TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

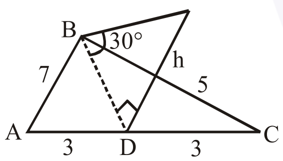
MATHEMATICS

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

Ans. (2)

Sol.



BD = hcot30° = $h\sqrt{3}$

So, $7^2 + 5^2 = 2((h\sqrt{3})^2 + 3^2)$

$\Rightarrow$ 37 = $3h^2$ + 9.

$\Rightarrow$ $3h^2$ = 28

$\Rightarrow h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$

2. Let f : R→R be a function such that $f(x) = x^3+x^2f'(1) + xf''(2)+f'''(3)$, $x \in R$. Then f(2) equal :

(1) 8 (2) –2 (3) –4 (4) 30

Ans. (2)

Sol. $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(x)$ .....(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5 \Rightarrow f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4 (3) $2\sqrt{5}$ (4) 5

Ans. (4)

Sol. $x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0) :

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$

$\Rightarrow \lambda = -4$

$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$

or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16+25-16} = 5$

4. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and theose whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 102 (2) 42 (3) 1 (4) 38

Ans. (4)

Sol. Let n(A) = number of students opted Mathematics = 70,

n(B) = number of students opted Physics = 46,

n(C) = number of students opted Chemistry = 28,

$n(A \cap B) = 23$,

$n(B \cap C) = 9$,
$n(A \cap C) = 14$,
$n(A \cap B \cap C) = 4$,
Now $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(A \cap C) + n(A \cap B \cap C)$
$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$
So number of students not opted for any course
$= \text{Total} - n(A \cup B \cup C)$
$= 140 - 102 = 38$

5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

(1) 1365 (2) 1256 (3) 1465 (4) 1356

Ans. **(4)**

Sol. $\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6 + 2\times 12$

$= 7 \times 90 + 24 = 654$

$\sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13 + 5\times 13 = 702$

Total = 654 + 702 = 1356

6. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is :-

(1) $\left(\frac{1}{2}, 4, -2\right)$ (2) $\left(-\frac{1}{2}, 4, 0\right)$

(3) (1,3,1) (4) (1,5,1)

Ans. **(2)**

Sol. $4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$ ....(1)

Given $\vec{a}.\vec{c} = 0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 2\lambda_1 + \lambda_3 = -1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

7. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x-y = 2$ is :

(1) $x-y+9 = 0$
(2) $x-y+7 = 0$
(3) $x-y+1 = 0$
(4) $x-y-3 = 0$

Ans. **(3)**

Sol. Hyperbola $\frac{x^2}{5} - \frac{y^2}{4} = 1$

slope of tangent = 1

equation of tangent $y = x \pm \sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

8. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, $(k>0)$, is 1 square unit. Then k is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Ans. **(1)**

Sol. Area bounded by $y^2 = 4ax$ & $x^2 = 4by$, $a, b \neq 0$

is $\left|\frac{16ab}{3}\right|$

by using formula : $4a = \frac{1}{k} = 4b, k > 0$

Area $= \left|\frac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right| = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \frac{1}{\sqrt{3}}$

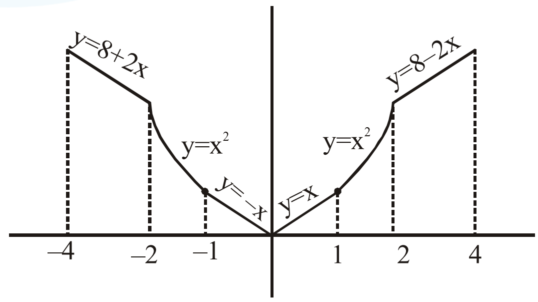
9. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S:

(1) is an empty set
(2) equals {–2, –1, 1, 2}
(3) equals {–2, –1, 0, 1, 2}
(4) equals {–2, 2}

Ans. **(3)**

Sol. $f(x) = \begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8-2x, & 2 < x \le 4 \end{cases}$



$f(x)$ is not differentiable at x = {–2,–1,0,1,2}

$\Rightarrow$ S = {–2, –1, 0, 1, 2}

10. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) (1, 1, 0) (2) $\left(\frac{1}{2},2,3\right)$

(3) $\left(\frac{1}{2},2,0\right)$ (4) (1, 1, 3)

Ans. (1,2,3,4)

Sol. Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b)\, m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b)\, m = (b - 2a)m^3$

We get all options are correct for m = 0 (common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$$m^2=\frac{c}{2a-b}-2>0\Rightarrow\frac{c}{2a-b}>2$$

Now according to options, option 4 is correct

11. The sum of all values of $\theta\in\left(0,\frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

Ans. (1)

Sol. $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$, $\theta\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta=\frac{1}{2}=\cos^2\frac{\pi}{4}$

$\Rightarrow 2\theta=n\pi\pm\frac{\pi}{4}$, $n \in I$

$\Rightarrow \theta=\frac{n\pi}{2}\pm\frac{\pi}{8}$

$\Rightarrow \theta=\frac{\pi}{8},\frac{\pi}{2}-\frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$

12. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$.

