TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then $|p|$ is :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{\sqrt{5}}$

(3) $\frac{1}{\sqrt{6}}$

(4) $\frac{1}{\sqrt{3}}$

Ans. **(1)**

Sol. A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$

2. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

(1) $\frac{5}{4}$

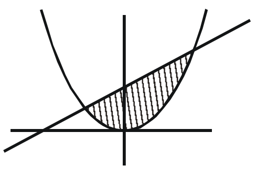
(2) $\frac{9}{8}$

(3) $\frac{3}{4}$

(4) $\frac{7}{8}$

Ans. **(2)**

Sol.



$x = 4y - 2$ & $x^2 = 4y$

$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$x = 2, -1$

So, $\int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx = \frac{9}{8}$

3. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

(1) 2 (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{3}$ (4) $\sqrt{2}$

Ans. **(4)**

Sol. Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

4. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

(1) $\frac{4}{9}$ (2) $\frac{2}{9}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. **(3)**

Sol. $\frac{a}{1-r} = 3$ ...(1)

$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ as $|r| < 1$

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is :

(1) $-14\hat{i} - 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-10\hat{i} + 5\hat{j}$ (4) $-14\hat{i} + 5\hat{j}$

Ans. (3)

Sol. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

Hence $\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$

$= -10\hat{i} + 5\hat{j}$

6. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27}\ (i = \sqrt{-1})$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85

(3) –91 (4) 91

Ans. (4)

Sol. $\left(-2 - \frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$

$= \frac{-198 - 107i}{27} = \frac{x + iy}{27}$

Hence, y – x = 198 – 107 = 91

7. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval (–2, 2), g is :-

(1) differentiable at all points

(2) not differentiable at two points

(3) not continuous

(4) not differentiable at one point

Ans. (4)

Sol. $|f(x)| = \begin{cases} 1 & , & -2 \le x < 0 \\ 1 - x^2 & , & 0 \le x < 1 \\ x^2 - 1 & , & 1 \le x \le 2 \end{cases}$

and $f(|x|) = x^2 - 1,\ x \in [-2, 2]$

Hence $g(x) = \begin{cases} x^2 & , & x \in [-2, 0) \\ 0 & , & x \in [0, 1) \\ 2(x^2 - 1) & , & x \in [1, 2] \end{cases}$

It is not differentiable at x = 1

8. Let f : R → R be defined by $f(x) = \frac{x}{1 + x^2}$, $x \in R$. Then the range of f is :

(1) (–1, 1) – {0} (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) R – [–1, 1]

Ans. (2)

Sol. $f(0) = 0$ & $f(x)$ is odd.

Further, if x > 0 then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

Hence, $f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

9. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :

(1) 6 (2) 8 (3) 0 (4) 4

Ans. (3)

Sol. $T_5 = {}^8C_4 \frac{x^{12}}{81} \times \frac{81}{x^4} = 5670$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

10. The value of r for which

${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + ... \,{}^{20}C_0\,{}^{20}C_r$ is maximum, is

(1) 20 (2) 15

(3) 11 (4) 10

Ans. (1)

Sol. Given sum = coefficient of x^r in the expansion of $(1 + x)^{20}(1 + x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when r = 20

11. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :

(1) $2(5^2)$ (2) $4(5^2)$

(3) 5^4 (4) 5^3

Ans. (3)

Sol. $a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1r^2}{a_1} = 25 \Rightarrow r^2 = 25$

$\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = 5^4$

12. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$

(3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

Ans. (2)

Sol. $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$

$\int \frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx$,

Put $\frac{1}{x^2} - 1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case-1 $x \geq 0$

$-\frac{1}{2}\int \sqrt{t}\,dt \Rightarrow -\frac{t^{3/2}}{3} + C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2}$

$\Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2} + C$

$A(x) = -\frac{1}{3x^3}$ and m = 3

$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$

Case-II $x \leq 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3} + C$

$A(x) = \frac{1}{-3x^3}$, m = 3

$(A(x))^m = \frac{-1}{27x^9}$

13. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

Ans. (2)

