## TEST PAPER OF JEE(MAIN) EXAMINATION - 2019

## (Held On Wednesday 09th JANUARY, 2019) TIME : 2:30 PM To 05:30 PM PHYSICS

1. Two plane mirrors arc inclined to each other such that a ray of light incident on the first mirror $\left(\mathrm{M}_{1}\right)$ and parallel to the second mirror $\left(\mathrm{M}_{2}\right)$ is finally reflected from the second mirror $\left(\mathrm{M}_{2}\right)$ parallel to the first mirror $\left(\mathrm{M}_{1}\right)$. The angle between the two mirrors will be :
(1) $90^{\circ}$
(2) $45^{\circ}$
(3) $75^{\circ}$
(4) $60^{\circ}$

Ans. (4)

Sol.


Assuming angles between two mirrors be $\theta$ as per geometry,
sum of anlges of $\Delta$
$3 \theta=180^{\circ}$
$\theta=60^{\circ}$
2. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda$ $=500 \mathrm{~nm}$ is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^{\circ} \leq \theta \leq 30^{\circ}$ is:
(1) 320
(2) 641
(3) 321
(4) 640

Ans. (2)

Sol.


Pam difference
$\mathrm{d} \sin \theta=\mathrm{n} \lambda$
where $d=$ seperation of slits
$\lambda=$ wave length
$\mathrm{n}=$ no. of maximas
$0.32 \times 10^{-3} \sin 30=\mathrm{n} \times 500 \times 10^{-9}$
$\mathrm{n}=320$

Hence total no. of maximas observed in angular range $-30^{\circ} \leq \theta \leq 30^{\circ}$ is
maximas $=320+1+320=641$
3. At a given instant, say $t=0$, two radioactive substances A and B have equal activities. The ratio $\frac{R_{B}}{R_{A}}$ of their activities after time $t$ itself decays with time $t$ as $e^{-3 t}$. [f the half-life of $A$ is $\mathrm{m}_{2}$, the half-life of B is :
(1) $\frac{\ln 2}{2}$
(2) $2 \ln 2$
(3) $\frac{\ln 2}{4}$
(4) $4 \ln 2$

Ans. (3)
Sol. Half life of $\mathrm{A}=\ell \mathrm{n} 2$
$\mathrm{t}_{1 / 2}=\frac{\ln 2}{\lambda}$
$\lambda_{\mathrm{A}}=1$
at $t=0 \quad R_{A}=R_{B}$
$N_{A} e^{-\lambda A T}=N_{B} e^{-\lambda B T}$
$N_{A}=N_{B}$ at $t=0$
at $\mathrm{t}=\mathrm{t} \quad \frac{\mathrm{R}_{\mathrm{B}}}{\mathrm{R}_{\mathrm{A}}}=\frac{\mathrm{N}_{0} \mathrm{e}^{-\lambda_{\mathrm{B}} \mathrm{t}}}{\mathrm{N}_{0} \mathrm{e}^{-\lambda_{\mathrm{A}} \mathrm{t}}}$
$\mathrm{e}^{-\left(\lambda_{B}-\lambda_{A}\right) t}=\mathrm{e}^{-\mathrm{t}}$
$\lambda_{\mathrm{B}}-\lambda_{\mathrm{A}}=3$
$\lambda_{\mathrm{B}}=3+\lambda_{\mathrm{A}}=4$
$\mathrm{t}_{1 / 2}=\frac{\ln 2}{\lambda_{\mathrm{B}}}=\frac{\ln 2}{4}$
4. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of $\mathrm{V}_{\mathrm{o}}$ changes by : (assume that the Ge diode has large breakdown voltage)

(1) 0.6 V
(2) 0.8 V
(3) 0.4 V
(4) 0.2 V

Ans. (3)

Sol. Initially Ge \& Si are both forward biased so current will effectivily pass through Ge diode with a drop of 0.3 V
if "Ge" is revesed then current will flow through "Si" diode hence an effective drop of $(0.7-0.3)$ $=0.4 \mathrm{~V}$ is observed.
5. A rod of mass ' M ' and length ' 2 L ' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by $20 \%$. The value of ratio $\mathrm{m} / \mathrm{M}$ is close to :
(1) 0.17
(2) 0.37
(3) 0.57
(4) 0.77

Ans. (2)
Sol. Frequency of torsonal oscillations is given by
$f=\frac{k}{\sqrt{I}}$
$f_{1}=\frac{k}{\sqrt{\frac{M(2 L)^{2}}{12}}}$
$\mathrm{f}_{2}=\frac{\mathrm{k}}{\sqrt{\frac{\mathrm{M}(2 \mathrm{~L})^{2}}{12}+2 \mathrm{~m}\left(\frac{L}{2}\right)^{2}}}$
$\mathrm{f}_{2}=0.8 \mathrm{f}_{1}$
$\frac{\mathrm{m}}{\mathrm{M}}=0.375$
6. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature $27^{\circ} \mathrm{C}$. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about :
[Take $\mathrm{R}=8.3 \mathrm{~J} / \mathrm{K}$ mole]
(1) 10 kJ
(2) 0.9 kJ
(3) 6 kJ
(4) 14 kJ

Ans. (1)
Sol. $\mathrm{Q}=\mathrm{nC}_{\mathrm{v}} \Delta \mathrm{T}$ as gas in closed vessel
$\mathrm{Q}=\frac{15}{28} \times \frac{5 \times \mathrm{R}}{2} \times(4 \mathrm{~T}-\mathrm{T})$
$\mathrm{Q}=10000 \mathrm{~J}=10 \mathrm{~kJ}$
7. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x -axis, about $x=0$. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :
(1) $\frac{\mathrm{A}}{2}$
(2) $\frac{\mathrm{A}}{2 \sqrt{2}}$
(3) $\frac{\mathrm{A}}{\sqrt{2}}$
(4) A

