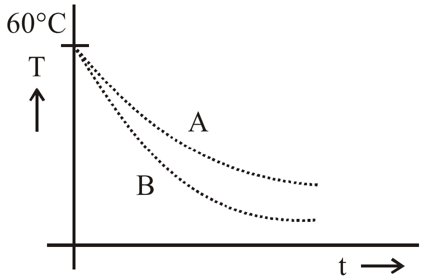
$Y \simeq 3\times10^6$ Nm^{-2}

6. Two identical breakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

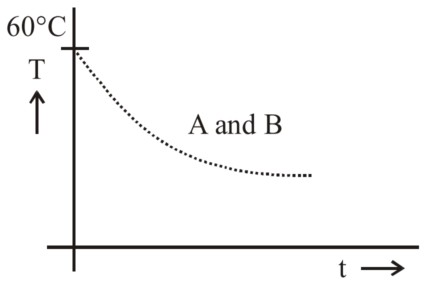
(1)



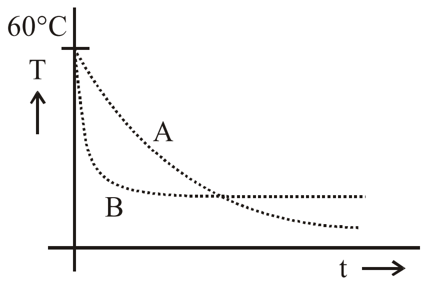
(2)



(3)



(4)



Official Ans. by NTA (1)

Sol. $-ms\frac{dT}{dt} = e\sigma A\left(T^4 - T_0^4\right)$

$$-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_0^4\right)$$

$$-\frac{dT}{dt} = \frac{4e\sigma A T_0^3}{ms}(\Delta T)$$

$T = T_0 + (T_i - T_0)e^{-kt}$

where $k = \frac{4e\sigma A T_0^3}{ms}$

$$k = \frac{4e\sigma A T_0^3}{\rho v s}$$

$$\left|\frac{dT}{dt}\right| \propto k$$

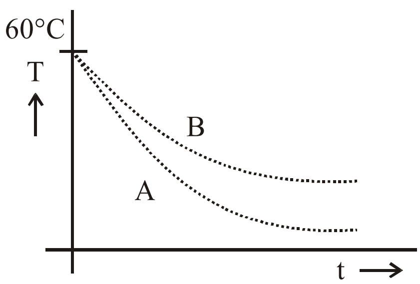
$$\therefore \left|\frac{dT}{dt}\right| \propto \frac{1}{\rho s}$$

$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

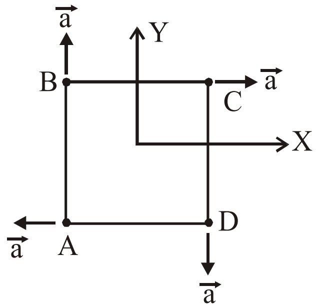
$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A s_A < \rho_B s_B$

$$\left|\frac{dT}{dt}\right|_A > \left|\frac{dT}{dt}\right|_B$$



7. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :

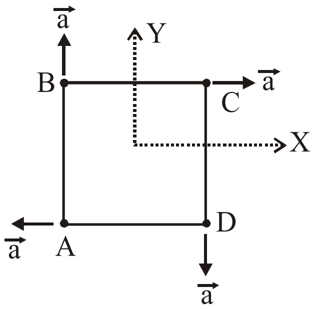


(1) $\frac{a}{5}\left(\hat{i} - \hat{j}\right)$ (2) $\frac{a}{5}\left(\hat{i} + \hat{j}\right)$

(3) Zero (4) $a\left(\hat{i} + \hat{j}\right)$

Official Ans. by NTA (1)

Sol.



