PART : MATHEMATICS

1. Let A = $[a_{ij}]$, B=$[b_{ij}]$ are two 3 × 3 matrices such that $b_{ij} = \lambda^{i+j-2}a_{ij}$ & |B| = 81. Find |A| if $\lambda = 3$.

माना A = $[a_{ij}]$, B=$[b_{ij}]$, 3 × 3 क्रम की दो आव्यूह इस प्रकार है कि $b_{ij} = \lambda^{i+j-2}a_{ij}$ तथा |B| = 81. यदि $\lambda = 3$ है तब |A|का मान है–

(1) $\frac{1}{9}$ (2) 3 (3) $\frac{1}{81}$ (4) $\frac{1}{27}$

Ans. **(1)**

Sol. $|B|=\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0a_{11} & 3^1a_{12} & 3^2a_{13} \\ 3^1a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{vmatrix} \Rightarrow 81 = 3^3.3.\ 3^2\,|A| \Rightarrow 3^4 = 3^6\,|A| \Rightarrow |A| = \frac{1}{9}$

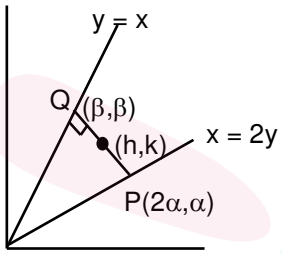
2. From any point P on the line x = 2y perpendicular is drawn on y = x. Let foot of perpendicular is Q. Find the locus of mid point of PQ.

रेखा x = 2y पर स्थित किसी बिन्दु P से रेखा y = x पर लम्ब खींचा जाता है। माना Q लम्बपाद है तब PQ के मध्यबिन्दु का बिन्दुपथ ज्ञात करो–

(1) 2x = 3y (2) 5x = 7y (3) 3x = 2y (4) 7x = 5y

Ans. (2)

Sol.



slope of PQ की प्रवणता = $\frac{k-\alpha}{h-2\alpha} = -1$

$\Rightarrow\ k - \alpha = -h + 2\alpha$

$\Rightarrow \alpha = \frac{h+k}{3}$ .....(1)

Also तथा $2h = 2\alpha + \beta$

$2k = \alpha + \beta$

$\Rightarrow 2h = \alpha + 2k$

$\Rightarrow \alpha = 2h - 2k$ .....(2)

from (1) और & (2) से

$\frac{h+k}{3} = 2(h-k)$

so locus is अतः बिन्दुपथ 6x –6y = x + y $\Rightarrow$ 5x = 7y

3. Pair of tangents are drawn from origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$ then square of length of chord of contact is

मूलबिन्दु से वृत्त $x^2 + y^2 - 8x - 4y + 16 = 0$ पर स्पर्श रेखाऐं खीची जाती है तब स्पर्शजीवा की लम्बाई का वर्ग है–

(1) $\frac{64}{5}$ (2) $\frac{24}{5}$ (3) $\frac{8}{5}$ (4) $\frac{8}{13}$

Ans. (1)

Sol. $L = \sqrt{S_1} = \sqrt{16} = 4$

$R = \sqrt{16 + 4 - 16} = 2$

Length of Chord of contact स्पर्शजीवा की लम्बाई $= \frac{2LR}{\sqrt{L^2 + R^2}} = \frac{2 \times 4 \times 2}{\sqrt{16+4}} = \frac{16}{\sqrt{20}}$

square of length of chord of contact स्पर्शजीवा की लम्बाई का वर्ग $= \frac{64}{5}$

4. Contrapositive of if $A \subset B$ and $B \subset C$ then $C \subset D$

(1) $C \not\subset D$ or $A \not\subset B$ or $B \not\subset C$ (2) $C \subset D$ and $A \not\subset B$ or $B \not\subset C$

(3) $C \subset D$ or $A \not\subset B$ and $B \not\subset C$ (4) $C \subset D$ or $A \not\subset B$ or $B \not\subset C$

यदि $A \subset B$ और $B \subset C$ तब $C \subset D$ का प्रतिपरिवर्तित है–

(1) $C \not\subset D$ या $A \not\subset B$ या $B \not\subset C$ (2) $C \subset D$ और $A \not\subset B$ या $B \not\subset C$