Ans. (3)

Sol. Potential energy $(U)=\frac{1}{2} \mathrm{kx}^{2}$

Kinetic energy $(K)=\frac{1}{2} \mathrm{kA}^{2}-\frac{1}{2} \mathrm{kx}^{2}$
According to the question, $\mathrm{U}=\mathrm{k}$
$\therefore \frac{1}{2} \mathrm{kx}^{2}=\frac{1}{2} \mathrm{kA}^{2}-\frac{1}{2} \mathrm{kx}^{2}$
$\mathbf{x}= \pm \frac{\mathrm{A}}{\sqrt{2}}$
$\therefore$ Correct answer is (3)
8. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of $10 \mathrm{~km} / \mathrm{h}$. If the wave speed is $330 \mathrm{~m} / \mathrm{s}$, the frequency heard by the running person shall be close to :
(1) 753 Hz
(2) 500 Hz
(3) 333 Hz
(4) 666 Hz

Ans. (4)
Sol. Frequency of the sound produced by flute,
$\mathrm{f}=2\left(\frac{\mathrm{v}}{2 \ell}\right)=\frac{2 \times 330}{2 \times 0.5}=660 \mathrm{~Hz}$

Velocity of observer, $\mathrm{v}_{0}=10 \times \frac{5}{18}=\frac{25}{9} \mathrm{~m} / \mathrm{s}$
$\therefore$ frequency detected by observer, $\mathrm{f}^{\prime}=$ $\left[\frac{v+v_{0}}{v}\right] f$
$\therefore \mathrm{f}^{\prime}=\left[\frac{\frac{25}{9}+330}{330}\right] 660$
$=335.56 \times 2=671.12$
$\therefore$ closest answer is (4)
9. In a communication system operating at wavelength 800 nm , only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $\mathrm{c}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}, \mathrm{h}=6.6 \times 10^{-34} \mathrm{~J}-\mathrm{s}$ )
(1) $3.75 \times 10^{6}$
(2) $4.87 \times 10^{5}$
(3) $3.86 \times 10^{6}$
(4) $6.25 \times 10^{5}$

Ans. (4)

Sol. $\mathrm{f}=\frac{3 \times 10^{8}}{8 \times 10^{-7}}=\frac{30}{8} \times 10^{14} \mathrm{~Hz}$
$=3.75 \times 10^{14} \mathrm{~Hz}$
$1 \%$ of $\mathrm{f}=0.0375 \times 10^{14} \mathrm{~Hz}$
$=3.75 \times 10^{12} \mathrm{~Hz}=3.75 \times 10^{6} \mathrm{MHz}$
number of channels $=\frac{3.75 \times 10^{6}}{6}=6.25 \times 10^{5}$
$\therefore$ correct answer is (4)
10. Two point charges $\mathrm{q}_{1}(\sqrt{10} \mu \mathrm{C})$ and $\mathrm{q}_{2}(-25 \mu \mathrm{C})$ are placed on the x -axis at $\mathrm{x}=1 \mathrm{~m}$ and $\mathrm{x}=4 \mathrm{~m}$ respectively. The electric field (in $\mathrm{V} / \mathrm{m}$ ) at a point $\mathrm{y}=3 \mathrm{~m}$ on y -axis is,
$\left[\right.$ take $\left.\frac{1}{4 \pi \varepsilon_{0}}=9 \times 10^{9} \mathrm{Nm}^{2} \mathrm{C}^{-2}\right]$
(1) $(-63 \hat{\mathrm{i}}+27 \hat{\mathrm{j}}) \times 10^{2}$
(2) $(81 \hat{\mathrm{i}}-81 \hat{\mathrm{j}}) \times 10^{2}$
(3) $(63 \hat{\mathrm{i}}-27 \hat{\mathrm{j}}) \times 10^{2}$
(4) $(-81 \hat{i}+81 \hat{j}) \times 10^{2}$

Ans. (3)

Sol.


Let $\vec{E}_{1} \& \vec{E}_{2}$ are the vaues of electric field due to $\mathrm{q}_{1} \& \mathrm{q}_{2}$ respectively magnitude of $\mathrm{E}_{2}=\frac{1}{4 \pi \epsilon_{0}} \frac{\mathrm{q}_{2}}{\mathrm{r}^{2}}$
$\mathrm{E}_{2}=\frac{9 \times 10^{9} \times(25) \times 10^{-6}}{\left(4^{2}+3^{2}\right)} \mathrm{V} / \mathrm{m}$
$\mathrm{E}_{2}=9 \times 10^{3} \mathrm{~V} / \mathrm{m}$
$\therefore \overrightarrow{\mathrm{E}}_{2}=9 \times 10^{3}\left(\cos \theta_{2} \hat{\mathrm{i}}-\sin \theta_{2} \hat{\mathrm{j}}\right)$
$\because \tan \theta_{2}=\frac{3}{4}$
$\therefore \overrightarrow{\mathrm{E}}_{2}=9 \times 10^{3}\left(\frac{4}{5} \hat{\mathrm{i}}-\frac{3}{5} \hat{\mathrm{j}}\right)=(72 \hat{\mathrm{i}}-54 \hat{\mathrm{j}}) \times 10^{2}$
Magnitude of $\mathrm{E}_{1}=\frac{1}{4 \pi \epsilon_{0}} \frac{\sqrt{10} \times 10^{-6}}{\left(1^{2}+3^{2}\right)}$
$=\left(9 \times 10^{9}\right) \times \sqrt{10} \times 10^{-7}$
$=9 \sqrt{10} \times 10^{2}$
$\therefore \overrightarrow{\mathrm{E}}_{1}=9 \sqrt{10} \times 10^{2}\left[\cos \theta_{1}(-\hat{\mathrm{i}})+\sin \theta_{1} \hat{\mathrm{j}}\right]$
$\therefore \tan \theta_{1}=3$