$\vec{a}_A = -a\hat{i}$

$\vec{a}_B = a\hat{j}$

$\vec{a}_C = a\hat{i}$

$\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a\vec{a}_a + m_b\vec{a}_b + m_c\vec{a}_c + m_d\vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m}$$

$$= \frac{2ma\hat{i} - 2ma\hat{j}}{10m}$$

$$= \frac{a}{5}\hat{i} - \frac{a}{5}\hat{j}$$

$$= \frac{a}{5}\left(\hat{i} - \hat{j}\right)$$

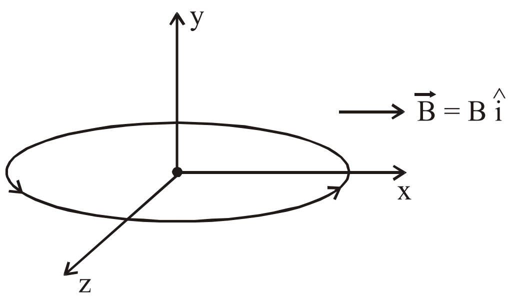
8. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is :

(1) $B\pi r^2 IN$ (2) $\frac{Br^2 I}{\pi N}$

(3) Zero (4) $\frac{B\pi r^2 I}{N}$

Official Ans. by NTA (1)

Sol.



Magnetic moment of coil = $NIA\hat{j}$

$= NI(\pi r^2)\hat{j}$

Torque on loop (coil) = $\vec{M} \times \vec{B}$

$= NI(\pi r^2)B\sin 90°(-\hat{k})$

$= NI\pi r^2 B(-\hat{k})$

9. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant is the capacitance is 15 pF?

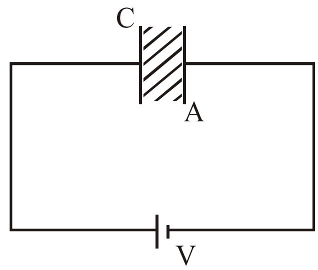
(given $\epsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(1) 3.8 (2) 4.5

(3) 6.2 (4) 8.5

Official Ans. by NTA (4)

Sol.



$A = 10^{-4}$ m^2

$E_{max} = 10^6$ V/m

$C = 15$ μF

$$C = \frac{k\varepsilon_0 A}{d}$$

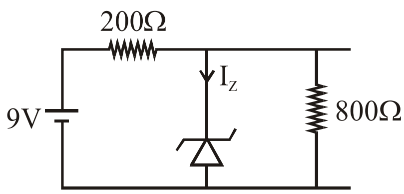
$$\frac{Cd}{\varepsilon_0 A} = k$$

$$k = \frac{15 \times 10^{-12} \times 500 \times 10^{-6}}{8.86 \times 10^{-12} \times 10^4}$$

$$= \frac{15 \times 5}{8.86} = 8.465$$

$k \approx 8.5$

10. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



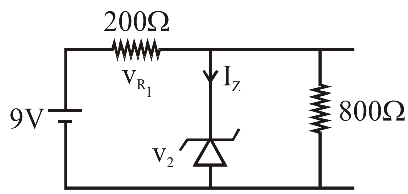
The current I_Z through the Zener is :

(1) 7 mA (2) 17 mA

(3) 10 mA (4) 15mA

Official Ans. by NTA (3)

Sol.



$9 = V_Z + V_{R_1}$

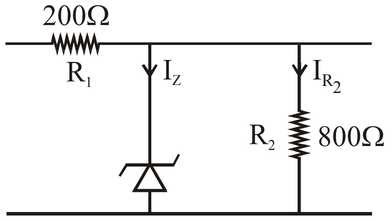
$V_Z = 5.6$ V

$V_{R_1} = 9 - 5.6$

$V_{R_1} = 3.4$

$$I_{R_1} = \frac{V_{R_1}}{R} = \frac{3.4}{200}$$

$I_{R_1} = 17$ mA



$$V_z = V_{R_2} = I_{R_2}(R_2)$$

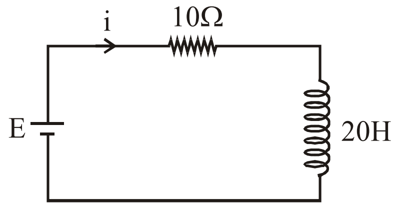
$$\frac{5.6}{800} = I_{R_2}$$

$I_{R_2} = 7\text{mA}$

$I_z = (17 - 7)$ mA

= 10 mA

11. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is :



