## TEST PAPER OF JEE(MAIN) EXAMINATION - 2019

## (Held On Wednesday 09th JANUARY, 2019) TIME : 9:30 AM To 12:30 PM PHYSICS

1. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A . The magnetic field at point O will be close to :

(1) $1.0 \times 10^{-5} \mathrm{~T}$
(2) $1.5 \times 10^{-5} \mathrm{~T}$
(3) $1.0 \times 10^{-7} \mathrm{~T}$
(4) $1.0 \times 10^{-7} \mathrm{~T}$

Ans. (1)

Sol.

$\overrightarrow{\mathrm{B}}=\frac{\mu_{0} \mathrm{i}}{4 \pi} \theta\left[\frac{1}{\mathrm{r}_{1}}-\frac{1}{\mathrm{r}_{2}}\right] \hat{\mathrm{k}}$
$\mathrm{r}_{1}=3 \mathrm{~cm}=3 \times 10^{-2} \mathrm{~m}$
$\mathrm{r}_{2}=5 \mathrm{~cm}=5 \times 10^{-2} \mathrm{~m}$
$\theta=\frac{\pi}{4}, \mathrm{i}=10 \mathrm{~A}$
$\Rightarrow \overrightarrow{\mathrm{B}}=\frac{4 \pi \times 10^{-7}}{16} \times 10\left[\frac{1}{3 \times 10^{-2}}-\frac{1}{5 \times 10^{-2}}\right] \hat{\mathrm{k}}$
$\Rightarrow|\overrightarrow{\mathrm{B}}|=\frac{\pi}{3} \times 10^{-5} \mathrm{~T}$
$\approx 1 \times 10^{-5} \mathrm{~T}$
2. $A$ gas can be taken from $A$ to $B$ via two different processes ACB and ADB .


When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J . The heat Flow into the system in path ADB is :
(1) 80 J
(2) 20 J
(3) 100 J
(4) 40 J

Ans. (4)

Sol.

$\Delta \mathrm{Q}_{\mathrm{ACB}}=\Delta \mathrm{W}_{\mathrm{ACB}}+\Delta \mathrm{U}_{\mathrm{ACB}}$
$\Rightarrow 60 \mathrm{~J}=30 \mathrm{~J}+\Delta \mathrm{U}_{\mathrm{ACB}}$
$\Rightarrow \Delta \mathrm{U}_{\mathrm{ACB}}=30 \mathrm{~J}$
$\Rightarrow \Delta \mathrm{U}_{\mathrm{ADB}}=\Delta \mathrm{U}_{\mathrm{ACB}}=30 \mathrm{~J}$
$\Delta \mathrm{Q}_{\mathrm{ACD}}=\Delta \mathrm{U}_{\mathrm{ACB}}+\Delta \mathrm{W}_{\mathrm{ADB}}$
$=10 \mathrm{~J}+30 \mathrm{~J}=40 \mathrm{~J}$
3. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x direction. At a particular point in space and time, $\quad \overrightarrow{\mathrm{E}}=6.3 \hat{\mathrm{j}} \mathrm{V} / \mathrm{m}$. The corresponding magnetic field $\overrightarrow{\mathrm{B}}$, at that point will be:
(1) $18.9 \times 10^{-8} \hat{\mathrm{k}} \mathrm{T}$
(2) $6.3 \times 10^{-8} \hat{\mathrm{k}} \mathrm{T}$
(3) $2.1 \times 10^{-8} \hat{\mathrm{k}} \mathrm{T}$
(4) $18.9 \times 10^{8} \mathrm{k} \mathrm{T}$

Ans. (3)
Sol. $|B|=\frac{|E|}{C}=\frac{6.3}{3 \times 10^{8}}=2.1 \times 10^{-8} \mathrm{~T}$ and $\hat{E} \times \hat{B}=\hat{C}$

$$
\hat{\mathrm{j}} \times \hat{\mathrm{B}}=\hat{\mathrm{i}}
$$

$\hat{\mathrm{B}}=\hat{\mathrm{k}}$
$\overrightarrow{\mathrm{B}}=|\mathrm{B}| \hat{\mathrm{B}}=2.1 \times 10^{8} \hat{\mathrm{k}} \mathrm{T}$