$E_{1}=9 \times \sqrt{10} \times 10^{2}\left[\frac{1}{\sqrt{10}}(-\hat{\mathrm{i}})+\frac{3}{\sqrt{10}} \hat{\mathrm{j}}\right]$
$\mathrm{E}_{1}=9 \times 10^{2}[-\hat{\mathrm{i}}+3 \hat{\mathrm{j}}]=[-9 \hat{\mathrm{i}}+27 \hat{\mathrm{j}}] 10^{2}$
$\therefore \overrightarrow{\mathrm{E}}=\overrightarrow{\mathrm{E}}_{1}+\overrightarrow{\mathrm{E}}_{2}=(63 \hat{\mathrm{i}}-27 \hat{\mathrm{j}}) \times 10^{2} \mathrm{~V} / \mathrm{m}$
$\therefore$ correct answer is (3)
11. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants $\mathrm{K}_{1}, \mathrm{~K}_{2}, \mathrm{~K}_{3}, \mathrm{~K}_{4}$ arranged as shown in the figure. The effective dielectric constant K will be :

$\rightarrow \mathrm{d} / 2 \rightarrow \mathrm{~d} / 2 \rightarrow$
(1) $\mathrm{K}=\frac{\left(\mathrm{K}_{1}+\mathrm{K}_{2}\right)\left(\mathrm{K}_{3}+\mathrm{K}_{4}\right)}{2\left(\mathrm{~K}_{1}+\mathrm{K}_{2}+\mathrm{K}_{3}+\mathrm{K}_{4}\right)}$
(2) $\mathrm{K}=\frac{\left(\mathrm{K}_{1}+\mathrm{K}_{2}\right)\left(\mathrm{K}_{3}+\mathrm{K}_{4}\right)}{\left(\mathrm{K}_{1}+\mathrm{K}_{2}+\mathrm{K}_{3}+\mathrm{K}_{4}\right)}$
(3) $\mathrm{K}=\frac{\left(\mathrm{K}_{1}+\mathrm{K}_{4}\right)\left(\mathrm{K}_{2}+\mathrm{K}_{3}\right)}{2\left(\mathrm{~K}_{1}+\mathrm{K}_{2}+\mathrm{K}_{3}+\mathrm{K}_{4}\right)}$
(4) $\mathrm{K}=\frac{\left(\mathrm{K}_{1}+\mathrm{K}_{3}\right)\left(\mathrm{K}_{2}+\mathrm{K}_{4}\right)}{\mathrm{K}_{1}+\mathrm{K}_{2}+\mathrm{K}_{3}+\mathrm{K}_{4}}$

## Ans. (Bonus)

Sol.


$C_{12}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}=\frac{\frac{k_{1} \in_{0} \frac{L}{2} \times L}{d / 2} \cdot \frac{k_{2}\left[\epsilon_{0} \frac{L}{2} \times L\right]}{\left(k_{1}+k_{2}\right)\left[\frac{\epsilon_{0} \cdot \frac{L}{2} \times L}{d / 2}\right]}}{\text { d/2 }}$
$\mathrm{C}_{12}=\frac{\mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{k}_{1}+\mathrm{k}_{2}} \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
in the same way we get, $C_{34}=\frac{\mathrm{k}_{3} \mathrm{k}_{4}}{\mathrm{k}_{3}+\mathrm{k}_{4}} \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
$\therefore C_{e q}=C_{12}+C_{34}=\left[\frac{\mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{k}_{1}+\mathrm{k}_{2}}+\frac{\mathrm{k}_{3} \mathrm{k}_{4}}{\mathrm{k}_{3}+\mathrm{k}_{4}}\right] \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
Now if $\mathrm{k}_{\mathrm{eq}}=\mathrm{k}, \mathrm{C}_{\mathrm{eq}}=\frac{\mathrm{k} \in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
on comparing equation (i) to equation (ii), we get
$\mathrm{k}_{\mathrm{eq}}=\frac{\mathrm{k}_{1} \mathrm{k}_{2}\left(\mathrm{k}_{3}+\mathrm{k}_{4}\right)+\mathrm{k}_{3} \mathrm{k}_{4}\left(\mathrm{k}_{1}+\mathrm{k}_{2}\right)}{\left(\mathrm{k}_{1}+\mathrm{k}_{2}\right)\left(\mathrm{k}_{3}+\mathrm{k}_{4}\right)}$
This does not match with any of the options so probably they have assumed the wrong combination
$C_{13}=\frac{\mathrm{k}_{1} \in_{0} \mathrm{~L} \frac{\mathrm{~L}}{2}}{\mathrm{~d} / 2}+\mathrm{k}_{3} \in_{0} \frac{\mathrm{~L} \cdot \frac{\mathrm{~L}}{2}}{\mathrm{~d} / 2}$
$=\left(\mathrm{k}_{1}+\mathrm{k}_{3}\right) \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
$\mathrm{C}_{24}=\left(\mathrm{k}_{2}+\mathrm{k}_{4}\right) \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
$\mathrm{C}_{\mathrm{eq}}=\frac{\mathrm{C}_{13} \mathrm{C}_{24}}{\mathrm{C}_{13} \mathrm{C}_{24}}=\frac{\left(\mathrm{k}_{1}+\mathrm{k}_{3}\right)\left(\mathrm{k}_{2}+\mathrm{k}_{4}\right)}{\left(\mathrm{k}_{1}+\mathrm{k}_{2}+\mathrm{k}_{3}+\mathrm{k}_{4}\right)} \frac{\in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
$=\frac{\mathrm{k} \in_{0} \mathrm{~L}^{2}}{\mathrm{~d}}$
$\mathrm{k}=\frac{\left(\mathrm{k}_{1}+\mathrm{k}_{3}\right)\left(\mathrm{k}_{2}+\mathrm{k}_{4}\right)}{\left(\mathrm{k}_{1}+\mathrm{k}_{2}+\mathrm{k}_{3}+\mathrm{k}_{4}\right)}$
However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)
12. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of $30^{\circ}$ from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in $\mathrm{rad} \mathrm{s}^{-1}$ ) will be $\left(\mathrm{g}=10 \mathrm{~ms}^{-2}\right)$