(1) $\frac{2}{\ell n2}$ (2) $\ell n2$

(3) $2\ell n2$ (4) $\frac{1}{2}\ell n2$

Official Ans. by NTA (3)

Sol. $LIdI = I^2R$

$$L \times \frac{E}{10}\left(-e^{-t/2}\right) \times \frac{-1}{2} = \frac{E}{10}\left(1 - e^{-t/2}\right) \times 10$$

$$e^{-t/2} = 1 - e^{-t/2}$$

$t = 2\ell n2$

12. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be :

(1) 40 cm from the convergent mirror, same size as the object

(2) 20 cm from the convergent mirror, same size as the object

(3) 20 cm from the convergent mirror, twice the size of the object

(4) 40 cm from the convergent lens, twice the size of the object

Official Ans. by NTA (2)

Allen Ans. is BONUS

Sol. **Note :**

There will be 3 phenomenon

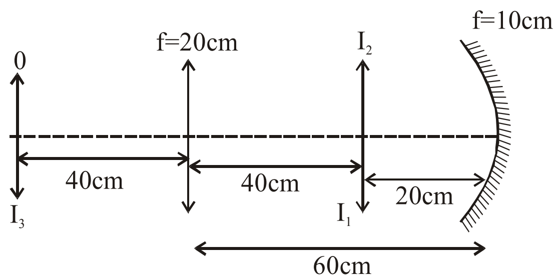
(i) Refraction from lens

(ii) Reflection from mirror

(iii) Refraction from lens

After these phenomena. Image will be on object and will have same size.

None of the option depicts so this question is Bonus



1st refraction u = –40cm; f = +20cm

⇒ v = +40 cm (image I_1)

and $m_1 = -1$

for reflection

u = –20cm; f = –10cm

⇒ v = –20cm (image I_2)

and $m_2 = -1$

2nd refraction

$u = -40cm$; $f = +20cm$

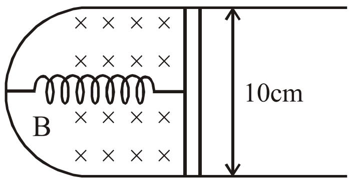
$\Rightarrow v = +40cm$ (image I_3)

and $m_3 = -1$

Total magnification = $m_1 \times m_2 \times m_3 = -1$

and final image is formed at distance 40cm from convergent lens and is of same size as the object

13. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to :



(1) 50000 (2) 5000

(3) 10000 (4) 1000

Official Ans. by NTA (2)

Sol. $T_0 = 2\pi\sqrt{\frac{m}{k}}$

$= \frac{2\pi}{\sqrt{10}}$

$A = A_0 e^{-t/\gamma}$

$\therefore$ for $A = \frac{A_0}{e}$, $t = \gamma$

$t = \gamma = \frac{2m}{b} = \frac{2m}{\frac{B^2\ell^2}{R}} = 10^4 s$

$\therefore$ No of oscillation $\frac{t}{T_0} = \frac{10^4}{2\pi/\sqrt{10}} \approx 5000$.

14. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is :

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{8}{5}$

Official Ans. by NTA (4)

Sol. $M = \int_0^R \rho_0 r(2\pi r dr) = \frac{\rho_0 \times 2\pi \times R^3}{3}$

$\underset{(\text{MOI about COM})}{I_0} = \int_0^R \rho_0 r(2\pi r dr) \times r^2 = \frac{\rho_0 \times 2\pi R^5}{5}$

by parallel axis theorem

$I = I_0 + MR^2$

$= \frac{\rho_0 \times 2\pi R^5}{5} + \frac{\rho_0 \times 2\pi R^3}{3} \times R^2 = \rho_0 2\pi R^5 \times \frac{8}{15}$