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4. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16 . The intensity of the waves are in the ratio:
(1) $4: 1$
(2) $25: 9$
(3) $16: 9$
(4) $5: 3$

Ans. (2)
Sol. $\frac{I_{\text {max }}}{I_{\text {min }}}=16$
$\Rightarrow \frac{\mathrm{A}_{\text {max }}}{\mathrm{A}_{\text {min }}}=4$
$\Rightarrow \frac{\mathrm{A}_{1}+\mathrm{A}_{2}}{\mathrm{~A}_{1}-\mathrm{A}_{2}}=\frac{4}{1}$
Using componendo \& dividendo.
$\frac{\mathrm{A}_{1}}{\mathrm{~A}_{2}}=\frac{5}{3} \Rightarrow \frac{\mathrm{I}_{1}}{\mathrm{I}_{2}}=\left(\frac{5}{3}\right)^{2}=\frac{25}{9}$
5. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $\mathrm{AB}=\mathrm{BC}$, and the angle made by AB with downward vertical is $\theta$, then :

(1) $\tan \theta=\frac{2}{\sqrt{3}}$
(2) $\tan \theta=\frac{1}{3}$
(3) $\tan \theta=\frac{1}{2}$
(4) $\tan \theta=\frac{1}{2 \sqrt{3}}$

Ans. (2)

Sol.


Let mass of one rod is m .
Balancing torque about hinge point.
$m g\left(\mathrm{C}_{1} \mathrm{P}\right)=m g\left(\mathrm{C}_{2} \mathrm{~N}\right)$
$m g\left(\frac{\mathrm{~L}}{2} \sin \theta\right)=\operatorname{mg}\left(\frac{\mathrm{L}}{2} \cos \theta-\mathrm{L} \sin \theta\right)$
$\Rightarrow \frac{3}{2} \mathrm{mgL} \sin \theta=\frac{\mathrm{mgL}}{2} \cos \theta$
$\Rightarrow \tan \theta=\frac{1}{3}$
6. A mixture of 2 moles of helium gas (atomic mass $=4 \mathrm{u}$ ), and 1 mole of argon gas (atomic mass $=40 \mathrm{u}$ ) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{\mathrm{V}_{\mathrm{rms}}(\text { helium })}{\mathrm{V}_{\mathrm{rms}}(\operatorname{(arg} \text { on })}\right]$, is close to
(1) 2.24
(2) 0.45
(3) 0.32
(4) 3.16

Ans. (4)
Sol. $\frac{\mathrm{V}_{\mathrm{rms}}(\mathrm{He})}{\mathrm{V}_{\mathrm{rms}}(\mathrm{Ar})}=\sqrt{\frac{\mathrm{M}_{\mathrm{Ar}}}{\mathrm{M}_{\mathrm{He}}}}=\sqrt{\frac{40}{4}}=3.16$
7. When the switch $S$, in the circuit shown, is closed, then the value of current $i$ will be :

(1) 3 A
(2) 5 A
(3) 4 A
(4) 2 A

Ans. (2)

Let voltage at $\mathrm{C}=\mathrm{xv}$
KCL: $\mathrm{i}_{1}+\mathrm{i}_{2}=\mathrm{i}$
$\frac{20-\mathrm{x}}{2}+\frac{10-\mathrm{x}}{4}=\frac{\mathrm{x}-0}{2}$
$\Rightarrow \mathrm{x}=10$
and $\mathrm{i}=5 \mathrm{amp}$.
8. A resistance is shown in the figure. Its value and tolerance are given respectively by:

(1) $27 \mathrm{~K} \Omega, 20 \%$
(2) $270 \mathrm{~K} \Omega, 5 \%$
(3) $270 \mathrm{~K} \Omega, 10 \%$
(4) $27 \mathrm{~K} \Omega, 10 \%$