(1) $\sqrt{30}$
(2) $\sqrt{\frac{30}{2}}$
(3) $\frac{\sqrt{30}}{2}$
(4) $\frac{\sqrt{20}}{3}$

Ans. (2)

Sol.


Work done by gravity from initial to final position is,

$$
\mathrm{W}=\operatorname{mg} \frac{\ell}{2} \sin 30^{\circ}
$$

$=\frac{\mathrm{mg} \ell}{4}$
According to work energy theorem
$\mathrm{W}=\frac{1}{2} \mathrm{I} \omega^{2}$
$\Rightarrow \frac{1}{2} \frac{\mathrm{~m} \ell^{2}}{3} \omega^{2}=\frac{\mathrm{mg} \ell}{4}$
$\omega=\sqrt{\frac{3 \mathrm{~g}}{2 \ell}}=\sqrt{\frac{3 \times 10}{2 \times 0.5}}$
$\omega=\sqrt{30} \mathrm{rad} / \mathrm{sec}$
$\therefore$ correct answer is (1)
13. One of the two identical conducting wires of length $L$ is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop $\left(\mathrm{B}_{\mathrm{L}}\right)$ to that at the centre of the coil $\left(B_{C}\right)$, i.e. $R \frac{B_{L}}{B_{C}}$ will be :
(1) $\frac{1}{\mathrm{~N}}$
(2) $\mathrm{N}^{2}$
(3) $\frac{1}{\mathrm{~N}^{2}}$
(4) N

Ans. (3)

Sol.

$\mathrm{L}=2 \pi \mathrm{R} \quad \mathrm{L}=\mathrm{N} \times 2 \pi \mathrm{r}$
$\mathrm{R}=\mathrm{Nr}$
$B_{L}=\frac{\mu_{0} \mathrm{i}}{2 R} \quad B_{C}=\frac{\mu_{0} \mathrm{Ni}}{2 \mathrm{r}}$
$B_{C}=\frac{\mu_{0} \mathrm{~N}^{2} \mathrm{i}}{2 \mathrm{R}}$
$\frac{\mathrm{B}_{\mathrm{L}}}{\mathrm{B}_{\mathrm{C}}}=\frac{1}{\mathrm{~N}^{2}}$
14. The energy required to take a satellite to a height ' h ' above Earth surface (radius of Earth $\left.=6.4 \times 10^{3} \mathrm{~km}\right)$ is $\mathrm{E}_{1}$ and kinetic energy required for the satellite to be in a circular orbit at this height is $E_{2}$. The value of $h$ for which $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ are equal, is:
(1) $1.28 \times 10^{4} \mathrm{~km}$
(2) $6.4 \times 10^{3} \mathrm{~km}$
(3) $3.2 \times 10^{3} \mathrm{~km}$
(4) $1.6 \times 10^{3} \mathrm{~km}$

Ans. (3)
Sol. $\mathrm{U}_{\text {surface }}+\mathrm{E}_{1}=\mathrm{U}_{\mathrm{h}}$
KE of satelite is zero at earth surface \& at height h
$-\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{\mathrm{R}_{\mathrm{e}}}+\mathrm{E}_{1}=-\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{(\mathrm{Re}+\mathrm{h})}$
$\mathrm{E}_{1}=\mathrm{GM}_{\mathrm{e}} \mathrm{m}\left(\frac{1}{\mathrm{R}_{\mathrm{e}}}-\frac{1}{\mathrm{R}_{\mathrm{e}}+\mathrm{h}}\right)$
$\mathrm{E}_{1}=\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)} \times \frac{\mathrm{h}}{\mathrm{R}_{\mathrm{e}}}$
Gravitational attraction $F_{G}=m a_{C}=\frac{m v^{2}}{\left(R_{e}+h\right)}$
$\mathrm{E}_{2} \Rightarrow \frac{\mathrm{mv}^{2}}{\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)}=\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)^{2}}$
$\mathrm{mv}^{2}=\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)}$
$\mathrm{E}_{2}=\frac{\mathrm{mv}^{2}}{2}=\frac{\mathrm{GM}_{\mathrm{e}} \mathrm{m}}{2\left(\mathrm{R}_{\mathrm{e}}+\mathrm{h}\right)}$
$\mathrm{E}_{1}=\mathrm{E}_{2}$
$\frac{\mathrm{h}}{\mathrm{R}_{\mathrm{e}}}=\frac{1}{2} \Rightarrow \mathrm{~h}=\frac{\mathrm{R}_{\mathrm{e}}}{2}=3200 \mathrm{~km}$
15. The energy associated with electric field is $\left(\mathrm{U}_{\mathrm{E}}\right)$ and with magnetic field is $\left(U_{B}\right)$ for an electromagnetic wave in free space. Then :
(1) $U_{E}=\frac{U_{B}}{2}$
(2) $\mathrm{U}_{\mathrm{E}}<\mathrm{U}_{\mathrm{B}}$
(3) $U_{E}=U_{B}$
(4) $U_{E}>U_{B}$