$= MR^2 \times \frac{8}{5}$

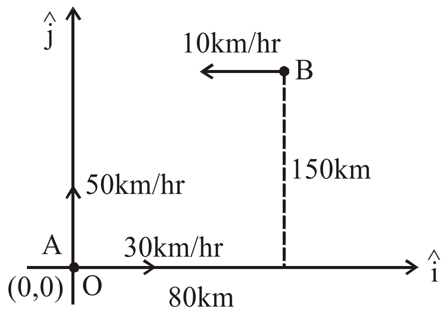
15. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in :

(1) 4.2 hrs. (2) 2.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

Official Ans. by NTA (4)

Sol. If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :



$\vec{v}_A = \left(30\hat{i} + 50\hat{j}\right) km / hr$

$\vec{v}_B = -10\hat{i}\ km / hr$

$\vec{r}_A = 0\hat{i} + 0\hat{j}$

$\vec{r}_B = \left(80\hat{i} + 150\hat{j}\right) km$

Time after which distance between them will be minimum

$t = -\dfrac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{\left|\vec{v}_{BA}\right|^2}$;

where $\vec{r}_{BA} = \left(80\hat{i} + 150\hat{j}\right) km$

$\vec{v}_{BA} = -10\hat{i} - \left(30\hat{i} + 50\hat{j}\right)$

$\left(-40\hat{i} - 50\hat{j}\right) km / hr$

$$\therefore t = -\frac{\left(80\hat{i} + 150\hat{j}\right) \cdot \left(-40\hat{i} - 50\hat{j}\right)}{\left|-40\hat{i} - 50\hat{j}\right|^2}$$

$$= \frac{3200 + 7500}{4100} hr = \frac{10700}{4100} hr = 2.6 hrs$$

16. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1 π ms^{-2}, what will be the tensile stress that would be developed in the wire ?

(1) $4.8 \times 10^6\ Nm^{-2}$

(2) $5.2 \times 10^6\ Nm^{-2}$

(3) $6.2 \times 10^6\ Nm^{-2}$

(4) $3.1 \times 10^6\ Nm^{-2}$

Official Ans. by NTA (4)

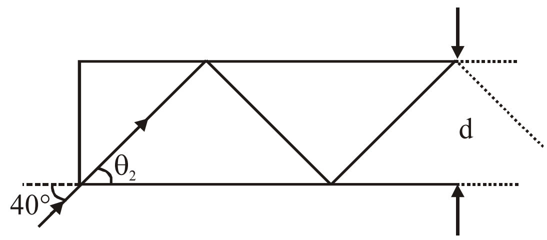
Sol. Tensile stresss in wire will be

$$= \frac{\text{Tensile force}}{\text{Cross section Area}}$$

$$= \frac{mg}{\pi R^2} = \frac{4 \times 3.1\pi}{\pi \times 4 \times 10^{-6}} Nm^{-2} = 3.1 \times 10^6\ Nm^{-2}$$

17. In figure, the optical fiber is ℓ = 2m long and has a diameter of d = 20 μm. If a ray of light is incident on one end of the fiber at angle θ_1 = 40°, the number of reflection it makes before emerging from the other end is close to:

(refractive index of fibre is 1.31 and sin 40° = 0.64)



(1) 55000 (2) 57000

(3) 66000 (4) 45000

Official Ans. by NTA (2)

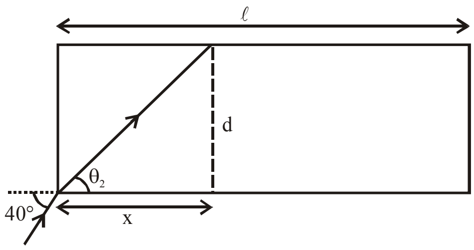
Allen Ans. is 1 or 2

Sol. Note :

If we approximate the angle θ_2 as 30° initially then answer will be closer to 57000. But if we solve thorougly, answer will be close to 55000.

So both the answers must be awarded.

Detailed solution is as following.