(3) $C \subset D$या $A \not\subset B$ और $B \not\subset C$ (4) $C \subset D$ या $A \not\subset B$ या $B \not\subset C$

Ans. (4)

Sol. Let $P = A \subset B$, $Q = B \subset C$, $R = C \subset A$

Contrapositive of $(P \wedge Q) \rightarrow R$ is $\sim R \rightarrow \sim (P \wedge Q)$

$R \vee (\sim P \vee \sim Q)$

Sol. माना $P = A \subset B$, $Q = B \subset C$, $R = C \subset A$

$(P \wedge Q) \rightarrow R$ का प्रतिपरिवर्तित $\sim R \rightarrow \sim (P \wedge Q)$

$R \vee (\sim P \vee \sim Q)$

5. Let y(x) is solution of differential equation $(y^2 - x)\frac{dy}{dx} = 1$ and y(0) = 1, then find the value of x where curve cuts the x-axis

माना y(x), अवकल समीकरण $(y^2 - x)\frac{dy}{dx} = 1$ का हल है तथा y(0) = 1, तब x का मान होगा, जहां वक्र x अक्ष को प्रतिच्छेद करता है–

(1) 2 – e (2) 2 +e (3) 2 (4) e

Ans. (1)

Sol. **Sol.** $\frac{dx}{dy} + x = y^2$

$I.F. = e^{\int I.dy} = e^y$

$x.e^y = \int y^2.e^y.dy$

$= y^2.e^y - \int 2y.e^y.dy$

$\Rightarrow y^2e^y - 2(y.e^y - e^y) + c$

$x.e^y = y^2e^y - 2ye^y + 2e^y + C$

$x = y^2 - 2y + 2 + c.\ e^{-y}$

$x = 0, \quad y = 1$

$0 = 1 - 2 + 2 + \frac{c}{e}$

$c = -e$

$y = 0, \quad x = 0 - 0 + 2 + (-e)\ (e^{-0})$

$x = 2 - e$

6. Let θ_1 and θ_2 (where$\theta_1 < \theta_2$)are two solutions of $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to

माना θ_1 और θ_2 (जहां $\theta_1 < \theta_2$) समीकरण $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0, \theta \in [0, 2\pi)$ के दो हल है तब $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ का मान है–

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3} + \frac{1}{6}$

Ans. (1)

Sol. $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$

$\frac{2\cos^2\theta}{\sin^2\theta} - \frac{5}{\sin\theta} + 4 = 0$

$2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0, \ \sin\theta \neq 0$

$2\sin^2\theta - 5\sin\theta + 2 = 0$

$(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta\ d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2}\ d\theta$

$= \frac{1}{2}\left[\theta + \frac{\sin 6\theta}{6}\right]_{\pi/6}^{5\pi/6} = \frac{1}{2}\left[\frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0-0)\right] = \frac{1}{2}.\frac{4\pi}{6} = \frac{\pi}{3}$

7. Let 3 + 4 + 8 + 9 + 13 + 14 + 18 +.....40 terms = S. If S = (102)m then m =

माना 3 + 4 + 8 + 9 + 13 + 14 + 18 +.....40 पद तक = S. यदि S = (102)m तब m =

(1) 20 (2) 25 (3) 10 (4) 5

Ans. (1)

Sol. $S = \underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19}$.....40 terms पद तक

$S = 7 + 17 + 27 + 37 + 47 +$.....20 terms पद तक

$S_{40} = \frac{20}{2}[2\times 7+(19)10] = 10[14 + 190] = 10[2040] = (102)\ (20)$

$\Rightarrow m = 20$

8. If $\left({}^{36}C_{r+1}\right)\times\left(k^2-3\right) = {}^{35}C_r$ 6 , then number of ordered pairs (r, k) are –(where $k\in I$).

यदि $\left({}^{36}C_{r+1}\right)\times\left(k^2-3\right) = {}^{35}C_r$ 6 , तो क्रमित युग्मों (r, k) की संख्या है –(जहाँ $k\in I$).