Ans. (4)
Sol. Color code :
Red violet orange silver
$\mathrm{R}=27 \times 10^{3} \Omega \pm 10 \%$
$=27 \mathrm{~K} \Omega \pm 10 \%$
9. A bar magnet is demagnetized by inserting it inside a solenoid of length $0.2 \mathrm{~m}, 100$ turas, and carrying a current of 5.2 A . The coercivitv of the bar magnet is :
(1) $1200 \mathrm{~A} / \mathrm{m}$
(2) $2600 \mathrm{~A} / \mathrm{m}$
(3) $520 \mathrm{~A} / \mathrm{jm}$
(4) $285 \mathrm{~A} / \mathrm{m}$

Ans. (2)
Sol. Coercivity $=\mathrm{H}=\frac{\mathrm{B}}{\mu_{0}}$
$=n i=\frac{\mathrm{N}}{\ell} \mathrm{i}=\frac{100}{0.2} \times 5.2$
$=2600 \mathrm{~A} / \mathrm{m}$
10. A rod, of length $L$ at room temperature and uniform area of cross section $A$, is made of a metal having coefficient of linear expansion $\alpha /$ ${ }^{\circ} \mathrm{C}$. It is observed that an external compressive force $F$, is applied on
each of its ends, prevents any change in the length of the rod, when its temperature rises by $\Delta \mathrm{T}$ K. Young's modulus, Y , for this metal is :
(1) $\frac{F}{2 A \alpha \Delta T}$
(2) $\frac{\mathrm{F}}{\mathrm{A} \alpha(\Delta \mathrm{T}-273)}$
(3) $\frac{F}{A \alpha \Delta T}$
(4) $\frac{2 F}{A \alpha \Delta T}$

Ans. (3)
Sol. Young's modulus $\mathrm{y}=\frac{\text { Stress }}{\text { Strain }}$

$$
\begin{aligned}
& =\frac{\mathrm{F} / \mathrm{A}}{(\Delta \ell / \ell)} \\
= & \frac{\mathrm{F}}{\mathrm{~A}(\alpha \Delta \mathrm{~T})}
\end{aligned}
$$

11. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant $k$. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F , the maximum speed of the block is :

(1) $\frac{\pi \mathrm{F}}{\sqrt{\mathrm{mk}}}$
(2) $\frac{2 \mathrm{~F}}{\sqrt{\mathrm{mk}}}$
(3) $\frac{\mathrm{F}}{\sqrt{\mathrm{mk}}}$
(4) $\frac{\mathrm{F}}{\pi \sqrt{\mathrm{mk}}}$

Ans. (3)
Sol. Maximum speed is at mean position (equilibrium). $\mathrm{F}=\mathrm{kx}$

$$
\begin{gathered}
\mathrm{x}=\frac{\mathrm{F}}{\mathrm{k}} \\
\mathrm{~W}_{\mathrm{F}}+\mathrm{W}_{\mathrm{sp}}=\Delta \mathrm{KE} \\
\mathrm{~F}(\mathrm{x})-\frac{1}{2} \mathrm{kx}^{2}=\frac{1}{2} \mathrm{mv}^{2}-0 \\
\mathrm{~F}\left(\frac{\mathrm{~F}}{\mathrm{k}}\right)-\frac{1}{2} \mathrm{k}\left(\frac{\mathrm{~F}}{\mathrm{k}}\right)^{2}=\frac{1}{2} \mathrm{mv}^{2} \\
\Rightarrow \mathrm{v}_{\max }=\frac{\mathrm{F}}{\sqrt{\mathrm{mk}}}
\end{gathered}
$$

12. Three charges $+\mathrm{Q}, \mathrm{q},+\mathrm{Q}$ are placed respectively, at distance, $0, \mathrm{~d} / 2$ and d from the origin, on the $x$-axis. If the net force experienced by +Q , placed at $\mathrm{x}=0$, Ls zero, then value of q is :
(1) $+Q / 2$
(2) $-Q / 2$
(3) $-\mathrm{Q} / 4$
(4) $+Q / 4$

Ans. (3)

Sol.