Exact solution

by Snells' law $1.\sin40° = (1.31)\sin\theta_2$

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

Now $\tan\theta_2 = \frac{64}{\sqrt{(131)^2-(64)^2}} = \frac{64}{\sqrt{13065}} \approx \frac{64}{114.3} = \frac{d}{x}$

Now no. of reflections

$$= \frac{2\times64}{114.3\times20\times10^{-6}} = \frac{64\times10^5}{114.3}$$

$\approx 55991 \approx 55000$

Approximate solution

By Snells' law $1.\sin40° = (1.31)\sin\theta_2$

$$\sin\theta_2 = \frac{.64}{1.31} = \frac{64}{131} \approx .49$$

If assume $\Rightarrow \theta_2 \approx 30°$

$$\tan30° = \frac{d}{x} \Rightarrow x = \sqrt{3}d$$

Now number of reflections

$$= \frac{\ell}{\sqrt{3}d} = \frac{2}{\sqrt{3}\times20\times10^{-6}} = \frac{10^5}{\sqrt{3}}$$

$\approx 57735 \approx 57000$

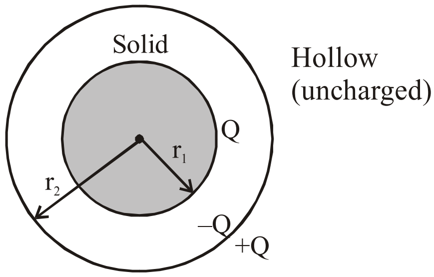
18. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of –4 Q, the new potential difference between the same two surfaces is :

(1) V (2) 2V

(3) –2V (4) 4V

Official Ans. by NTA (1)

Sol. As given in the first condition :

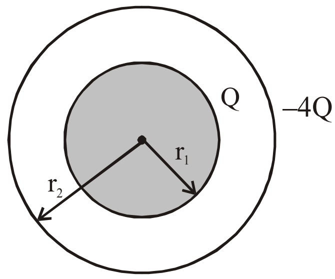


Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

In the second condition :



Shell is now given charge –4Q.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

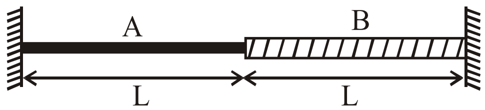
$$= \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

Hence, we also obtain that potential difference does not depend on charge of outer sphere.

$\therefore$ P.d. remains same

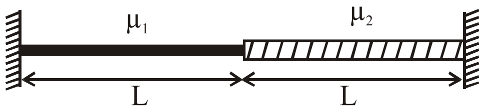
19. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is :



(1) 4 : 9 (2) 3 : 5 (3) 1 : 4 (4) 1 : 2

Official Ans. by NTA (4)

Sol.



radius = r radius = 2r

Let mass per unit length of wires are μ_1 and μ_2 respectively.

$\because$ Materials are same, so density ρ is same.

$$\therefore \ \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu \ \text{ and } \ \mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension in both are same = T, let speed of wave in wires are V_1 and V_2

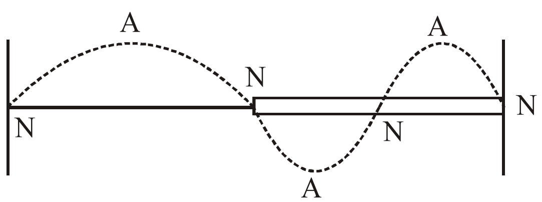
$$V_1 = \sqrt{\frac{T}{\mu}} = V \ ; \ V_2 = \sqrt{\frac{T}{4\mu}} = \frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_{01} = \frac{V_1}{2L} = \frac{V}{2L} \ \& \ f_{02} = \frac{V_2}{2L} = \frac{V}{4L}$$

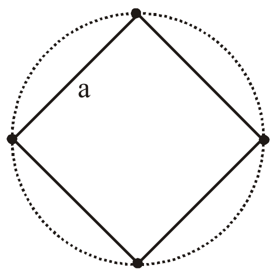
Frequency at which both resonate is L.C.M of both frequencies i.e. $\frac{V}{2L}$.

Hence no. of loops in wires are 1 and 2 respectively.



So, ratio of no. of antinodes is 1 : 2.

20. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square?