(1) 6 (2) 2 (3) 3 (4) 4

Ans. (4)

Sol. $\frac{36}{r+1}\times\frac{35}{C_r}\left(k^2-3\right) = \frac{35}{C_r}$

$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$

r can be 5, 35

for r = 5, k = ±2

r = 35, k = ±3

Hence number of order pair = 4

Hindi. $\frac{36}{r+1}\times\frac{35}{C_r}\left(k^2-3\right) = \frac{35}{C_r}$

$k^2 - 3 = \frac{r+1}{6} \Rightarrow k^2 = 3 + \frac{r+1}{6}$

r 5, 35 हो सकता है।

r = 5, के लिए k = ±2

r = 35, के लिएk = ±3

अतः क्रमित युग्मों की संख्या = 4

9. Let $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$ then α =

माना कि $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$ तो α =

(1) $\ell n2$ (2) $\ell n\sqrt{2}$ (3) $\ell n\frac{3}{4}$ (4) $\ell n\frac{4}{3}$

Ans. (1)

Sol. $4\alpha\left\{\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right\} = 5$

$\Rightarrow 4\alpha\left\{\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + \frac{e^{-\alpha x}}{-\alpha}\Big|_0^2\right\} = 5$

$\Rightarrow 4\alpha\left\{\left(\frac{1-e^{-\alpha}}{\alpha}\right)-\left(\frac{e^{-2\alpha}-1}{\alpha}\right)\right\}=5$ $\Rightarrow 4(2-e^{-\alpha}-e^{-2\alpha})=5$ Put $e^{-\alpha}=t$

$\Rightarrow 4t^2+4t-3=0$ $\Rightarrow (2t+3)(2t-1)=0$

$\Rightarrow e^{-\alpha}=\frac{1}{2}$ $\Rightarrow \alpha=\ln 2$

10. Let f(x) is a five degree polynomial which has critical points x = ±1 and $\lim\limits_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$ then which one is incorrect.

(1) f(x) has minima at x = 1 & maxima at x = –1

(2) f(1) –4f(–1) = 4

(3) f(x) is maxima at x = 1 and minima at x = –1

(4) f(x) is odd

माना कि f(x) एक 5 घात का बहुपद है जिसके क्रांतिक बिन्दु x = ±1 है तथा $\lim\limits_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$ तो निम्न में से कौनसा गलत है।

(1) f(x) का x = 1 पर निम्ननिष्ठ तथा x = –1 पर उच्चिष्ठ है।

(2) f(1) –4f(–1) = 4

(3) f(x) का x = 1 उच्चिष्ठ तथा x = –1 पर निम्निष्ठि है।

(4) f(x) विषम घात है।

Ans. (1)

Sol. $f(x)=ax^5+bx^4+cx^3$

$\lim\limits_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right)=4 \Rightarrow 2+c=4 \Rightarrow c=2$

$f'(x)=5ax^4+4bx^3+6x^2$

$=x^2\left(5ax^2+4bx+6\right)$

$f'(1)=0 \Rightarrow 5a+4b+6=0$

$f'(-1)=0 \Rightarrow 5a-4b+6=0$

$b=0$

$a=-\frac{6}{5}$

$f(x)=\frac{-6}{5}x^5+2x^3$

$f'(x)=-6x^4+6x^2$

$=6x^2(-x^2+1)$

$=-6x^2(x+1)(x-1)$

$\frac{-1 \quad + \quad 1-}{1- \qquad\qquad 1}$

Minimal at x = –1

Maxima at x = 1

x = –1 पर निम्ननिष्ठ

x = 1 पर उच्चिष्ठ

11. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then $(\lambda,\vec{d}) =$

यदि $\vec{a},\vec{b},\vec{c}$ इकाई सदिश इस प्रकार है कि $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ तथा $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ और $\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ तब $(\lambda,\vec{d}) =$

(1) $\left(\frac{3}{2},3\vec{a}\times\vec{b}\right)$ (2) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$ (3) $\left(-\frac{3}{2},3\vec{a}\times\vec{c}\right)$ (4) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$

Ans. (4)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$3 + 2\,(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

$(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = \frac{-3}{2}$

$\Rightarrow \lambda = \frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+(-\vec{a}-\vec{b})\times\vec{a}$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}$