For equilibrium,
$\overrightarrow{\mathrm{F}}_{\mathrm{a}}+\overrightarrow{\mathrm{F}}_{\mathrm{B}}=0$
$\vec{F}_{a}=-\vec{F}_{B}$
$\frac{k Q Q}{d^{2}}=-\frac{k Q q}{(d / 2)^{2}}$
$\Rightarrow \mathrm{q}=-\frac{\mathrm{Q}}{4}$
13. A conducting circular loop made of a thill wire, has area $3.5 \times 10^{-3} \mathrm{~m}^{2}$ and resistance $10 \Omega$. It is placed perpendicular to a time dependent magnetic field
$\mathrm{B}(\mathrm{t})=(0.4 \mathrm{~T}) \sin (50 \pi \mathrm{t})$. The field is uniform in space. Then the net charge flowing through the loop during $\mathrm{t}=0 \mathrm{~s}$ and $\mathrm{t}=10 \mathrm{~ms}$ is close to:
(1) 14 mC
(2) 21 mC
(3) 6 mC
(4) 7 mC

Ans. (1)
Sol. $\mathrm{Q}=\frac{\Delta \phi}{\mathrm{R}}=\frac{1}{10} \mathrm{~A}\left(\mathrm{~B}_{\mathrm{f}}-\mathrm{B}_{\mathrm{i}}\right)=\frac{1}{10} \times 3.5 \times 10^{-3}\left(0.4 \sin \frac{\pi}{2}-0\right)$
$=\frac{1}{10}\left(3.5 \times 10^{-3}\right)(0.4-0)$
$=1.4 \times 10^{-4}=0.14 \mathrm{mC}$
14. Two masses $m$ and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length $l$. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant $k$, the restoring torque is $\tau=k \theta$ for angular displacement 0 . If the rod is rota ted by $\theta_{0}$ and released, the tension in it when it passes through its mean position will be:

(1) $\frac{3 \mathrm{k} \theta_{0}^{2}}{l}$
(2) $\frac{\mathrm{k} \theta_{0}^{2}}{2 l}$
(3) $\frac{2 \mathrm{k} \theta_{0}^{2}}{l}$
(4) $\frac{\mathrm{k} \theta_{0}^{2}}{l}$

Ans. (4)
Sol. $\omega=\sqrt{\frac{k}{\mathrm{I}}}$

$$
\omega=\sqrt{\frac{3 \mathrm{k}}{\mathrm{~m} \ell^{2}}}
$$


$\Omega=\omega \theta_{0}=$ average velocity
$\mathrm{T}=\mathrm{m} \Omega^{2} \mathrm{r}_{1}$
$\mathrm{T}=\mathrm{m} \Omega^{2} \frac{\ell}{3}$
$=m \omega^{2} \theta_{0}^{2} \frac{\ell}{3}$
$=\mathrm{m} \frac{3 \mathrm{k}}{\mathrm{m} \ell^{2}} \theta_{0}^{2} \frac{\ell}{3}$
$=\frac{\mathrm{k} \theta_{0}^{2}}{\ell}$
$\mathrm{I}=\mu \ell^{2}=\frac{\frac{\mathrm{m}^{2}}{2}}{\frac{3 \mathrm{~m}}{2}} \ell^{2}$
$=\frac{\mathrm{m} \ell^{2}}{3}$

$\frac{\mathrm{r}_{1}}{\mathrm{r}_{2}}=\frac{1}{2} \Rightarrow \mathrm{r}_{1}=\frac{\ell}{3}$
15. A copper wire is stretched to make it $0.5 \%$ longer. The percentage change in its electrical resistance if its volume remains unchanged is:
(1) $2.5 \%$
(2) $0.5 \%$
(3) $1.0 \%$
(4) $2.0 \%$

Ans. (3)
Sol. $\mathrm{R}=\frac{\rho \ell}{\mathrm{A}}$ and volume $(\mathrm{V})=\mathrm{A} \ell$.
$\mathrm{R}=\frac{\rho \ell^{2}}{\mathrm{~V}}$
$\Rightarrow \frac{\Delta \mathrm{R}}{\mathrm{R}}=\frac{2 \Delta \ell}{\ell}=1 \%$
16. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d ( $\mathrm{d} \ll \mathrm{a}$ ). The lower triangular portion is filled with a dielectric of dielectric constant K , as shown in the figure.
Capacitance of this capacitor is :
(1) $\frac{1}{2} \frac{\mathrm{k} \in_{0} \mathrm{a}^{2}}{\mathrm{~d}}$
(2) $\frac{k \in_{0} a^{2}}{d} \ln K$
(3) $\frac{\mathrm{k} \in_{0} \mathrm{a}^{2}}{\mathrm{~d}(\mathrm{~K}-1)} \ln \mathrm{K}$
(4) $\frac{\mathrm{k} \in_{0} \mathrm{a}^{2}}{2 \mathrm{~d}(\mathrm{~K}+1)}$

Ans. (3)

Sol.