Sol. Given a + b = x and ab = y

If $x^2 - c^2 = y \Rightarrow (a+b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2}$$

$$\Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

14. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$

(where [x] denotes the greatest integer less than ^{20}Cr or equal to x) is :

(1) 4 (2) 4 – sin4

(3) sin 4 (4) 0

Ans. (4)

Sol. $$I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right] = -1 \text{ as } \quad x \neq n\pi\right)$$

$$I = \int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} + \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx = 0$$

15. If the system of linear equations

2x + 2y + 3z = a

3x – y + 5z = b

x – 3y + 2z = c

where a, b, c are non-zero real numbers, has more then one solution, then :

(1) b – c – a = 0 (2) a + b + c = 0

(3) b + c – a = 0 (4) b – c + a = 0

Ans. (1)

Sol. P_1 : 2x + 2y + 3z = a

P_2 : 3x – y + 5z = b

P_3 : x – 3y + 2z = c

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

16. A square is inscribed inthe circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the corrdinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2) $\sqrt{137}$

(3) 6 (4) $\sqrt{41}$

Ans. (4)

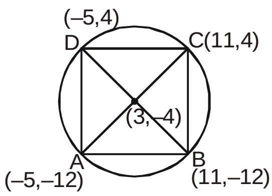
Sol. $R = \sqrt{9+16+103} = 8\sqrt{2}$

OA = 13

$OB = \sqrt{265}$

$OC = \sqrt{137}$

$OD = \sqrt{41}$



17. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all x ∈ R, the value of $f_4(x) - f_6(x)$ is equal to :-

(1) $\frac{5}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

Ans. (4)

Sol. $f_4(x) - f_6(x)$

$$= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

18. Let [x] denote the greatest integer less than or equal to x. Then :-

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$$

(1) equals π

(2) equals 0

(3) equals $\pi + 1$

(4) does not exist

Ans. (4)

Sol. R.H.L. $=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$$=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+x^2}{x^2}$$

$$=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}+1=\pi+1$$

L.H.L. $=\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$$\lim_{x\to 0+}\frac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x}.\frac{\pi\sin^2 x}{x^2}+\left(-1+\frac{\sin x}{x}\right)^2\Rightarrow\pi$$

R.H.L. $\neq$ L.H.L.

19. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an anlge $\frac{\pi}{4}$ with the plane y–z+5=0 are:

(1) $2\sqrt{3}$, 1, –1

(2) 2, $\sqrt{2}$, $-\sqrt{2}$

(3) 2, –1, 1

(4) $\sqrt{2}$, 1, –1

Ans. (2, 4)

Sol. Let the equation of plane be

$a(x - 0) + b(y + 1) + c(z - 0) = 0$

It passes through (0,0,1) then

$b + c = 0$ ...(1)

Now $\cos\frac{\pi}{4}=\frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow a=\pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $\left(\sqrt{2},-1,1\right)$ or $\left(\sqrt{2},1,-1\right)$

20. If $x\log_e(\log_e x) - x^2 + y^2 = 4(y > 0)$, then dy/dx at x = e is equal to :

(1) $\frac{e}{\sqrt{4+e^2}}$

(2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$

(4) $\frac{(1+2e)}{\sqrt{4+e^2}}$

Ans. (3)

Sol. Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \text{ as } y(e)=\sqrt{4+e^2}$$

21. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(1) $\frac{\sqrt{5}}{4}$

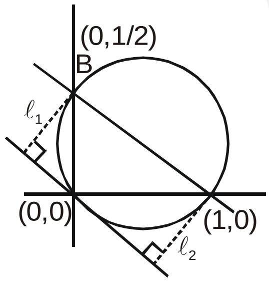
(2) $\frac{\sqrt{5}}{2}$

(3) $2\sqrt{5}$

(4) $4\sqrt{5}$

Ans. **(2)**

Sol.