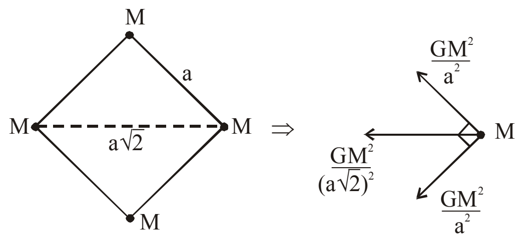


(1) $1.21\sqrt{\frac{GM}{a}}$ (2) $1.41\sqrt{\frac{GM}{a}}$

(3) $1.16\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

Official Ans. by NTA (3)

Sol.



Net force on particle towards centre of circle is $F_C = \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\sqrt{2}$

$$= \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force. Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

$$r = \frac{a}{\sqrt{2}}, \ F_C = \frac{mv^2}{r}$$

$$\frac{mv^2}{\frac{a}{\sqrt{2}}} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$v^2 = \frac{GM}{a}\left(\frac{1}{2\sqrt{2}}+1\right)$$

$$v^2 = \frac{GM}{a}(1.35)$$

$$v = 1.16\sqrt{\frac{GM}{a}}$$

21. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is E = 6 V m^{-1} along y-direction. Its corresponding magnetic field component, B would be :

(1) 6×10^{-8} T along z-direction
(2) 6×10^{-8} T along x-direction
(3) 2×10^{-8} T along z-direction
(4) 2×10^{-8} T along y-direction

Official Ans. by NTA (3)

Sol. The direction of propogation of an EM wave is direction of $\vec{E}\times\vec{B}$.

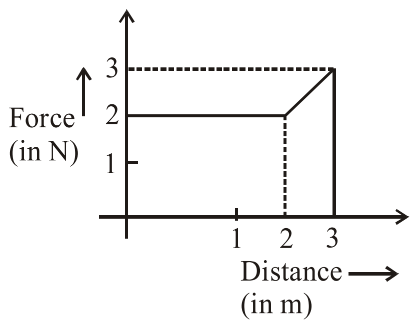
$$\hat{i} = \hat{j}\times\hat{B}$$

$$\Rightarrow \hat{B} = \hat{k}$$

$$C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{6}{3\times10^8}$$

B = 2×10^{-8} T along z direction.

22. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3m is :



(1) 6.5 J (2) 2.5 J (3) 4 J (4) 5 J

Official Ans. by NTA (1)

Sol. According to work energy theorem

Work done by force on the particle = Change in KE

Work done = Area under F-x graph

$$= \int F.dx$$

$$= 2\times2+\frac{(2+3)\times1}{2}$$

W = $KE_{final} - KE_{initial}$ = 6.5

$KE_{initial}$ = 0

KE_{final} = 6.5 J

23. In SI units, the dimesions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is :

(1) $A^{-1}TML^3$ (2) $A^2T^3M^{-1}L^{-2}$
(3) $AT^2M^{-1}L^{-1}$ (4) $AT^{-3}ML^{3/2}$

Official Ans. by NTA (2)

Sol. dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

$$\text{dimensions of } \sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$$

$= [M^{-2}L^{-4}T^6A^4]^{1/2}$

$= [M^{-1}L^{-2}T^3A^2]$

24. Radiation coming from transitions n = 2 to n = 1 of hydrogen atoms fall on He^+ ions in n = 1 and n = 2 states. The possible transition of helium ions as they absorb energy from the radiation is :

(1) n = 1 → n = 4 (2) n = 2 → n = 4
(3) n = 2 → n = 5 (4) n = 2 → n = 3

Official Ans. by NTA (2)

Sol. Energy released for transition n = 2 to n = 1 of hydrogen atom

$$E = 13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

Z = 1, n_1 = 1, n_2 = 2

$$E = 13.6\times1\times\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$E = 13.6 \times \frac{3}{4} eV$$

For He^{+} ion z = 2

(1) n = 1 to n = 4

$$E = 13.6 \times 2^2 \times \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{15}{4} eV$$

(2) n = 2 to n = 4

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 13.6 \times \frac{3}{4} eV$$

(3) n = 2 to n = 5

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 13.6 \times \frac{21}{25} eV$$

(4) n = 2 to n = 3

$$E = 13.6 \times 2^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{9} eV$$

Energy required for transition of He^{+} for n = 2 to n = 4 matches exactly with energy released in transition of H for n = 2 to n = 1.

25. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ, of the final particle, is given by :

(1) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (2) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(3) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (4) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

Official Ans. by NTA (4)

Sol.  $\frac{h}{\lambda_1} = P_1$  $P_2 = \frac{h}{\lambda_2}$

$$\vec{P}_1 = \frac{h}{\lambda_1}\hat{i}$$

& $\vec{P}_2 = \frac{h}{\lambda_2}\hat{j}$

Using momentum conservation

$$\vec{P} = \vec{P}_1 + \vec{P}_2$$

$$= \frac{h}{\lambda_1}\hat{i} + \frac{h}{\lambda_2}\hat{j}$$

$$|\vec{P}| = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{h}{\lambda} = \sqrt{\left(\frac{h}{\lambda_1}\right)^2 + \left(\frac{h}{\lambda_2}\right)^2}$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

26. A thermally insulated vessel contains 150g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to :

(Latent heat of vaporization of water = 2.10 × 10^6 J kg^{-1} and Latent heat of Fusion of water = 3.36 × 10^5 J kg^{-1})

(1) 130 g (2) 35 g

(3) 20 g (4) 150 g

Official Ans. by NTA (3)

Sol. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{req} = mL_v$

Mass that converts into ice = (150 – m)

So, heat released in this process

$\Delta Q_{rel} = (150 - m) L_f$

Now,

$\Delta Q_{rel} = \Delta Q_{req}$

$(150 - m) L_f = mL_V$

$m(L_f + L_v) = 150 L_f$

$$m = \frac{150L_f}{L_f + L_v}$$

m = 20g

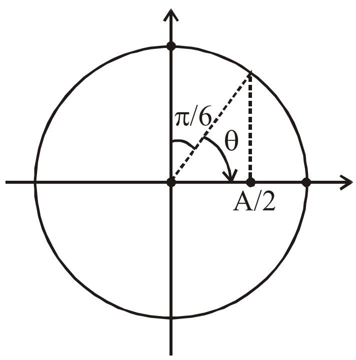
27. An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is :

(1) 2.2 ms (2) 5 ms

(3) 3.3 ms (4) 7.2 ms

Official Ans. by NTA (3)

Sol.



V(t) = 220 sin(100πt) volt

time taken,

$$t = \frac{\theta}{\omega} = \frac{\frac{\pi}{3}}{100\pi} = \frac{1}{300} \text{sec}$$

= 3.3 ms

28. The wavelength of the carrier waves in a modern optical fiber communication network is close to :

(1) 600 nm (2) 900 nm

(3) 2400 nm (4) 1500 nm

Official Ans. by NTA (4)

Sol. To minimise attenuation, wavelength of carrier waves is close to 1500 nm

29. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be :

(1) 4 (2) 2

(3) 9 (4) 18

Official Ans. by NTA (1)

Sol. Given $\frac{a_1}{a_2} = \frac{1}{3}$

Ratio of intensities, $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{9}$

Now, $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{1+3}{1-3}\right)^2 = 4$

30. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of :

(1) 10^8 N/m^2 (2) 10^4 N/m^2

(3) 10^3 N/m^2 (4) 10^{16} N/m^2

Official Ans. by NTA (3)

Allen Ans. is BONUS

Sol. Note :

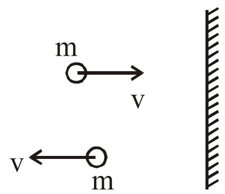
Pressure is defined as normal force per unit area.

Force is calculated as change in momentum/time

By this answer is 2N/m^2

None of the option matches so this question must be Bonus

Detailed solution is as following.



Magnitude of change in momentum per collision = 2mv

Pressure = $\frac{\text{Force}}{\text{Area}} = \frac{N(2mv)}{1}$

$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1}$

= 2N/m^2