$\frac{y}{x}=\frac{d}{a}$
$y=\frac{d}{a} x$
$d y=\frac{d}{a}(d x)$
$\frac{1}{d c}=\frac{y}{K E \cdot a d x}+\frac{(d-y)}{\epsilon_{0} \operatorname{adx}}$
$\frac{1}{d c}=\frac{1}{\epsilon_{0} \operatorname{adx}}\left(\frac{y}{k}+d-y\right)$
$\int \mathrm{dc}=\int \frac{\epsilon_{0} \mathrm{adx}}{\frac{\mathrm{y}}{\mathrm{k}}+\mathrm{d}-\mathrm{y}}$
$c=\epsilon_{0} a \cdot \frac{a}{d} \int_{0}^{d} \frac{d y}{d+y\left(\frac{1}{k}-1\right)}$
$=\frac{\epsilon_{0} a^{2}}{\left(\frac{1}{k}-1\right) d}\left[\ln \left(d+y\left(\frac{1}{k}-1\right)\right)\right]_{0}^{d}$
$=\frac{k \in_{0} a^{2}}{(1-k) d} \ell n\left(\frac{d+d\left(\frac{1}{k}-1\right)}{d}\right)$
$=\frac{\mathrm{k} \in_{0} \mathrm{a}^{2}}{(1-\mathrm{k}) \mathrm{d}} \ln \left(\frac{1}{\mathrm{k}}\right)=\frac{\mathrm{k} \in_{0} \mathrm{a}^{2} \ell \mathrm{nk}}{(\mathrm{k}-1) \mathrm{d}}$

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17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19} \mathrm{~m}^{-3}$ and their mobility is $1.6 \mathrm{~m}^{2} /(\mathrm{V} . \mathrm{s})$ then the resistivity of the semiconductor (since it is an n-type semiconductor
contribution of holes is ignored) is close to:
(1) $2 \Omega \mathrm{~m}$
(2) $0.4 \Omega \mathrm{~m}$
(3) $4 \Omega \mathrm{~m}$
(4) $0.2 \Omega \mathrm{~m}$

Ans. (2)
Sol. $\mathrm{j}=\sigma \mathrm{E}=\mathrm{nev}_{\mathrm{d}}$
$\sigma=n e \frac{\mathrm{v}_{\mathrm{d}}}{\mathrm{E}}$
$=n e \mu$
$\frac{1}{\sigma}=\rho=\frac{1}{n_{e} e \mu_{e}}$
$=\frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6}$
$=0.4 \Omega \mathrm{~m}$
18. If the angular momentum of a planet of mass m , moving around the Sun in a circular orbit is $L$, about the center of the Sun, its areal velocity is :
(1) $\frac{4 \mathrm{~L}}{\mathrm{~m}}$
(2) $\frac{L}{m}$
(3) $\frac{L}{2 m}$
(4) $\frac{2 \mathrm{~L}}{\mathrm{~m}}$

Ans. (3)
Sol. $\frac{\mathrm{dA}}{\mathrm{dt}}=\frac{\mathrm{L}}{2 \mathrm{~m}}$
19. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6 . What should be the minimum value of force P , such that the block doesnot move downward ?