If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$ then :

(1) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$ (2) Re(z) = 0

(3) $|z|=\sqrt{\frac{5}{2}}$ (4) Im(z) = 0

Ans. (Bonus)

Sol. $3|z_1| = 4|z_2|$

$\Rightarrow \frac{|z_1|}{|z_2|}=\frac{4}{3}$

$\Rightarrow \frac{|3z_1|}{|2z_2|}=2$

Let $\frac{3z_1}{2z_2}=a=2\cos\theta+2i\sin\theta$

$z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}=a+\frac{1}{a}$

$=\frac{5}{2}\cos\theta+\frac{3}{2}i\sin\theta$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = 2|z_2|$.

If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\frac{3z_1}{2z_2}\right|=1$

Let $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$$

$$= a + \frac{1}{a} = 2\cos\theta$$

$\therefore$ Im(z) = 0

Now option (4) is correct.

13. If the system of equations

$x+y+z = 5$

$x+2y+3z = 9$

$x+3y+\alpha z = \beta$

has infinitely many solutions, then $\beta-\alpha$ equals:

(1) 5 (2) 18 (3) 21 (4) 8

Ans. **(4)**

Sol. $$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1)-4 = (\alpha-5)$$

for infinite solutions D = 0 $\Rightarrow$ α = 5

$$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2+\beta-15 = 0 \Rightarrow \beta-13 = 0$$

on β = 13 we get $D_y = D_z = 0$

$\alpha = 5$, $\beta = 13$

14. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$ is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{5}{4}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

Ans. **(1)**

Sol. Let points $\left(\frac{3}{2}, 0\right)$, (t^2, t), $t > 0$

$$\text{Distance} = \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$$

$$= \sqrt{t^4 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2-1)^2 + \frac{5}{4}}$$

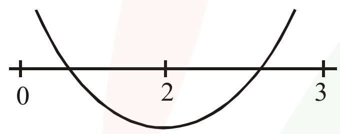
So minimum distance is $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

15. Consider the quadratic equation $(c-5)x^2-2cx + (c-4) = 0$, $c\neq5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

(1) 11 (2) 18 (3) 10 (4) 12

Ans. **(1)**

Sol.



Let $f(x) = (c-5)x^2 - 2cx + c - 4$

$\therefore$ $f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c-4)(c-24) < 0$

& $(c-24)(4c-49) < 0$

$$\Rightarrow \frac{49}{4} < c < 24$$

$\therefore$ s = {13, 14, 15, ... 23}

Number of elements in set S = 11

16. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right) = \frac{k}{21}$, then k equals :

(1) 200 (2) 50 (3) 100 (4) 400

Ans. **(3)**

Sol. $$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$$

$\Rightarrow$ 100 = k

17. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0,2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7 (2) $2\left(\sqrt{2}+2\right)$

(3) –5 (4) $2\left(\sqrt{2}+1\right)$

Ans. (3)

Sol. $\det A = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$

$(R_1 \to R_1 + R_3 - 2R_2)$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$

$=(2+\sin\theta)(2+2d-\sin\theta)-d(2\sin\theta-d)$

$=4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$=d^2 + 4d + 4 - \sin^2\theta$

$=(d + 2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d + 2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

18. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is :

(1) $2\sqrt{2}$ (2) $\frac{1}{8}$ (3) $4\sqrt{2}$ (4) $\frac{1}{4}$

Ans. (4)

Sol. $(1+x^{\log_2 x})^5$

$T_3 = {}^5C_2.(x^{\log_2 x})^2 = 2560$

$\Rightarrow 10.x^{2\log_2 x} = 2560$

$\Rightarrow x^{2\log_2 x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \quad \Rightarrow \log_2 x = 2$ or -2

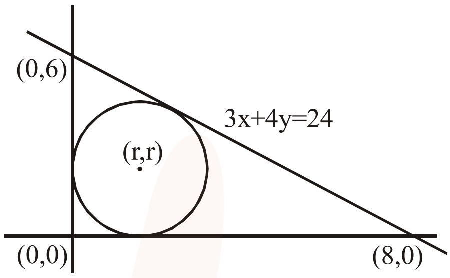
$x = 4$ or $\frac{1}{4}$

19. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

Ans. (2)

Sol.



