FINAL JEE–MAIN EXAMINATION – SEPTEMBER, 2020

(Held On Wednesday 02nd SEPTEMBER, 2020) TIME : 3 PM to 6 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

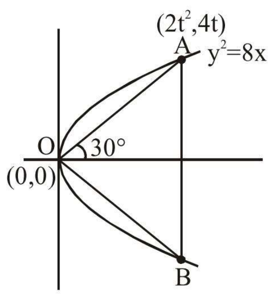
1. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $192\sqrt{3}$ (4) $128\sqrt{3}$

Official Ans. by NTA (3)

Sol.



$$\tan 30° = \frac{4t}{2t^2} = \frac{2}{t} \Rightarrow t = 2\sqrt{3}$$

$$AB = 8t = 16\sqrt{3}$$

$$\text{Area} = 256.3\cdot\frac{\sqrt{3}}{4} = 192\sqrt{3}$$

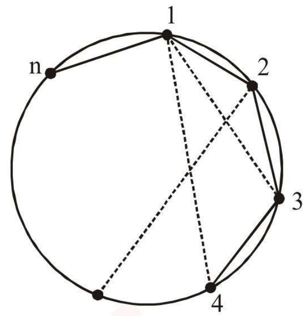
2. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-

(1) 199 (2) 101

(3) 201 (4) 200

Official Ans. by NTA (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$$= {}^nC_2 - n$$

$${}^nC_2 - n = 99\,n \Rightarrow \frac{n(n-1)}{2} - n = 99\,n$$

$$\frac{n-1}{2} - 1 = 99 \Rightarrow n = 201$$

3. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (2) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(3) $\left(-\frac{5}{4}, -1\right)$ (4) $\left[-1, -\frac{1}{2}\right]$

Official Ans. by NTA (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

4. Let f(x) be a quadratic polynomial such that f(–1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in :

(1) (–3, –1) (2) (1, 3)

(3) (–1, 0) (4) (0, 1)

Official Ans. by NTA (3)

Sol. $f(x) = a(x - 3)(x - \alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$$\alpha = -\frac{2}{5} = -0.4$$

$\alpha \in (-1, 0)$

5. Let f : R → R be a function which satisfies $f(x + y) = f(x) + f(y) \forall x, y \in R$. If f(1) = 2 and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which g(n) = 20, is :

(1) 5 (2) 9

(3) 20 (4) 4

Official Ans. by NTA (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$$

$g(n) = 20 \Rightarrow n(n - 1) = 20$

$n = 5$

6. Let $a, b, c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be :

(1) $\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) 3 (4) $\frac{1}{3}$

Official Ans. by NTA (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 (1 - 0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$$\Rightarrow abc = 1, \frac{1}{3}$$

7. Let $f : (-1, \infty) \to R$ be defined by f(0) = 1 and $f(x) = \frac{1}{x}\log_e(1+x), x \neq 0$. Then the function f:

(1) decreases in (–1, ∞)

(2) decreases in (–1, 0) and increases in (0, ∞)

(3) increases in (–1, ∞)

(4) increases in (–1, 0) and decreases in (0, ∞)

Official Ans. by NTA (1)

Sol. $f'(x)=\dfrac{\dfrac{x}{1+x}-\ell n(1+x)}{x^2}$

$=\dfrac{x-(1+x)\ell n(1+x)}{x^2(1+x)}$

Suppose $h(x) = x - (1 + x)\ \ell n(1+x)$

$\Rightarrow h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$

$h'(x) > 0,\ \forall\ x \in (-1, 0)$

$h'(x) < 0,\ \forall\ x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) < 0\ \forall\ x \in (-1, \infty)$

$\Rightarrow f'(x) < 0\ \forall\ x\in(-1, \infty)$

$\Rightarrow$ f(x) is a decreasing function for all $x\in(-1, \infty)$

8. If the sum of first 11 terms of an A.P., a_1 a_2, a_3,... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5,...,a_{23} is ka_1, where k is equal to :

(1) $\dfrac{121}{10}$ (2) $-\dfrac{72}{5}$

(3) $\dfrac{72}{5}$ (4) $-\dfrac{121}{10}$

Official Ans. by NTA (2)

Sol. $a_1 + a_2 + a_3 + \ldots + a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \dfrac{11}{2}=0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1=-5d}$