(take $\mathrm{g}=10 \mathrm{~ms}^{-2}$ )
(1) 32 N
(2) 25 N
(3) 23 N
(4) 18 N

Ans. (1)

$m g \sin 45^{\circ}=\frac{100}{\sqrt{2}}=50 \sqrt{2}$
$\mu \mathrm{mg} \cos \theta=0.6 \times \mathrm{mg} \times \frac{1}{\sqrt{2}}=0.6 \times 50 \sqrt{2}$
$\mathrm{P}=31.28 \simeq 32 \mathrm{~N}$

$73.7=3+\mathrm{mgsin} \theta$
20. Temperature difference of $120^{\circ} \mathrm{C}$ is maintained between two ends of a uniform $\operatorname{rod} \mathrm{AB}$ of length 2 L . Another bent rod PQ , of same crosssection as $A B$ and length $\frac{3 L}{2}$, is connected across $A B$ (See figure). In steady state, temperature difference between P and Q will be close to :

(1) $60^{\circ} \mathrm{C}$
(2) $75^{\circ} \mathrm{C}$
(3) $35^{\circ} \mathrm{C}$
(4) $45^{\circ} \mathrm{C}$

Ans. (4)

Sol.

$\frac{\Delta T}{R_{\text {eq. }}}=I=\frac{(120) 5}{8 R}=\frac{120 \times 5}{8 R}$
$\Delta \mathrm{T}_{\mathrm{PQ}}=\frac{120 \times 5}{8 \mathrm{R}} \times \frac{3}{5} \mathrm{R}=\frac{360}{8}=45^{\circ} \mathrm{C}$

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21. A heavy ball of mass $M$ is suspended from the ceiling of a car by a light string of mass $m$ ( $\mathrm{m} \ll \mathrm{M}$ ). When the car is at rest, the speed of transverse waves in the string is $60 \mathrm{~ms}^{-1}$. When the car has acceleration a, the wave-speed increases to $60.5 \mathrm{~ms}^{-1}$. The value of a, in terms of gravitational acceleration g , is closest to :
(1) $\frac{g}{5}$
(2) $\frac{\mathrm{g}}{20}$
(3) $\frac{\mathrm{g}}{10}$
(4) $\frac{\mathrm{g}}{30}$

Ans. (1)
Sol. $60=\sqrt{\frac{\mathrm{Mg}}{\mu}}$
$60.5=\sqrt{\frac{M\left(g^{2}+a^{2}\right)^{1 / 2}}{\mu}} \Rightarrow \frac{60.5}{60}=\sqrt{\sqrt{\frac{\mathrm{g}^{2}+\mathrm{a}^{2}}{\mathrm{~g}^{2}}}}$
$\left(1+\frac{0.5}{60}\right)^{4}=\frac{\mathrm{g}^{2}+\mathrm{a}^{2}}{\mathrm{~g}^{2}}=1+\frac{2}{60}$
$\Rightarrow \mathrm{g}^{2}+\mathrm{a}^{2}=\mathrm{g}^{2}+\mathrm{g}^{2} \times \frac{2}{60}$
$a=g \sqrt{\frac{2}{60}}=\frac{g}{\sqrt{30}}=\frac{g}{5.47}$
$\simeq \frac{\mathrm{g}}{5}$
22. A sample of radioactive material A , that has an activity of $10 \mathrm{mCi}\left(1 \mathrm{Ci}=3.7 \times 10^{10}\right.$ decays $\left./ \mathrm{s}\right)$, has twice the number of nuclei as another sample of a different radioactive maternal B which has an activity of 20 mCi . The correct choices for hall-lives of A and B would then be respectively :
(1) 20 days and 5 days (2) 20 days and 10 days (3) 5 days and 10 days (4) 10 days and 40 days
Ans. (1)
Sol. Activity A $=\lambda \mathrm{N}$
For A $\quad 10=\left(2 \mathrm{~N}_{0}\right) \lambda_{\mathrm{A}}$
For B $\quad 20=N_{0} \lambda_{B}$
$\therefore \lambda_{\mathrm{B}}=4 \lambda_{\mathrm{A}} \Rightarrow\left(\mathrm{T}_{1 / 2}\right)_{\mathrm{A}}=4\left(\mathrm{~T}_{1 / 2}\right)_{\mathrm{B}}$
23. Consider a tank made of glass(reiractive index $1.5)$ with a thick bottom. It is filled with a liquid of refractive index $\mu$,. A student finds that, irrespective of what the incident angle $i$ (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of $\mu$ is :

(1) $\frac{3}{\sqrt{5}}$
(2) $\frac{5}{\sqrt{3}}$
(3) $\sqrt{\frac{5}{3}}$
(4) $\frac{4}{3}$