$\vec{d} = 3\,(\vec{a}\times\vec{b})$

12. Coefficient of x^7 in $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8$+.....+$x^{10}$ is-

$(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8$+.....+$x^{10}$ में x^7 का गुणांक है–

(1) 330 (2) 210 (3) 420 (4) 260

Ans. (1)

Sol. $\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$

$\frac{(1+x)^{10}\left[(1+x)^{11}-x^{11}\right]}{(1+x)^{11}\times\frac{1}{(1+x)}}$

$= (1 + x)^{11} - x^{11}$

Coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_4 = 330$

x^7 का गुणांक ${}^{11}C_7 = {}^{11}C_4 = 330$

13. Let α and β are the roots of $x^2 - x - 1 = 0$ such that $P_k = \alpha^k + \beta^k$, $k \geq 1$ then which one is incorrect?

माना कि α तथा β समीकरण $x^2 - x - 1 = 0$ के मूल इस प्रकार है कि $P_k = \alpha^k + \beta^k$, $k \geq 1$ तो निम्न में से कौनसा गलत है–

(1) $P_5 = P_2 \times P_3$ (2) $P_1 + P_2 + P_3 + P_4 + P_5 = 26$

(3) $P_3 = P_5 - P_4$ (4) $P_4 = 11$

Ans. (1)

Sol. $\alpha^5 = 5\alpha + 3$

$\beta^5 = 5\beta + 3$

$P_5 = 5(\alpha + \beta) + 6$

$= 5(1) + 6$

$P_5 = 11$and तथा $P_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$

$P_2 = 3$ and तथा $P_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1 = 2(1) + 2 = 4$

$P_2 \times P_3 = 12$ and तथा $P_5 = 11 \quad \Rightarrow \quad P_5 \neq P_2 \times P_3$

14. Let $f(x) = x^3 - 4x^2 + 8x + 11$, if LMVT is applicable on f(x) in [0, 1], value of c is :

माना कि $f(x) = x^3 - 4x^2 + 8x + 11$, f(x) के लिए LMVT अन्तराल [0, 1] के लिए लागू होती तो c है।

(1) $\frac{4-\sqrt{7}}{3}$ (2) $\frac{4-\sqrt{5}}{3}$ (3) $\frac{4+\sqrt{7}}{3}$ (4) $\frac{4+\sqrt{5}}{3}$

Ans. **(1)**

Sol. f(x) is a polynomial function

$\therefore$ it is continuous and differentiable in [0, 1]

Here $f(0) = 11$, $f(1) = 1 - 4 + 8 + 11 = 16$

$f'(x) = 3x^2 - 8x + 8$

$\therefore \quad f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1} = 3c^2 - 8c + 8$

$\Rightarrow \quad 3c^2 - 8c + 3 = 0$

$C = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$

$\therefore \quad c = \frac{4-\sqrt{7}}{3} \in (0, 1)$

Hindi. f(x) एक बहुपदीय फलन है।

$\therefore$ यह अन्तराल [0, 1] में सतत तथा अन्तराल (0, 1) में अवकलनीय है।

यहां $f(0) = 11$, $f(1) = 1 - 4 + 8 + 11 = 16$

$f'(x) = 3x^2 - 8x + 8$

$\therefore \quad f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{16-11}{1} = 3c^2 - 8c + 8$

$\Rightarrow \quad 3c^2 - 8c + 3 = 0$

$C = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$

$\therefore \quad c = \frac{4-\sqrt{7}}{3} \in (0, 1)$

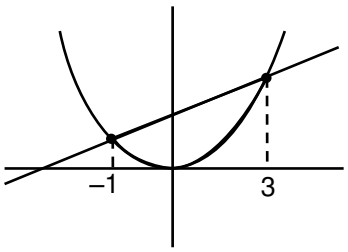
15. The area bounded by $4x^2 \le y \le 8x+12$ is -

$4x^2 \le y \le 8x+12$ द्वारा घिरा हुआ क्षेत्रफल है–

(1) $\frac{127}{3}$ (2) $\frac{128}{3}$ (3) $\frac{124}{3}$ (4) $\frac{125}{3}$

Ans. (2)

Sol.