Ans. (3)
Sol. Average energy density of magnetic field,
$u_{B}=\frac{B_{0}^{2}}{2 \mu_{0}}, B_{0}$ is maximum value of magnetic field.
Average energy density of electric field,
$\mathrm{u}_{\mathrm{E}}=\frac{\varepsilon_{0} \in_{0}^{2}}{2}$
now, $\epsilon_{0}=\mathrm{CB}_{0}, \mathrm{C}^{2}=\frac{1}{\mu_{0} \in_{0}}$
$\mathrm{u}_{\mathrm{E}}=\frac{\epsilon_{0}}{2} \times \mathrm{C}^{2} \mathrm{~B}_{0}^{2}$
$=\frac{\in_{0}}{2} \times \frac{1}{\mu_{0} \epsilon_{0}} \times \mathrm{B}_{0}^{2}=\frac{\mathrm{B}_{0}^{2}}{2 \mu_{0}}=\mathrm{u}_{\mathrm{B}}$
$\mathrm{u}_{\mathrm{E}}=\mathrm{u}_{\mathrm{B}}$
since energy density of electric \& magnetic field is same, energy associated with equal volume will be equal.
$u_{E}=u_{B}$
16. A series AC circuit containing an inductor ( 20 $\mathrm{mH})$, a capacitor ( $120 \mu \mathrm{~F}$ ) and a resistor ( $60 \Omega$ ) is driven by an AC source of $24 \mathrm{~V} / 50 \mathrm{~Hz}$. The energy dissipated in the circuit in 60 s is :
(1) $2.26 \times 10^{3} \mathrm{~J}$
(2) $3.39 \times 10^{3} \mathrm{~J}$
(3) $5.65 \times 10^{2} \mathrm{~J}$
(4) $5.17 \times 10^{2} \mathrm{~J}$

Ans. (4)
Sol. $R=60 \Omega \quad \mathrm{f}=50 \mathrm{~Hz}, \omega=2 \pi \mathrm{f}=100 \pi$
$\mathrm{x}_{\mathrm{C}}=\frac{1}{\omega \mathrm{C}}=\frac{1}{100 \pi \times 120 \times 10^{-6}}$
$\mathrm{x}_{\mathrm{C}}=26.52 \Omega$
$\mathrm{x}_{\mathrm{L}}=\omega \mathrm{L}=100 \pi \times 20 \times 10^{-3}=2 \pi \Omega$
$\mathrm{x}_{\mathrm{C}}-\mathrm{x}_{\mathrm{L}}=20.24 \approx 20$

$\mathrm{z}=\sqrt{\mathrm{R}^{2}+\left(\mathrm{x}_{\mathrm{C}}-\mathrm{x}_{\mathrm{L}}\right)^{2}}$
$\mathrm{z}=20 \sqrt{10} \Omega$
$\cos \phi=\frac{\mathrm{R}}{\mathrm{z}}=\frac{3}{\sqrt{10}}$
$\mathrm{P}_{\text {avg }}=\mathrm{VI} \cos \phi, \mathrm{I}=\frac{\mathrm{v}}{\mathrm{z}}$
$=\frac{\mathrm{v}^{2}}{\mathrm{z}} \cos \phi$
$=8.64$ watt
$\mathrm{Q}=\mathrm{P} . \mathrm{t}=8.64 \times 60=5.18 \times 10^{2}$
17. Expression for time in terms of $G$ (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :
(1) $\sqrt{\frac{G h}{c^{3}}}$
(2) $\sqrt{\frac{\mathrm{hc}^{5}}{\mathrm{G}}}$
(3) $\sqrt{\frac{c^{3}}{G h}}$
(4) $\sqrt{\frac{G h}{c^{5}}}$

Ans. (4)