Ans. (1)
Sol. $\mathrm{C}<\mathrm{i}_{\mathrm{b}}$
here $\mathrm{i}_{\mathrm{b}}$ is "brewester angle" and $c$ is critical angle
$\sin _{\mathrm{c}}<\operatorname{sini}_{\mathrm{b}} \quad$ since $\tan \mathrm{i}_{\mathrm{b}}=\mu_{0_{\text {rel }}}=\frac{1.5}{\mu}$
$\frac{1}{\mu}<\frac{1.5}{\sqrt{\mu^{2}+(1.5)^{2}}} \quad \therefore \sin i_{b}=\frac{1.5}{\sqrt{\mu^{2}+(1.5)^{2}}}$
$\sqrt{\mu^{2} \times(1.5)^{2}}<1.5 \times \mu$
$\mu^{2}+(1.5)^{2}<(\mu \times 1.5)^{2}$
$\mu<\frac{3}{\sqrt{5}}$

slab $\mu=1.5$
24. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d(d » a)$. If the loop applies a force F on the wire then :

(1) $F \propto\left(\frac{a^{2}}{d^{3}}\right)$
(2) $F \propto\left(\frac{\mathrm{a}}{\mathrm{d}}\right)$
(3) $\mathrm{F} \propto\left(\frac{\mathrm{a}}{\mathrm{d}}\right)^{2}$
(4) $\mathrm{F}=0$

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Ans. (3)

Sol.

$\infty$ long wire
Eqvilent dipole of given loop
$\mathrm{F}=\mathrm{m} \cdot \frac{\mathrm{dB}}{\mathrm{dr}}$
Now $\frac{\mathrm{dB}}{\mathrm{dx}}=\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\mu_{0} \mathrm{I}}{2 \pi \mathrm{x}}\right)$
$\propto \frac{1}{\mathrm{x}^{2}}$
$\Rightarrow$ So $\mathrm{F} \propto \frac{\mathrm{M}}{\mathrm{x}^{2}}[\because \mathrm{M}=\mathrm{NIA}]$
$\therefore \mathrm{F} \propto \frac{\mathrm{a}^{2}}{\mathrm{~d}^{2}}$
25. Surface of certain metal is first illuminated with light of wavelength $\lambda_{1}=350 \mathrm{~nm}$ and then, by light of wavelength $\lambda_{2}=54 \mathrm{D} \mathrm{nm}$. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2 . The work function of the metal (in eV ) is close to :
$\left(\right.$ Energjr of photon $=\frac{1240}{\lambda(\text { innm })} \mathrm{eV}$ )
(1) 1.8
(2) 1.4
(3) 2.5
(4) 5.6

Ans. (1)
Sol. $\frac{\mathrm{hc}}{\lambda_{1}}=\phi+\frac{1}{2} \mathrm{~m}(2 \mathrm{v})^{2}$
$\frac{\mathrm{hc}}{\lambda_{2}}=\phi+\frac{1}{2} \mathrm{mv}^{2}$
$\Rightarrow \frac{\frac{\mathrm{hc}}{\lambda_{1}}-\phi}{\frac{\mathrm{hc}}{\lambda_{2}}-\phi}=4 \Rightarrow \frac{\mathrm{hc}}{\lambda_{1}}-\phi=\frac{4 \mathrm{hc}}{\lambda_{2}}-4 \phi$
$\Rightarrow \frac{4 \mathrm{hc}}{\lambda_{2}}-\frac{\mathrm{hc}}{\lambda_{1}}=3 \phi$
$\Rightarrow \phi=\frac{1}{3} \mathrm{hc}\left(\frac{4}{\lambda_{2}}-\frac{1}{\lambda_{1}}\right)$
$=\frac{1}{3} \times 1240\left(\frac{4 \times 350-540}{350 \times 540}\right)$
$=1.8 \mathrm{eV}$
26. A particle is moving with a velocity $\bar{v}=K(y \hat{i}+x \hat{j})$, where $K$ is a constant. The general equation for its path is:
(1) $x y=$ constant
(2) $y^{2}=x^{2}+$ constant
(3) $y=x^{2}+$ constant
(4) $y^{2}=x+$ constant

Ans. (2)
Sol. $\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{ky}, \frac{\mathrm{dy}}{\mathrm{dt}}=\mathrm{kx}$
Now, $\frac{d y}{d x}=\frac{\frac{d y}{d t}}{\frac{d x}{d t}}=\frac{x}{y}$
$\Rightarrow \mathrm{ydy}=\mathrm{xdx}$
Integrating both side
$y^{2}=x^{2}+c$
27. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5 ) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance $d$. Then $d$ is :
(1) 0.55 cm away from the lens
(2) 1.1 cm away from the lens
(3) 0.55 cm towards the lens
(4) 0

Ans. (1)

Sol.

