TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

1. Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) R(z) > 0 and I(z) > 0

(2) R(z) < 0 and I(z) > 0

(3) R(z) = –3

(4) I(z) = 0

Ans. (4)

Sol. $z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$

$z = \left(e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5$

$= e^{i5\pi/6} + e^{-i5\pi/6}$

$= \cos\frac{5\pi}{6} + i\frac{\sin 5\pi}{6} + \cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)$

$= 2\cos\frac{5\pi}{6} < 0$

I(z) = 0 and Re(z) < 0

Option (4)

2. Let $a_1, a_2, a_3, ..., a_{10}$ be in G.P. with $a_i > 0$ for i = 1,2,...., 10 and S be the set of pairs (r,k), r k∈N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

Ans. (1)

Sol. Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get D = 0

Option (1)

3. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :

(1) $\sqrt{5}$ (2) 4

(3) $2\sqrt{2}$ (4) 3

Ans. (2)

Sol. $x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$

$x^2\left[{}^{10}C_r (x)^{\frac{10-r}{2}} (\lambda)^r (x)^{-2r}\right]$

$x^2\left[{}^{10}C_r \lambda^r x^{\frac{10-5r}{2}}\right]$

∴ r = 2

Hence, ${}^{10}C_2\, \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

4. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot ..... \cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

Ans. (3)

Sol. $2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}......\cos\frac{\pi}{2^2}$

$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$

Option (3)

5. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

Ans. (4)

Sol. $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$

$$= \int_{\frac{-\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4}$$

$$+\int_{0}^{1} \frac{dx}{0+0+4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{1+0+4}$$

$$\int_{\frac{-\pi}{2}}^{-1} \frac{dx}{1} + \int_{-1}^{0} \frac{dx}{2} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\frac{\pi}{2}} \frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20}+\frac{6\pi}{10}$$

$$\frac{-9}{20}+\frac{3\pi}{5}$$

Option (4)

6. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $1 - {}^{n}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$

$n_{min} = 5 \Rightarrow$ Option (2)

7. If mean and standard deviation of 5 observations x_1, x_2, x_3,x_4,x_5 are 10 and 3, respectively, then the variance of 6 observations x_1,x_2, ...,x_5 and –50 is equal to :

(1) 582.5 (2) 507.5

(3) 586.5 (4) 509.5

Ans. (2)

Sol. $\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$$

$$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$$

$$\text{variance} = \frac{\sum_{i=1}^{5} (x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^{5} \frac{x_i - 50}{6}\right)$$

= 507.5

Option (2)

8. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

Ans. (3)

Sol. $x^2 = 4y$

$x-\sqrt{2}y+4\sqrt{2}=0$

Solving together we get

$$x^2=4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$

$$\sqrt{2}x^2+4x+16\sqrt{2}$$

$$\sqrt{2}x^2-4x-16\sqrt{2}=0$$

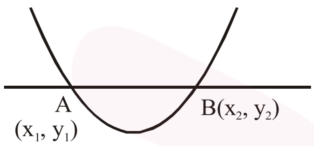
$x_1 + x_2 = 2\sqrt{2}$; $x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$

Similarly,

$$\left(\sqrt{2}y-4\sqrt{2}\right)^2=4y$$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0$ → $y_1 + y_2 = 10$, $y_1 y_2 = 16$



$$\ell_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(2\sqrt{2}\right)^2+64+(10)^2-4(16)}$$

$$=\sqrt{8+64+100-64}$$

$$=\sqrt{108}=6\sqrt{3}$$

Option (3)

9. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (4)

Sol. $A = \begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ $(b > 0)$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1 (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$$\frac{|A|}{b}=b+\frac{3}{b} \Rightarrow \frac{b+\frac{3}{b}}{2}\geq\sqrt{3}$$

$$b+\frac{3}{b}\geq 2\sqrt{3}$$

Option (4)