Sol. $\mathrm{F}=\frac{\mathrm{GM}^{2}}{\mathrm{R}^{2}} \Rightarrow \mathrm{G}=\left[\mathrm{M}^{-1} \mathrm{~L}^{3} \mathrm{~T}^{-2}\right]$
$\mathrm{E}=\mathrm{h} \nu \Rightarrow \mathrm{h}=\left[\mathrm{ML}^{2} \mathrm{~T}^{-1}\right]$
$\mathrm{C}=\left[\mathrm{LT}^{-1}\right]$
$\mathrm{t} \propto \mathrm{G}^{x} \mathrm{~h}^{y} \mathrm{C}^{z}$
$[\mathrm{T}]=\left[\mathrm{M}^{-1} \mathrm{~L}^{3} \mathrm{~T}^{-2}\right]^{x}\left[\mathrm{ML}^{2} \mathrm{~T}^{-1}\right] y\left[\mathrm{LT}^{-1}\right]^{\mathrm{z}}$
$\left[M^{0} L^{0} T^{1}\right]=\left[M^{-x}+y L^{3 x}+2 y+z T^{-2 x-y-z}\right]$
on comparing the powers of $\mathrm{M}, \mathrm{L}, \mathrm{T}$
$-\mathrm{x}+\mathrm{y}=0 \Rightarrow \mathrm{x}=\mathrm{y}$
$3 \mathrm{x}+2 \mathrm{y}+\mathrm{z}=0 \Rightarrow 5 \mathrm{x}+\mathrm{z}=0$
$-2 \mathrm{x}-\mathrm{y}-\mathrm{z}=1 \Rightarrow 3 \mathrm{x}+\mathrm{z}=-1$
on solving (i) \& (ii) $\mathrm{x}=\mathrm{y}=\frac{1}{2}, \mathrm{z}=-\frac{5}{2}$
$\mathrm{t} \propto \sqrt{\frac{\mathrm{Gh}}{\mathrm{C}^{5}}}$
18. The magnetic field associated with a light wave is given, at the origin, by $B=B_{0}\left[\sin \left(3.14 \times 10^{7}\right) \mathrm{ct}+\sin \left(6.28 \times 10^{7}\right) \mathrm{ct}\right]$. If this light falls on a silver plate having a work function of 4.7 eV , what will be the maximum kinetic energy of the photo electrons ?
(c $=3 \times 10^{8} \mathrm{~ms}^{-1}, \mathrm{~h}=6.6 \times 10^{-34} \mathrm{~J}-\mathrm{s}$ )
(1) 7.72 eV
(2) 8.52 eV
(3) 12.5 eV
(4) 6.82 eV

Ans. (1)
Sol. $\quad \mathrm{B}=\mathrm{B}_{0} \sin \left(\pi \times 10^{7} \mathrm{C}\right) \mathrm{t}+\mathrm{B}_{0} \sin \left(2 \pi \times 10^{7} \mathrm{C}\right) \mathrm{t}$ since there are two EM waves with different frequency, to get maximum kinetic energy we take the photon with higher frequency
$\mathrm{B}_{1}=\mathrm{B}_{0} \sin \left(\pi \times 10^{7} \mathrm{C}\right) \mathrm{t} \quad \mathrm{v}_{1}=\frac{10^{7}}{2} \times \mathrm{C}$
$\mathrm{B}_{2}=\mathrm{B}_{0} \sin \left(2 \pi \times 10^{7} \mathrm{C}\right) \mathrm{t} \mathrm{v}_{2}=10^{7} \mathrm{C}$
where C is speed of light $\mathrm{C}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ $v_{2}>v_{1}$
so KE of photoelectron will be maximum for photon of higher energy.
$v_{2}=10^{7} \mathrm{C} \mathrm{Hz}$
$h \nu=\phi+\mathrm{KE}_{\text {max }}$
energy of photon
$\mathrm{E}_{\mathrm{ph}}=\mathrm{h} v=6.6 \times 10^{-34} \times 10^{7} \times 3 \times 10^{9}$
$\mathrm{E}_{\mathrm{ph}}=6.6 \times 3 \times 10^{-19} \mathrm{~J}$
$=\frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}} \mathrm{eV}=12.375 \mathrm{eV}$
$\mathrm{KE}_{\text {max }}=\mathrm{E}_{\mathrm{ph}}-\phi$
$=12.375-4.7=7.675 \mathrm{eV} \approx 7.7 \mathrm{eV}$
19. Charge is distributed within a sphere of radius $R$ with a volume charge density $\rho(r)=\frac{A}{r^{2}} e^{-2 r / a}$, where $A$ and a are constants. If $Q$ is the total charge of this charge distribution, the radius R is :
(1) $\frac{\mathrm{a}}{2} \log \left(1-\frac{\mathrm{Q}}{2 \pi \mathrm{aA}}\right)$
(2) $a \log \left(1-\frac{Q}{2 \pi a \mathrm{~A}}\right)$
(3) $a \log \left(\frac{1}{1-\frac{\mathrm{Q}}{2 \pi \mathrm{aA}}}\right)$
(4) $\frac{\mathrm{a}}{2} \log \left(\frac{1}{1-\frac{\mathrm{Q}}{2 \pi \mathrm{aA}}}\right)$

Ans. (4)

Sol.

$Q=\int \rho d v$
$=\int_{0}^{\mathrm{R}} \frac{\mathrm{A}}{\mathrm{r}^{2}} \mathrm{e}^{-2 \mathrm{r} / \mathrm{a}}\left(4 \pi \mathrm{r}^{2} \mathrm{dr}\right)$
$=\int_{0}^{\mathrm{R}} \frac{\mathrm{A}}{\mathrm{r}^{2}} \mathrm{e}^{-2 \mathrm{r} / \mathrm{a}}\left(4 \pi \mathrm{r}^{2} \mathrm{dr}\right)$
$=4 \pi \mathrm{~A} \int_{0}^{\mathrm{R}} \mathrm{e}^{-2 \mathrm{r} / \mathrm{a}} \mathrm{dr}$
$=4 \pi \mathrm{~A}\left(\frac{\mathrm{e}^{-2 \mathrm{r} / \mathrm{a}}}{-\frac{2}{\mathrm{a}}}\right)_{0}^{\mathrm{R}}$
$=4 \pi \mathrm{~A}\left(-\frac{\mathrm{a}}{2}\right)\left(\mathrm{e}^{-2 \mathrm{R} / \mathrm{a}}-1\right)$
$\mathrm{Q}=2 \pi \mathrm{a} \mathrm{A}\left(1-\mathrm{e}^{-2 \mathrm{R} / \mathrm{a}}\right)$
$\mathrm{R}=\frac{\mathrm{a}}{2} \log \left(\frac{1}{1-\frac{\mathrm{Q}}{2 \pi \mathrm{aA}}}\right)$
20. Two Carrnot engines $A$ and $B$ are operated in series. The first one, A, receives heat at $T_{1}(=600$ K ) and rejects to a reservoir at temperature $\mathrm{T}_{2}$. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at $\mathrm{T}_{3}(=400 \mathrm{~K})$. Calculate the temperature $\mathrm{T}_{2}$ if the work outputs of the two engines are equal :
(1) 400 K
(2) 600 K
(3) 500 K
(4) 300 K

Ans. (3)

Sol.