$a_1 + a_3 + a_5 + \ldots + a_{23}$

$= (a_1+a_{23})\times\dfrac{12}{2}=(a_1+a_1+22d)\times 6$

$=\left(2a_1+22\left(\dfrac{-a_1}{5}\right)\right)\times 6$

$=-\dfrac{72}{5}a_1 \Rightarrow K=\dfrac{-72}{5}$

9. The imaginary part of

$\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be :

(1) $-2\sqrt{6}$ (2) 6

(3) $\sqrt{6}$ (4) $-\sqrt{6}$

Official Ans. by NTA (1)

Sol. $\left(3+2\sqrt{-54}\right)=3+2\times 3\times\sqrt{6}\ i$

$=\left(3+\sqrt{6}\ i\right)^2$

$\left(3-2\sqrt{54}\ \right)=\left(3-\sqrt{6}\ i\right)^2$

$\left(3+2\sqrt{-54}\ \right)^{1/2}+\left(3-2\sqrt{-54}\right)^{1/2}$

$=\pm\left(3+\sqrt{6}\ i\ \right)\pm\left(3-\sqrt{6}\ i\right)$

$= 6, -6,\ 2\sqrt{6}i, -2\sqrt{6}i,$

10. $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{1/x}$ is equal to :

(1) 2 (2) e

(3) 1 (4) e^2

Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\left\{\tan\left(\dfrac{\pi}{4}+x\right)\right\}^{1/x}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

11. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y = 4x + 2$ (2) $x + 4y = 8$

(3) $y + 4x = 2$ (4) $2y + x = 4$

Official Ans. by NTA (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x=0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y=2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$

Put $x = 0$ & $y = 2$

$y' = e^{2\times 2\ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve = $-\frac{1}{4}$ $\{m_1m_2=-1\}$

Hence equation of normal at (0, 2) is

$y - 2 = -\frac{1}{4}(x-0)$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x+4y=8}$

12. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

(1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$

(3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$

Official Ans. by NTA (2)

Sol. Given $\theta \in \left(0, \frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$\Rightarrow \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$

Hence eccentricity of hyperbola

$(e_H) = \sqrt{1 + \frac{10\cos^2\theta}{10}}$ ...(1)

$\left\{e = \sqrt{1 + \frac{b^2}{a^2}}\right\}$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$\Rightarrow \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1$ $\left\{e = \sqrt{1 - \frac{a^2}{b^2}}\right\}$

Hence eccenticity of ellipse

$(e_E) = \sqrt{1 - \frac{5\cos^2\theta}{5}}$

$(e_E) = \sqrt{1-\cos^2\theta} = |\sin\theta| = \sin\theta$ ...(2)

$\left\{\because \theta \in \left(0, \frac{\pi}{2}\right)\right\}$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \dfrac{2}{3}$ ...(3)

Now length of latus rectum of ellipse

$$= \frac{2a^2}{b} = \frac{10\cos^2\theta}{\sqrt{5}} = \frac{20}{3\sqrt{5}} = \frac{4\sqrt{5}}{3}$$

13. Which of the following is a tautology ?

(1) $(\sim p) \wedge (p \vee q) \to q$ (2) $(q \to p) \vee \sim(p \to q)$

(3) $(p \to q) \wedge (q \to p)$ (4) $(\sim q) \vee (p \wedge q) \to q$

Official Ans. by NTA (1)

Sol. Option (1) is

$\sim p \wedge (p \vee q) \to q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \to q$

$\equiv C \vee (\sim p \wedge q) \to q$

$\equiv (\sim p \wedge q) \to q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee t$

so $\sim p \wedge (p \vee q) \to q$ is a tautology

14. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to:

(1) –10 (2) 5

(3) 10 (4) –5

Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through (α, –3, 5) so this point satisfies the equation of plane so put in equation (1)

$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

15. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$.

Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3^C) - P(E_2)$ (2) $P(E_2^C) + P(E_3)$

(3) $P(E_3^C) - P(E_2^C)$ (4) $P(E_3) - P(E_2^C)$

Official Ans. by NTA (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events

so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$= \frac{P(E_1) - \left[P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)\right]}{P(E_1)}$

$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$

= 1 – P(E_2) – P(E_3)

= [1 – P(E_3)] – P(E_2)

$= P(E_3^C) - P(E_2)$

16. Let A = {X = (x, y, z)^T: PX = 0 and

$x^2 + y^2 + z^2 = 1$} where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$,

then the set A :

(1) is a singleton

(2) contains exactly two elements

(3) contains more than two elements

(4) is an empty set

Official Ans. by NTA (2)

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \left.\begin{matrix} x + 2y + z = 0 \\ -2x + 3y - 4z = 0 \\ x + 9y - z = 0 \end{matrix}\right\}$ D = 0, so system have

infinite many solutions,

By solving these equation

we get $x = \frac{-11\lambda}{2}; y = \lambda; z = \frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$

$\Rightarrow \lambda = \pm \frac{1}{\sqrt{\frac{121}{4} + 1 + \frac{49}{4}}}$

so, there are 2 values of λ.

∴ so, there are 2 solution set of (x,y,z).

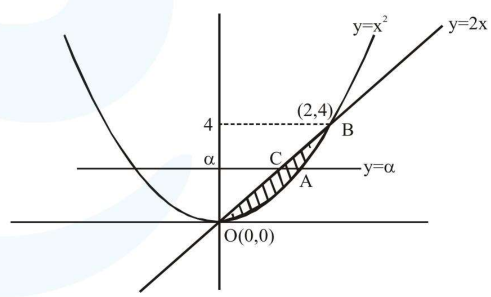
17. Consider a region R = {(x, y) ∈R^2 : $x^2 \le y \le 2x$}. If a line y = α divides the area of region R into two equal parts, then which of the following is true?

(1) $\alpha^3 - 6\alpha^2 + 16 = 0$ (2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$ (4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

Official Ans. by NTA (4)

Sol.



* $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of y = 2x

According to ques, area of OABC = 2 area of OAC

$\Rightarrow \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) dy = 2\int_0^\alpha \left(\sqrt{y} - \frac{y}{2}\right).dy$

$$\Rightarrow \frac{4}{3}=2\left[\frac{2}{3}\alpha^{3/2}-\frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2-8\alpha^{3/2}+8=0}$$

18. If a curve y = f(x), passing through the point (1,2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$

(3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$

Official Ans. by NTA (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx}=\frac{2xy+y^2}{2x^2} \quad \{\text{Homogeneous D.E.}\}$$

$$\left\{\begin{array}{l}\text{let } y = xt\\ \Rightarrow \frac{dy}{dx}=t+x\frac{dt}{dx}\end{array}\right\}$$

$$\Rightarrow t+x\frac{dt}{dx}=\frac{2x^2t+x^2t^2}{2x^2}$$

$$\Rightarrow t+x\frac{dt}{dx}=t+\frac{t^2}{2}$$

$$\Rightarrow x\frac{dt}{dx}=\frac{t^2}{2}$$

$$\Rightarrow 2\int\frac{dt}{t^2}=\int\frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right)=\ell n(x)+C \quad \left\{\text{Put } t=\frac{y}{x}\right\}$$

$$\Rightarrow -\frac{2x}{y}=\ell n x+C \quad \left\{\begin{array}{l}\text{Put } x=1\ \&\ y=2\\ \text{then we get } C=-1\end{array}\right\}$$

$$\Rightarrow \frac{-2x}{y}=\ell n(x)-1$$

$$\Rightarrow y=\frac{2x}{1-\ell n x}$$

$$\Rightarrow f(x)=\frac{2x}{1-\log_e x}$$

so, $\boxed{f\left(\frac{1}{2}\right)=\frac{1}{1+\log_e 2}}$

19. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3+(k+4)a\}+ \{x^4+(k + 6)a\}+.....$ where $a \neq 0$ and $x \neq 1$. If $S=\frac{x^{10}-x+45a(x-1)}{x-1}$, then k is equal to :

(1) –5 (2) 1

(3) –3 (4) 3

Official Ans. by NTA (3)

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] +......9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4+.....9 \text{ terms}) + (ka + ka + ka + ka +.......9 \text{ terms}) + (0 + 2a + 4a + 6a + ...9 \text{ terms})$

$$\Rightarrow S=x\left[\frac{x^9-1}{x-1}\right]+9ka+72a$$

$$\Rightarrow S=\frac{(x^{10}-x)+(9k+72)a(x-1)}{(x-1)}$$

Compare with given sum, then we get, $(9k + 72) = 45$

$$\Rightarrow \boxed{k=-3}$$

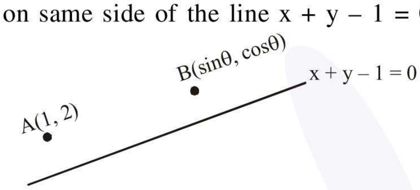
20. The set of all possible values of θ in the interval (0, π) for which the points (1, 2) and (sin θ, cosθ) lie on the same side of the line x + y = 1 is :