10. The tangent to the curve, $y=xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\frac{4}{3},2e\right)$ (2) (2,3e)

(3) $\left(\frac{5}{3},2e\right)$ (4) (3,6e)

Ans. (1)

Sol. $y=xe^{x^2}$

$$\left.\frac{dy}{dx}\right|_{(1,\,e)}=\left.\left(e\cdot e^{x^2}\cdot 2x+e^{x^2}\right)\right|_{(1,\,e)} = 2\cdot e + e = 3e$$

T : $y - e = 3e (x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

$\left(\frac{4}{3},\,2e\right)$ lies on it

Option (1)

11. The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta) y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three

(3) Four (4) Two

Ans. (4)

Sol. $\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta)$
$+ 7(-\cos 2\theta - 4\sin 3\theta) = 0$
$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta$
$- 28\sin 3\theta = 0$
$14 - 7\sin 3\theta - 14\cos 2\theta = 0$
$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$
$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$
$7\sin\theta[-3 + 4\sin^2\theta + 4\sin\theta] = 0$
$\sin\theta,$
$4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$
$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2}$; $\sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$
Option (4)

12. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then f'(1/2) is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

Ans. **(2)**

Sol. $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$ $f'\left(\frac{1}{2}\right) = ?$

Differentiate w.r.t. 'x'
$f(x) = 2x + 0 - x^2 f(x)$

$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$

$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

13. Let $f : (-1,1) \to R$ be a function defined by $f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

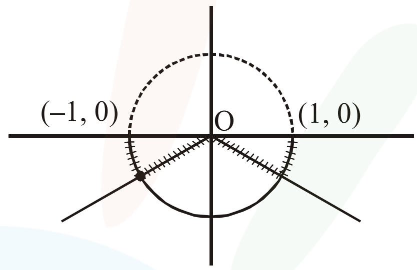
(1) Three elements (2) One element

(3) Five elements (4) Two elements

Ans. **(1)**

Sol. $f : (-1, 1) \to R$

$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$



Non-derivable at 3 points in (–1, 1)
Option (1)

14. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

Ans. (4)

Sol. $\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for r > 1, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$=\sqrt{\frac{(r+1)-(r-1)}{(r+1)}}$

$=\sqrt{\frac{2}{r+1}}=\sqrt{\frac{2}{r+1}}$

Option (4)

15. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to :

(1) $2^{25}-1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

Ans. (3)

Sol. $\sum_{r=0}^{25}{}^{50}C_r\cdot{}^{50-r}C_{25-r}$

$=\sum_{r=0}^{25}\frac{50!}{r!\,(50-r)!}\times\frac{(50-r)!}{(25)!\,(25-r)!}$

$=\sum_{r=0}^{25}\frac{50!}{25!\,25!}\times\frac{25!}{(25-r)!\,(r!)}$

$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r=\left(2^{25}\right){}^{50}C_{25}$

$\therefore$ K = 2^{25}

Option (3)

16. Let N be the set of natural numbers and two functions f and g be defined as f,g : N→N such that : $f(n)=\begin{cases}\frac{n+1}{2} & \text{if n is odd}\\ \frac{n}{2} & \text{if n is even}\end{cases}$

and g(n) = $n-(-1)^n$. The fog is :

(1) Both one-one and onto
(2) One-one but not onto
(3) Neither one-one nor onto
(4) onto but not one-one

Ans. (4)

Sol. $f(x)=\begin{cases}\frac{n+1}{2} & \text{n is odd}\\ n/2 & \text{n is even}\end{cases}$

$g(x)=n-(-1)^n\begin{cases}n+1\ ; & \text{n is odd}\\ n-1\ ; & \text{n is even}\end{cases}$

$f(g(n))=\begin{cases}\frac{n}{2}; & \text{n is even}\\ \frac{n+1}{2}; & \text{n is odd}\end{cases}$