$\mathrm{w}_{1}=\mathrm{w}_{2}$
$\Delta \mathrm{u}_{1}=\Delta \mathrm{u}_{2}$
$\mathrm{T}_{3}-\mathrm{T}_{2}=\mathrm{T}_{2}-\mathrm{T}_{1}$
$2 \mathrm{~T}_{2}=\mathrm{T}_{1}+\mathrm{T}_{3}$
$\mathrm{T}_{2}=500 \mathrm{~K}$
21. A carbon resistance has a following colour code. What is the value of the resistance ?

(1) $1.64 \mathrm{M} \Omega \pm 5 \%$
(2) $530 \mathrm{k} \Omega \pm 5 \%$
(3) $64 \mathrm{k} \Omega \pm 10 \%$
(4) $5.3 \mathrm{M} \Omega \pm 5 \%$

Ans. (2)

Sol.

$\mathrm{R}=53 \times 10^{4} \pm 5 \%=530 \mathrm{k} \Omega \pm 5 \%$
22. A force acts on a 2 kg object so that its position is given as a function of time as $\mathrm{x}=3 \mathrm{t}^{2}+5$. What is the work done by this force in first 5 seconds ?
(1) 850 J
(2) 900 J
(3) 950 J
(4) 875 J

Ans. (2)
Sol. $\mathrm{x}=3 \mathrm{t}^{2}+5$
$\mathrm{v}=\frac{\mathrm{dx}}{\mathrm{dt}}$
$v=6 t+0$
at $t=0 \quad v=0$
$\mathrm{t}=5 \mathrm{sec} \quad \mathrm{v}=30 \mathrm{~m} / \mathrm{s}$
W.D. $=\Delta \mathrm{KE}$
W.D. $=\frac{1}{2} \mathrm{mv}^{2}-0=\frac{1}{2}(2)(30)^{2}=900 \mathrm{~J}$
23. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$$
\begin{aligned}
& x=a \cos \omega t \\
& y=a \sin \omega t
\end{aligned}
$$

and $\mathrm{z}=\mathrm{a} \omega \mathrm{t}$
The speed of the particle is :
(1) $\mathrm{a} \omega$
(2) $\sqrt{3} \mathrm{a} \omega$
(3) $\sqrt{2} \mathrm{a} \omega$
(4) $2 \mathrm{a} \omega$

Ans. (3)
Sol. $\mathrm{v}_{\mathrm{x}}=-\mathrm{a} \omega \sin \omega \mathrm{t} \Rightarrow \mathrm{v}_{\mathrm{y}}=\mathrm{a} \omega \cos \omega \mathrm{t}$
$\mathrm{v}_{\mathrm{z}}=\mathrm{a} \omega \quad \Rightarrow \mathrm{v}=\sqrt{\mathrm{v}_{\mathrm{x}}^{2}+\mathrm{v}_{\mathrm{y}}^{2}+\mathrm{v}_{\mathrm{z}}^{2}}$
$\mathrm{v}=\sqrt{2} \mathrm{a} \omega$
24. In the given circuit the internal resistance of the 18 V cell is negligible. If $\mathrm{R}_{1}=400 \Omega, \mathrm{R}_{3}=100 \Omega$ and $\mathrm{R}_{4}=500 \Omega$ and the reading of an ideal voltmeter across $R_{4}$ is 5 V , then the value $R_{2}$ will be :

(1) $300 \Omega$
(2) $230 \Omega$
(3) $450 \Omega$
(4) $550 \Omega$

Ans. (1)

Sol.

$\mathrm{V}_{4}=5 \mathrm{~V}$
$\mathrm{i}_{1}=\frac{\mathrm{V}_{4}}{\mathrm{R}_{4}}=0.01 \mathrm{~A}$
$\mathrm{V}_{3}=\mathrm{i}_{1} \mathrm{R}_{3}=1 \mathrm{~V}$
$V_{3}+V_{4}=6 V=V_{2}$
$\mathrm{V}_{1}+\mathrm{V}_{3}+\mathrm{V}_{4}=18 \mathrm{~V}$
$\mathrm{V}_{1}=12 \mathrm{~V}$
$\mathrm{i}=\frac{\mathrm{V}_{1}}{\mathrm{R}_{1}}=0.03 \mathrm{Amp}$.
$\mathrm{i}_{2}=0.02 \mathrm{Amp} \quad \mathrm{V}_{2}=6 \mathrm{~V}$
$\mathrm{R}_{2}=\frac{\mathrm{V}_{2}}{\mathrm{i}_{2}}=\frac{6}{0.02}=300 \Omega$
25. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of $45^{\circ}$ at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is $\left(\mathrm{g}=10 \mathrm{~ms}^{-2}\right)$
(1) 200 N
(2) 100 N
(3) 140 N
(4) 70 N

Ans. (2)

Sol.