(1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$

(3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

Official Ans. by NTA (4)

Sol. Given that both points (1, 2) & (sinθ, cosθ) lie on same side of the line x + y – 1 = 0



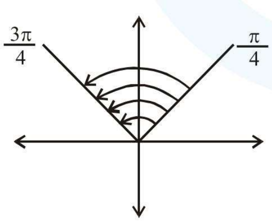
So, $\begin{pmatrix}\text{Put } (1,2) \text{ in} \\ \text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta, \cos\theta \text{ in} \\ \text{given line}\end{pmatrix} > 0$

$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \ \{\div \text{by} \sqrt{2}\}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta > \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$



$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$

$\Rightarrow \boxed{0 < \theta < \frac{\pi}{2}}$

21. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \ldots b_{11}$ is 90, then the common difference of this A.P. is_______.

Official Ans. by NTA (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$\overline{X} = a + \frac{0 + d + 2d + \ldots + 10d}{11}$

= a + 5d

$\Rightarrow$ varience = $\frac{\Sigma(\overline{X} - x_i)^2}{11}$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

22. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$,

then $\frac{dy}{dx}$ at x = 0 is________.

Official Ans. by NTA (91)

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}$ $0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

= cos α . cos kx – sin α . sin kx

= cos(α + kx)

As we have to find derivate at x = 0

We have $\cos^{-1}(\cos(\alpha + kx))$

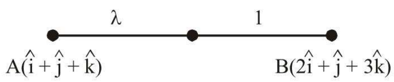
= (α + kx)

$\Rightarrow y = \sum_{k=1}^{6}(\alpha + kx)$

$\Rightarrow \left.\frac{dy}{dx}\right|_{\text{at } x=0} = \sum_{k=x}^{6} k = \frac{6 \times 7 \times 13}{6} = 91$

23. Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio λ : 1 (λ > 0). If O is the origin and $\overrightarrow{OB} \cdot \overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6$, then λ is equal to______.

Official Ans. by NTA (0.8)

Sol. 

Using section formula we get

$$\overline{OP}=\frac{2\lambda+1}{\lambda+1}\hat{i}+\frac{\lambda+1}{\lambda+1}\hat{j}+\frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB}\cdot\overline{OP}=\frac{4\lambda+2+\lambda+1+9\lambda+3}{\lambda+1}$

$$=\frac{14\lambda+6}{\lambda+1}$$

$$\overline{OA}\times\overline{OP}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ \frac{2\lambda+1}{\lambda+1} & 1 & \frac{3\lambda+1}{\lambda+1}\end{vmatrix}$$

$$=\frac{2\lambda+1}{\lambda+1}\hat{i}+\frac{-\lambda}{\lambda+1}\hat{j}+\frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA}\times\overline{OP}|^2=\frac{(2\lambda+1)^2+\lambda^2+\lambda^2}{(\lambda+1)^2}$$

$$=\frac{6\lambda^2+1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1}-3\times\frac{(6\lambda^2+1)}{(\lambda+1)^2}=6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0, \frac{8}{10}=0.8$

$\Rightarrow \lambda = 0.8$

24. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

Official Ans. by NTA (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\frac{{}^nC_{r-1}}{{}^nC_r}=\frac{2}{5}$

$\Rightarrow 7r = 2n + 2$ ...(1)

$$\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{5}{12}$$

$\Rightarrow 17r = 5n - 12$ ...(2)

On solving (1) & (2)

$\Rightarrow n = 118$

25. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is_______.

Official Ans. by NTA (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\int_1^2 |2x-[3x]|\,dx$

$$=\int_1^{4/3}(3-2x)dx+\int_{4/3}^{5/3}(4-2x)\,dx+\int_{5/3}^{2}(5-2x)dx$$

$$=\frac{2}{9}+\frac{3}{9}+\frac{4}{9}=1$$