$\therefore$ many one but onto

Option (4)

17. The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

Ans. (1)

Sol. $\alpha+\beta=\lambda-3$

$\alpha\beta=2-\lambda$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(\lambda-3)^2-2(2-\lambda)$

$=\lambda^2+9-6\lambda-4+2\lambda$

$=\lambda^2-4\lambda+5$

$=(\lambda-2)^2+1$

$\therefore$ λ = 2

Option (1)

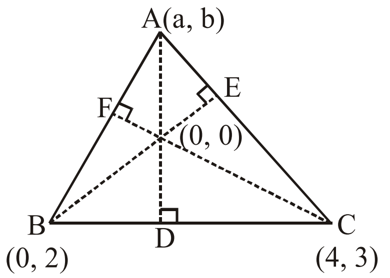
18. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) Fourth
(2) Second
(3) Third
(4) First

Ans. (2)

Sol. $m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right)\times\left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right)\times\left(\frac{3}{4}\right) = -1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

$\therefore$ II[nd] quadrant.

Option (2)

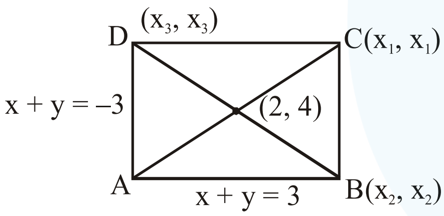
19. Two sides of a parallelogram are along the lines, x + y = 3 and x – y + 3 = 0. If its diagonals intersect at (2,4), then one of its vertex is :

(1) (2,6) (2) (2,1)

(3) (3,5) (4) (3,6)

Ans. **(4)**

Sol.



Solving x + y = 3 and x – y = –3 $\Rightarrow$ A(0, 3)

$\frac{x_1+0}{2} = 2$; $x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is x – y = –1

and equation of CD is x + y = 9

Solving x + y = 9 and x – y = –3

Point D is (3, 6)

Option (4)

20. Let $\vec{\alpha} = (\lambda-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda-2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4

(3) 3 (4) –4

Ans. **(4)**

Sol. $\vec{\alpha} = (\lambda-2)\vec{\alpha} + \vec{b}$

$\vec{\beta} = (4\lambda-2)\vec{\alpha} + 3\vec{b}$

$\frac{\lambda-2}{4\lambda-2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda = -4}$

$\therefore$ Option (4)

21. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

Ans. **(3)**

Sol. $\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1)\right)$

$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$

$\sum_{n=1}^{19}(\tan^{-1}(n+1) - \tan^{-1}n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A\cot\beta+1}{\cot\beta - \cot A}$

(Where tanA=20 , tanB=1) $\frac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}} = \frac{21}{19}$

$\therefore$ Option (3)

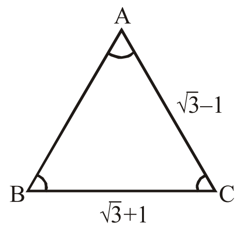
22. With the usual notation, in ΔABC, if $\angle A + \angle B = 120^\circ$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is :

(1) 7 : 1 (2) 5 : 3

(3) 9 : 7 (4) 3 : 1

Ans. **(1)**

Sol. A + B = 120°



$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°)=\frac{1}{\sqrt{3}}.\sqrt{3}=1$$

$$\frac{A-B}{2}=45° \quad\Rightarrow\quad A-B=90°$$

$$A+B=120°$$

2A = 210°

A = 105°

B = 15°

∴ Option (1)

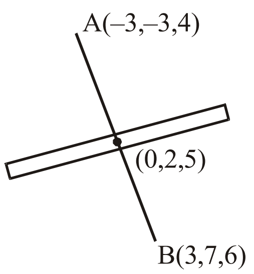
23. The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)

(3) (–2,3,5) (4) (2,1,3)

Ans. **(2)**

Sol.



$$\vec{n}=3\hat{i}+5\hat{j}+\hat{k}$$

p : 3(x – 0) + 5 (y – 2) + 1 (z – 5) = 0

3x + 5y + z = 15

∴ Option (2)

24. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q)\vee(\sim R)$

(2) $(\sim P)\wedge(\sim Q\wedge R)$

(3) $(\sim P)\vee(Q\wedge R)$

(4) $P\vee(\sim Q\wedge R)$

Ans. **(4)**

Sol. It is obvious

∴ Option (4)

25. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, x + y + z = 2 ?