$\left|\frac{3r + 4r - 24}{5}\right| = r$

$7r - 24 = \pm 5r$

$2r = 24$ or $12r + 24$

$r = 14, \quad r = 2$

then incentre is (2, 2)

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

Ans. (1)

Sol. Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

21. A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1,4)$ and $R(3,-2)$ are fixed points, then the locus of the centroid of ΔPQR is a line :

(1) parallel to x-axis
(2) with slope $\frac{2}{3}$
(3) with slope $\frac{3}{2}$
(4) parallel to y-axis

Ans. (2)

Sol. Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$\frac{\alpha+1+3}{3}=h$ and $\frac{\beta+4-2}{3}=k$

$\alpha = (3h - 4)$ $\beta = (3k - 4)$

Point $P(\alpha, \beta)$ lies on line $2x - 3y + 4 = 0$

$\therefore\ 2(3h - 4) - 3(3k - 2) + 4 = 0$

$\Rightarrow$ locus is $6x - 9y + 2 = 0$

22. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x \in \left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(1) $\frac{1}{3}+e^6$
(2) $\frac{1}{3}$
(3) $-\frac{4}{3}$
(4) $\frac{1}{3}+e^3$

Ans. (1)

Sol. $\frac{dy}{dx}+3\sec^2 x.y=\sec^2 x$

I.F. $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

or $y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x}dx$

or $y.e^{3\tan x} = \frac{1}{3}e^{3\tan x}+C$ ....(1)

Given

$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$

$\therefore\ \frac{4}{3}.e^3=\frac{1}{3}e^3+C$

$\therefore\ C = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$\therefore\ y.e^{-3}=\frac{1}{3}e^{-3}+e^3$

$\therefore\ y=\frac{1}{3}+e^6$

$\therefore\ y\left(-\frac{\pi}{4}\right)=\frac{1}{3}+e^6$

23. The plane passing through the point $(4, -1, 2)$ and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

(1) $(-1, -1, -1)$
(2) $(-1, -1, 1)$
(3) $(1, 1, -1)$
(4) $(1, 1, 1)$

Ans. (4)

Sol. Let $\vec{n}$ be the normal vector to the plane passing through $(4, -1, 2)$ and parallel to the lines L_1 & L_2

then $\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore\ \vec{n}=-7\hat{i}-7\hat{j}+7\hat{k}$

$\therefore$ Equation of plane is

$-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$

$\therefore\ x + y - z - 1 = 0$

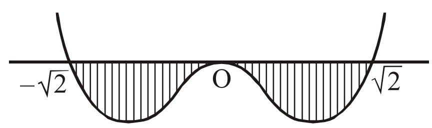
Now check options

24. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $(-\sqrt{2},0)$
(2) $(-\sqrt{2},\sqrt{2})$
(3) $(0,\sqrt{2})$
(4) $(\sqrt{2},-\sqrt{2})$

Ans. (2)

Sol. Let $f(x) = x^2(x^2 - 2)$



As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2},\sqrt{2})$ for minimum of I

25. If 5, 5r, 5r^2 are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) $\frac{7}{4}$

Ans. (4)

Sol. r = 1 is obviously true.

Let 0 < r < 1

$\Rightarrow\ r + r^2 > 1$

$\Rightarrow\ r^2 + r - 1 > 0$

$$\left(r - \frac{-1-\sqrt{5}}{2}\right)\left(r - \left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow\ r - \frac{-1-\sqrt{5}}{2} \text{ or } r > \frac{-1+\sqrt{5}}{2}$$

$$r \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$$

$$\frac{\sqrt{5}-1}{2} < r < 1$$

When r > 1

$$\Rightarrow\ \frac{\sqrt{5}+1}{2} > \frac{1}{r} > 1$$

$$\Rightarrow\ r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

Now check options

26. Consider the statement : "P(n): n^2 – n + 41 is prime." Then which one of the following is true?

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true

Ans. (4)

Sol. P(n) : n^2 – n + 41 is prime

P(5) = 61 which is prime

P(3) = 47 which is also prime

27. Let A be a point on the line

$\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overline{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Ans. (3)

Sol. Let point A is

$(1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\overline{AB} = (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

which is parallel to the plane x – 4y + 3z = 1

$\therefore\ 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$

$8\mu = 2$

$\mu = \frac{1}{4}$

28. For each t∈R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals –1 (2) equals 1

(3) does not exist (4) equals 0

Ans. (4)

Sol. $$\lim_{x\to 1^+} \frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$= \lim_{x\to 1^+} \frac{(1-x)+\sin(x-1)}{(x-1)(-1)} \sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x\to 1^+} \left(1 - \frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$$

29. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

Ans. **(3)**

Sol. Start

$\xrightarrow{1/2}$ $H \to \text{Sum } 7 \text{ or } 8 \Rightarrow \frac{11}{36}$

$\xrightarrow{1/2}$ $T \to \text{Number is } 7 \text{ or } 8 = \frac{2}{9}$

$$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

30. Let $n \geq 2$ be a natural number and $0 < \theta < \pi/2$.

Then $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2+1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2-1}\left(1 + \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

Ans. **(3)**

Sol. $\int \frac{(\sin^n \theta - \sin \theta)^{1/n} \cos \theta}{\sin^{n+1} \theta} d\theta$

$$= \int \frac{\sin \theta \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{1/n}}{\sin^{n+1} \theta} d\theta$$

Put $1 - \frac{1}{\sin^{n-1} \theta} = t$

So $\frac{(n-1)}{\sin^n \theta} \cos \theta d\theta = dt$

Now $\frac{1}{n-1} \int (t)^{1/n} dt$

$$= \frac{1}{(n-1)} \frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{1}{n}+1} + C$$