$4x^2 = y$

$y = 8x + 12$

$4x^2 = 8x + 12$

$x^2 - x - 3 = 0$

$x^2 - 2x - 3 = 0]$

$x^2 - 3x + x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$

$A = \int_{-1}^{3} (8x+12-4x^2)\,dx$

$A = \frac{8x^2}{2} + 12x - \frac{4x^3}{3}\Big|_{-1}^{3} = (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right) = 36 + 8 - \frac{4}{3}$

$= 44 - \frac{4}{3} = \frac{132-4}{3} = \frac{128}{3}$

16. There are 5 machines. Probability of a machine being faulted is $\frac{1}{4}$. Probability of atmost two machines is faulted, is $\left(\frac{3}{4}\right)^3$ k then value of k is

5 मशीन इस प्रकार है कि किसी एक मशीन के खराब होने की प्रायिकता $\frac{1}{4}$ है, यदि अधिकतम दो मशीनों के खराब होने की प्रायिकता $\left(\frac{3}{4}\right)^3$ k है, तो k का मान होगा।

(1) $\frac{17}{2}$ (2) 4 (3) $\frac{17}{8}$ (4) $\frac{17}{4}$

Ans. (3)

Sol. Required probability = when no. machine has fault + when only one machine has fault + when only two machines have fault.

अभीष्ट प्रायिकता = जब कोई भी मशीन खराब नहीं है + जब केवल एक मशीन खराब है + जब केवल दो मशीन खराब है

$$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$$

$$= \frac{243}{1024} + \frac{405}{1024} + \frac{270}{1024} = \frac{918}{1024} = \frac{459}{512} = \frac{27\times17}{64\times8}$$

$$= \left(\frac{3}{4}\right)^3 \times k = \left(\frac{3}{4}\right)^3 \times \frac{17}{8}$$

$$\therefore k = \frac{17}{8}$$

17. $3x + 4y = 12\sqrt{2}$ is the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ then the distance between focii of ellipse is-

यदि $3x + 4y = 12\sqrt{2}$, दीर्घवृत्त $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ की स्पर्श रेखा है तो दीर्घवृत्त की नाभियों के मध्य दूरी होगी–

(1) $2\sqrt{5}$ (2) $2\sqrt{3}$ (3) $2\sqrt{7}$ (4) 4

Ans. (3)

Sol. $3x + 4y = 12\sqrt{2}$

$\Rightarrow 4y = -3x + 12\sqrt{2}$

$\Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$

condition of tangency स्पर्शता का प्रतिबन्ध $c^2 = a^2m^2 + b^2$

$18 = a^2.\frac{9}{16} + 9$

$a^2.\frac{9}{16} = 9$

$a^2 = 16$

$1a = 4$

$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore ae = \frac{\sqrt{7}}{4}.4 = \sqrt{7}$

$\therefore$ focus are नाभियां $(\pm\sqrt{7}, 0)$

$\therefore$ distance between foci नाभियों के मध्य दूरी $= 2\sqrt{7}$

18. If $z = \left(\frac{3+i\sin\theta}{4-i\cos\theta}\right)$ is purely real and $\theta \in \left(\frac{\pi}{2},\pi\right)$ then arg(sinθ + i cosθ) is -

यदि $z = \left(\frac{3+i\sin\theta}{4-i\cos\theta}\right)$ पूर्णतया वास्तविक है तथा $\theta \in \left(\frac{\pi}{2},\pi\right)$ तो arg(sinθ + i cosθ) का मान होगा–

(1) $-\tan^{-1}\frac{3}{4}$ (2) $\pi-\tan^{-1}\frac{3}{4}$ (3) $\pi-\tan^{-1}\frac{4}{3}$ (4) $\tan^{-1}\frac{4}{3}$

Ans. (3)

Sol. $z = \frac{(3+i\sin\theta)}{(4-i\cos\theta)} \times \frac{(4+i\cos\theta)}{(4+i\cos\theta)}$

as चूंकि z is purely real पूर्णतः वास्तविक है $\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$

$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) = \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(-\frac{4}{3}\right)$

19. $a_1, a_2, a_3 \ldots..a_9$ are in GP where $a_1 < 0$,

$a_1 + a_2 = 4$, $a_3 + a_4 = 16$, if $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to