Equation of circle

$(x - 1)(x - 0) + (y - 0)\left(y - \frac{1}{2}\right) = 0$

$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$

Equation of tangent of origin is 2x + y = 0

$\ell_1 + \ell_2 = \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}}$

$= \frac{4+1}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$

22. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(1) $(p \vee r) \rightarrow (p \wedge r)$

(2) $p \vee r$

(3) $p \wedge r$

(4) $(p \wedge r) \rightarrow (p \vee r)$

Ans. **(4)**

Sol. Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow$ q is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

23. If y(x) is the solution of the differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, \; x > 0,$$

where $y(1) = \frac{1}{2}e^{-2}$, then :

(1) y(x) is decreasing in (0, 1)

(2) y(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

(3) $y(\log_e 2) = \frac{\log_e 2}{4}$

(4) $y(\log_e 2) = \log_e 4$

Ans. **(2)**

Sol. $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$

$$\text{I.F.} = e^{\int\left(\frac{2x+1}{x}\right)dx} = e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x+\ell nx} = e^{2x}.x$$

So, $y(xe^{2x}) = \int e^{-2x}.xe^{2x} + C$

$\Rightarrow xye^{2x} = \int x\,dx + C$

$\Rightarrow 2xye^{2x} = x^2 + 2C$

It passess through $\left(1, \frac{1}{2}e^{-2}\right)$ we get C = 0

$y = \frac{xe^{-2x}}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

$$y(\log_e 2) = \frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

$= \frac{1}{8}\log_e 2$

24. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R : x^2 + 30 \le 11x\}$ is :

(1) 122 (2) –222
(3) –122 (4) 222

Ans. (1)

Sol. $S = \{x \in R, x^2 + 30 - 11x \le 0\}$
$= \{x \in R, 5 \le x \le 6\}$
Now $f(x) = 3x^3 - 18x^2 + 27x - 40$
$\Rightarrow f'(x) = 9(x - 1)(x - 3)$,
which is positive in [5, 6]
$\Rightarrow f(x)$ increasing in [5, 6]
Hence maximum value = $f(6) = 122$

25. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –81 (2) 100 (3) –300 (4) 144

Ans. (3)

Sol. $81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$$

$$\text{Now } -\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27}$$

$\Rightarrow k = \pm 300$

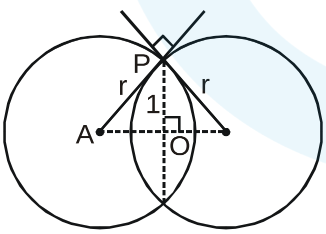
26. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) $\sqrt{2}$

(3) $2\sqrt{2}$ (4) 2

Ans. (4)

Sol. In ΔAPO



$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$

So distance between centres = $\sqrt{2}\,r = 2$

27. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbole $xy = 2$ is :

(1) $x + 2y + 4 = 0$

(2) $x - 2y + 4 = 0$

(3) $x + y + 1 = 0$

(4) $4x + 2y + 1 = 0$

Ans. (1)

Sol. Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \frac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$$

$$\Rightarrow x^2 m + \frac{x}{m} - 2 = 0$$

$$D = 0 \Rightarrow m = -\frac{1}{2}$$

So tangent is $2y + x + 4 = 0$

28. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(1) (2, 0, –2) (2) (–2, 2, 2)
(3) (0, –2, 2) (4) (2, 2, 0)

Ans. (1)

Sol. The normal vector of required plane

$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right) \times \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)
So required plane is
$-(x - 3) + (y + 2) + (z - 1) = 0$
$\Rightarrow -x + y + z + 4 = 0$
Which is satisfied by (2, 0, –2)

29. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted betwen the coordinate axes lie on the curve :

(1) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (2) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(3) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (4) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

Ans. **(3)**

Sol. Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta}+\frac{y}{b\operatorname{cosec}\theta}=1$$

$a=\sqrt{2}$, b = 1

$$\Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\operatorname{cosec}\theta}=1$$

Let the midpoint be (h, k)

$$h=\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$$

and $k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta=\frac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$$

$$\Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$$

30. Two integers are selected at random from the set {1, 2,...., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$

(2) $\frac{1}{2}$

(3) $\frac{3}{5}$

(4) $\frac{7}{10}$

Ans. **(1)**

Sol. Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^5C_2 + {}^6C_2$

so required probability $= \frac{{}^5C_2}{{}^5C_2+{}^6C_2}$