at equation
$\tan 45^{\circ}=\frac{100}{\mathrm{~F}}$
$\mathrm{F}=100 \mathrm{~N}$
26. In a car race on straight road, car $A$ takes a time $t$ less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration $a_{1}$ and $a_{2}$ respectively. Then ' $v$ ' is equal to
(1) $\frac{a_{1}+a_{2}}{2} t$
(2) $\sqrt{2 \mathrm{a}_{1} \mathrm{a}_{2}} t$
(3) $\frac{2 a_{1} a_{2}}{a_{1}+a_{2}} t$
(4) $\sqrt{a_{1} a_{2}} t$

Ans. (4)
Sol. For A \& B let time taken by A is $t_{0}$

from ques.
$\mathrm{v}_{\mathrm{A}}-\mathrm{v}_{\mathrm{B}}=\mathrm{v}=\left(\mathrm{a}_{1}-\mathrm{a}_{2}\right) \mathrm{t}_{0}-\mathrm{a}_{2} \mathrm{t}$
$\mathrm{x}_{\mathrm{B}}=\mathrm{x}_{\mathrm{A}}=\frac{1}{2} \mathrm{a}_{1} \mathrm{t}_{0}^{2}=\frac{1}{2} \mathrm{a}_{2}\left(\mathrm{t}_{0}+\mathrm{t}\right)^{2}$
$\Rightarrow \sqrt{\mathrm{a}_{1}} \mathrm{t}_{0}=\sqrt{\mathrm{a}_{2}}\left(\mathrm{t}_{0}+\mathrm{t}\right)$
$\Rightarrow\left(\sqrt{\mathrm{a}_{2}}-\sqrt{\mathrm{a}_{2}}\right) \mathrm{t}_{0}=\sqrt{\mathrm{a}_{2}} \mathrm{t}$
putting $t_{0}$ in equation
$v=\left(a_{1}-a_{2}\right) \frac{\sqrt{a_{2}} t}{\sqrt{a_{1}}-\sqrt{a_{2}}}-a_{2} t$
$=\left(\sqrt{\mathrm{a}_{1}}+\sqrt{\mathrm{a}_{2}}\right) \sqrt{\mathrm{a}_{2}} \mathrm{t}-\mathrm{a}_{2} \mathrm{t} \Rightarrow \mathrm{v}=\sqrt{\mathrm{a}_{1} \mathrm{a}_{2}} \mathrm{t}$
$\Rightarrow \sqrt{\mathrm{a}_{1} \mathrm{a}_{2}} \mathrm{t}+\mathrm{a}_{2} \mathrm{t}-\mathrm{a}_{2} \mathrm{t}$
27. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is $90 \%$, the output current would be :
(1) 25 A
(2) 50 A
(3) 35 A
(4) 45 A

Ans. (4)
Sol. $\eta=\frac{P_{\text {out }}}{P_{\text {in }}}=\frac{V_{S} I_{S}}{V_{P} I_{P}}$
$\Rightarrow 0.9=\frac{23 \times \mathrm{I}_{\mathrm{S}}}{230 \times 5}$
$\Rightarrow \mathrm{I}_{\mathrm{S}}=45 \mathrm{~A}$
28. The top of a water tank is open to air and its water level is maintained. It is giving out $0.74 \mathrm{~m}^{3}$ water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :
(1) 9.6 m
(2) 4.8 m
(3) 2.9 m
(4) 6.0 m

Ans. (2)
Sol. In flow volume = outflow volume
$\Rightarrow \frac{0.74}{60}=\left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2 \mathrm{gh}}$
$\Rightarrow \sqrt{2 \mathrm{gh}}=\frac{74 \times 100}{240 \pi}$
$\Rightarrow \sqrt{2 \mathrm{gh}}=\frac{740}{24 \pi}$
$\Rightarrow 2 \mathrm{gh}=\frac{740 \times 740}{24 \times 24 \times 10}\left(\pi^{2}=10\right)$
$\Rightarrow \mathrm{h}=\frac{74 \times 74}{2 \times 24 \times 24}$
$\Rightarrow \mathrm{h} \approx 4.8 \mathrm{~m}$
29. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.
The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is :
(1) 5.755 m
(2) 5.725 mm
(3) 5.740 m
(4) 5.950 mm

Ans. (2)
Sol. $L C=\frac{\text { Pitch }}{\text { No. of division }}$
$\mathrm{LC}=0.5 \times 10^{-2} \mathrm{~mm}$

+ ve error $=3 \times 0.5 \times 10^{-2} \mathrm{~mm}$
$=1.5 \times 10^{-2} \mathrm{~mm}=0.015 \mathrm{~mm}$
Reading $=$ MSR + CSR $-(+v e$ error $)$
$=5.5 \mathrm{~mm}+\left(48 \times 0.5 \times 10^{-2}\right)-0.015$
$=5.5+0.24-0.015=5.725 \mathrm{~mm}$

30. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T . If an electric field of $100 \mathrm{~V} / \mathrm{m}$ makes it to move in a straight path, then the mass of the particle is (Given charge of electron $=1.6 \times 10^{-19} \mathrm{C}$ )
(1) $2.0 \times 10^{-24} \mathrm{~kg}$
(2) $1.6 \times 10^{-19} \mathrm{~kg}$
(3) $1.6 \times 10^{-27} \mathrm{~kg}$
(4) $9.1 \times 10^{-31} \mathrm{~kg}$

Ans. (1)
Sol. $\frac{\mathrm{mv}^{2}}{\mathrm{R}}=\mathrm{qvB}$
$m v=q B R$
Path is straight line
it $q E=q v B$
$\mathrm{E}=\mathrm{vB}$
From equation (i) \& (ii)
$m=\frac{q B^{2} R}{E}$
$\mathrm{m}=2.0 \times 10^{-24} \mathrm{~kg}$