$a_1, a_2, a_3 \ldots..a_9$ गुणोत्तर श्रेणी में है जहां $a_1 < 0$,

$a_1 + a_2 = 4$, $a_3 + a_4 = 16$, if $\sum_{i=1}^{9} a_i = 4\lambda$, तो λ का मान होगा–

(1) –513 (2) $-\frac{511}{3}$ (3) -171 (4) 171

Ans. **(3)**

Sol. $a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4$ .....(i)

$a_3 + a_4 = 16 \Rightarrow a_1r^2 + a_1r^3 = 16$ .....(ii)

$\frac{1}{r^2} + \frac{1}{4} \Rightarrow r^2 = 4$

$r = \pm 2$

$r = 2$, $a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$

$r = -2$, $a_1(1-2) = 4 \Rightarrow a_1 = -4$

$\sum_{i=1}^{a} a_i = \frac{a_1(r^q-1)}{r-1} = \frac{(-4)((-2)^9-1)}{-2-1} = \frac{4}{3}(-513) = 4\lambda$

$\lambda = -171$

20. If यदि $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ and और $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then तो $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is होगा -

(1) $\frac{2}{\sqrt{5}}$ (2) $-\frac{\sqrt{5}}{4}$ (3) $-\frac{\sqrt{5}}{2}$ (4) $\frac{\sqrt{5}}{2}$

Ans. (3)

Sol. $x = \frac{1}{2},\ y = \frac{-1}{4} \Rightarrow \quad xy = \frac{-1}{8}$

$$y.\frac{1.(=2x)}{2\sqrt{-x^2}} + y'.\sqrt{1-x^2} = -\left\{1.\sqrt{1-y^2} + \frac{x.(-2y)}{2\sqrt{1-y^2}}y'\right\}$$

$$-\frac{xy}{\sqrt{1-x^2}} + y'\sqrt{1-x^2} = -\sqrt{1-y^2} + \frac{xy.y'}{\sqrt{1-y^2}}$$

$$y'\left(\sqrt{1-x^2} - \frac{xy}{\sqrt{1-y^2}}\right) = \frac{xy}{\sqrt{1-x^2}} - \sqrt{1-y^2}$$

$$y'\left(\frac{\sqrt{3}}{2} + \frac{1}{8.\frac{\sqrt{15}}{4}}\right) = \frac{-1}{8.\sqrt{\frac{3}{2}}} - \frac{\sqrt{15}}{4}$$

$$y'\left(\frac{\sqrt{45}+1}{2\sqrt{15}}\right) = -\frac{\left(1+\sqrt{45}\right)}{4\sqrt{3}}$$

$$y' = -\frac{\sqrt{5}}{2}$$

21. Let $X = \{x : 1 \le x \le 50, x \in N\}$
$A = \{x: x \text{ is multiple of } 2\}$
$B = \{x: x \text{ is multiple of } 7\}$
Then find number of elements in the smallest subset of X which contain elements of both A and B

माना $X = \{x : 1 \le x \le 50, x \in N\}$
$A = \{x: x, 2 \text{ का गुणज है}\}$
$B = \{x: x, 7 \text{ का गुणज है }\}$
तो X के सबसे छोटे उपसमुच्चय, जो A तथा B दोनों के अवयवों को समाहित करता है, में अवयवों की संख्या है–

Ans. **29**

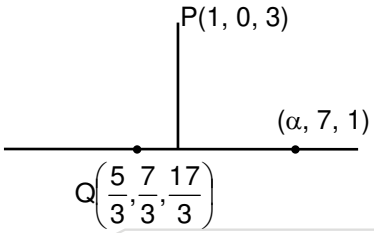
Sol. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 25 + 7 - 3$
$= 29$

22. If $Q\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$ is foot of perpendicular drawn from P(1, 0, 3) on a line L and if line L is passing through $(\alpha, 7, 1)$, then value of α is

यदि P(1, 0, 3) से किसी रेखा L पर लम्ब का पाद $Q\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$ है तथा रेखा L $(\alpha, 7, 1)$ से गुजरती है तो α का मान होगा–