$\frac{1}{v}-\frac{1}{u}=\frac{1}{\mathrm{f}} \Rightarrow \frac{1}{10}-\frac{1}{-10}=\frac{1}{\mathrm{f}} \Rightarrow \mathrm{f}=5 \mathrm{~cm}$
Shift due to slab $=t\left(1-\frac{1}{\mu}\right)$ in the direction of incident ray
$=1.5\left(1-\frac{2}{3}\right)=0.5$
again, $\frac{1}{\mathrm{v}}-\frac{1}{-9.5}=\frac{1}{5}$
$\Rightarrow \frac{1}{\mathrm{u}}=\frac{1}{5}-\frac{2}{19}=\frac{9}{95}$
$\Rightarrow \mathrm{v}=\frac{95}{9}=10.55 \mathrm{~cm}$
28. For a uniformly charged ring of radius $R$, the electric field on its axis has the largest magnitude at a distance $h$ from its centre. Then value of $h$ is :
(1) $\frac{\mathrm{R}}{\sqrt{5}}$
(2) R
(3) $\frac{R}{\sqrt{2}}$
(4) $\mathrm{R} \sqrt{2}$

Ans. (3)
Sol. Electric field on axis of ring
$E=\frac{k Q h}{\left(h^{2}+R^{2}\right)^{3 / 2}}$
for maximum electric field
$\frac{\mathrm{dE}}{\mathrm{dh}}=0$
$\Rightarrow \mathrm{h}=\frac{\mathrm{R}}{\sqrt{2}}$
29. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and $B$ have equal masses, $m$ while $C$ has mass $M$. Block A is given an brutal speed v towards B due to which it collides with $B$ perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of $\mathrm{M} / \mathrm{m}$ ?

(1) 4
(2) 5
(3) 3
(4) 2

Ans. (1)
Sol. $\mathrm{k}_{\mathrm{i}}=\frac{1}{2} \mathrm{mv}_{0}^{2}$
From linear momentum conservation
$\mathrm{mv}_{0}=(2 \mathrm{~m}+\mathrm{M}) \mathrm{v}_{\mathrm{f}}$
$\Rightarrow \mathrm{v}_{\mathrm{f}}=\frac{\mathrm{mv}_{0}}{2 \mathrm{~m}+\mathrm{M}}$
$\frac{\mathrm{k}_{\mathrm{i}}}{\mathrm{k}_{\mathrm{f}}}=6$
$\Rightarrow \frac{\frac{1}{2} \mathrm{mv}_{0}^{2}}{\frac{1}{2}(2 \mathrm{~m}+\mathrm{M})\left(\frac{\mathrm{mv}_{0}}{2 \mathrm{~m}+\mathrm{M}}\right)^{2}}=6$
$\Rightarrow \frac{2 \mathrm{~m}+\mathrm{M}}{\mathrm{m}}=6$
$\Rightarrow \frac{\mathrm{M}}{\mathrm{m}}=4$
30. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm 2 , is $v$. If the electron density in copper is $9 \times 10^{28}$ $/ \mathrm{m}^{3}$ the value of v in $\mathrm{mm} / \mathrm{s}$ is close to (Take charge of electron to be $=1.6 \times 10^{-19} \mathrm{C}$ )
(1) 0.2
(2) 3
(3) 2
(4) 0.02

Ans. (4)
Sol. $I=n e A v_{d}$
$\Rightarrow \mathrm{v}_{\mathrm{d}}=\frac{\mathrm{I}}{\text { neA }}=\frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$
$=0.02 \mathrm{~m} / \mathrm{s}$