(1) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

(2) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

(3) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

(4) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

Ans. **(3)**

Sol. General point on the given line is

x = 2λ + 4

y = 2λ + 5

z = λ + 3

Solving with plane,

2λ + 4 + 2λ + 5 + λ + 3 = 2

5λ + 12 = 2

5λ = –10

$\boxed{\lambda=-2}$

∴ Option (3)

26. Let f be a differentiable function such that

$f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, (x > 0)$ and $f(1) \neq 4$.

Then $\lim_{x \to 0^+} x f\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

Ans. **(1)**

Sol. $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x} \qquad (x > 0)$

Given $f(1) \neq 4 \qquad \lim_{x \to 0^+} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx} + \frac{3}{4}\frac{y}{x} = 7$ (This is LDE)

IF = $e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln|x|} = x^{\frac{3}{4}}$

$y.x^{\frac{3}{4}} = \int 7.x^{\frac{3}{4}}dx$

$y.x^{\frac{3}{4}} = 7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$

f(x) = $4x + C.x^{-\frac{3}{4}}$

$f\left(\frac{1}{x}\right) = \frac{4}{x} + C.x^{\frac{3}{4}}$

$\lim_{x \to 0^+} xf\left(\frac{1}{x}\right) = \lim_{x \to 0^+}\left(4 + C.x^{\frac{7}{4}}\right) = 4$

$\therefore$ Option (1)

27. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(1) $\frac{1}{2}$

(2) $\frac{1}{3}\sqrt{\frac{7}{3}}$

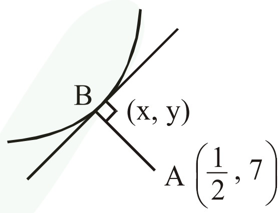
(3) $\frac{1}{6}\sqrt{\frac{7}{3}}$

(4) $\frac{\sqrt{5}}{6}$

Ans. **(3)**

Sol. $y - x^{3/2} = 7 \ (x \geq 0)$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$

$\frac{3}{2}\cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore$ x = –1 (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9 \times 12}}$

$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

28. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

Ans. (1)

Sol. $\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$

Put $x^3 = t$

$3x^2\, dx = dt$

$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$

$\frac{1}{3}\int t \cdot e^{-4t} dt$

$\frac{1}{3}\left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt\right]$

$-\frac{e^{-4t}}{48}[4t+1] + c$

$\frac{-e^{-4x^3}}{48}[4x^3+1] + c$

$\therefore$ $f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

29. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy\, dy = 0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

Ans. (2)

Sol. $(x^2 - y^2)\, dx + 2xy\, dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Solving we get,

$\int \frac{2v}{v^2+1} dv = \int -\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2 + x^2 = 2x}$

$\therefore$ Option (2)

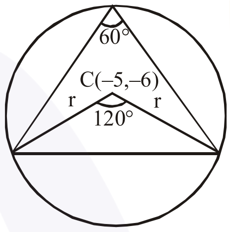
30. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25

(3) 13 (4) –25

Ans. (2)

Sol. $3\left(\frac{1}{2}r^2.\sin 120°\right) = 27\sqrt{3}$



$\frac{r^2}{2}\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$

$r^2 = \frac{108}{3} = 36$

Radius = $\sqrt{25+36-C} = \sqrt{36}$

$\boxed{C = 25}$

$\therefore$ Option (2)