Ans. **4**

Sol. Since PQ is perpendicular to L, therefore
चूंकि PQ रेखा L के लम्बवत् है इसलिए



$$\left(1-\frac{5}{3}\right)\left(\alpha-\frac{5}{3}\right)+\left(\frac{-7}{3}\right)\left(7-\frac{7}{3}\right)+\left(3-\frac{17}{3}\right)\left(1-\frac{17}{3}\right)=0$$

$$\Rightarrow \frac{-2\alpha}{3}+\frac{10}{9}-\frac{98}{9}+\frac{112}{9}=0$$

$$\Rightarrow \frac{2\alpha}{3}=\frac{24}{9} \Rightarrow \alpha = 4$$

23. If f(x) is defined in $x \in \left(-\frac{1}{3},\frac{1}{3}\right)$

$$f(x) = \begin{cases} \left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$ Find k such that f(x) is continuous

यदि f(x), $x \in \left(-\frac{1}{3},\frac{1}{3}\right)$ में परिभाषित है

$$f(x) = \begin{cases} \left(\frac{1}{x}\right)\log_e\left(\frac{1+3x}{1-2x}\right) & x \neq 0 \\ k & x = 0 \end{cases}$$ यदि f(x) सतत् है तो k का मान होगा–

Ans. 5

Sol. $\lim\limits_{x\to0} f(x)\ \lim\limits_{x\to0}\left(\frac{1}{x}\ell n\left(\frac{1+3x}{1-2x}\right)\right) = \lim\limits_{x\to0}\left(\frac{\ell n(1+3x)}{x}-\frac{\ell n(1-2x)}{x}\right)$

$$= \lim_{x\to0}\left(\frac{3\ell n(1+3x)}{3x}-\frac{2\ell n(1-2x)}{-2x}\right) = 3 + 2 = 5$$

$\therefore$ f(x) will be continuous सतत होगा if यदि $f(0) = \lim\limits_{x\to0} f(x)$

24. If system of equation यदि समीकरणों का निकाय

$x + y + z = 6$
$x + 2y + 3z = 10$
$3x + 2y + \lambda z = \mu$
has more than two solutions. Find $(\mu - \lambda^2)$
के दो से अधिक हल है तब $(\mu - \lambda^2)$ का मान होगा–

Ans. **13**

Sol. $x + y + z = 6$ …….. (1)

$x + 2y + 3z = 10$ …….. (2)

$3x + 2y + \lambda z = \mu$ …….. (3)

from (1) and तथा (2) से

if $z = 0 \Rightarrow x + y = 6$ and और $x + 2y = 10$

$\Rightarrow y = 4, x = 2$

(2, 4, 0)

if $y = 0 \Rightarrow x + z = 6$ and और $x + 3z = 10$

$\Rightarrow z = 2$ and और $x = 4$

(4, 0, 2)

so इसलिए $3x + 2y + \lambda z = \mu$

must pass through गुजरती है (2, 4, 0) and तथा (4, 0, 2)

so इसलिए$6 + 8 = \mu \Rightarrow \mu = 14$

and तथा $12 + 2\lambda = \mu$

$12 + 2\lambda = 14 \Rightarrow \lambda = 1$

so इसलिए $\mu - \lambda^2 = 14 - 1$

$= 13$

25. If mean and variance of 2, 3, 16, 20, 13, 7, x, y are 10 and 25 respectively then find xy

यदि चरों 2, 3, 16, 20, 13, 7, x, y का माध्य और प्रसरण क्रमशः 10 और 25 है तब xy का मान है–

Ans. **124**

Sol. mean माध्य $= \bar{x} = \frac{2+3+16+20+13+7+x+y}{8} = 10 \quad \Rightarrow x + y = 19$……(i)

variance प्रसरण $\sigma^2 = \frac{\sum (x_i)^2}{8} - (\bar{x})^2 = 25$

$$\frac{4+9+256+400+169+49+x^2+y^2}{8} - 100 = 25$$

$\Rightarrow \quad x^2 + y^2 = 113$……(ii)

$(x+y)^2 = (19)^2 \Rightarrow x^2 + y^2 + 2xy = 361 \Rightarrow xy = 124$

(exact data is not retrieved so ans. can vary)