FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

Official Ans. by NTA (4)

Sol. $\vec{a}.\vec{b}=0$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b}=-|\vec{a}|^2\vec{b}$

Now $\vec{a}\times\left(\vec{a}\times\left(-|\vec{a}|^2\vec{b}\right)\right)$

$=-|\vec{a}|^2\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=-|\vec{a}|^2\left(-|\vec{a}|^2\vec{b}\right)=|\vec{a}|^4\vec{b}$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

Official Ans. by NTA (1)

Sol. Let the coin be tossed n-times

$P(H) = P(T) = \frac{1}{2}$

$P(7\text{ heads}) = {}^nC_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^7=\frac{{}^nC_7}{2^n}$

$P(9\text{ heads}) = {}^nC_9\left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^9=\frac{{}^nC_9}{2^n}$

P(7 heads) = P(9 heads)

${}^nC_7 = {}^nC_9 \Rightarrow n = 16$

$P(2\text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^2=\frac{15\times8}{2^{16}}$

$P(2\text{ heads}) = \frac{15}{2^{13}}$

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

Official Ans. by NTA (1)

Sol. $A=\begin{pmatrix}a & b\\ b & c\end{pmatrix}$, a, b, c ∈ I

$A^2=\begin{pmatrix}a & b\\ b & c\end{pmatrix}\begin{pmatrix}a & b\\ b & c\end{pmatrix}=\begin{pmatrix}a^2+b^2 & b(a+c)\\ b(a+c) & b^2+c^2\end{pmatrix}$

Sum of the diagonal entries of

$A^2 = a^2 + 2b^2 + c^2$

Given $a^2 + 2b^2 + c^2 = 1$, a, b, c ∈ I

$b = 0$ & $a^2 + c^2 = 1$

Case–1 : $a = 0 \Rightarrow c = \pm1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm1$ (2-matrices)

Total = 4 matrices

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

Official Ans. by NTA (3)

Sol. a, ar, ar^2, ...

$T_2+T_6=\frac{25}{2}\Rightarrow ar\left(1+r^4\right)=\frac{25}{2}$

$a^2r^2(1+r^4)^2=\frac{625}{4}$ .... (1)

$T_3 . T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$

$a^2r^6 = 25$ .... (2)

On dividing (1) by (2)

$\frac{\left(1+r^4\right)^2}{r^4}=\frac{25}{4}$

$4r^8 - 17r^4 + 4 = 0$

$(4r^4 - 1)(r^4 - 4) = 0$

$r^4 = \frac{1}{4}, 4 \Rightarrow r^4 = 4$

(an increasing geometric series)

$a^2r^6 = 25 \Rightarrow (ar^3)^2 = 25$

$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 5(1 + 2 + 4) = 35$

5. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\leq$ x, is

(1) 100(e – 1) (2) 100(1 – e)

(3) 100e (4) 100 (1 + e)

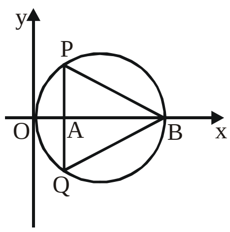
Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}dx$, period of {x} = 1

$$\sum_{n=1}^{100} \int_{0}^{1} e^{\{x\}}\, dx = \sum_{n=1}^{100} \int_{0}^{1} e^{x}dx$$

$$\sum_{n=1}^{100} (e-1) = 100(e-1)$$

6. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Official Ans. by NTA (2)

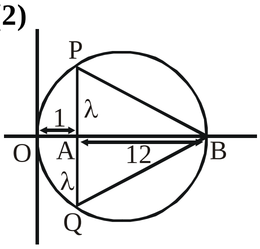
Sol. PA = AQ = λ

OA·AB

= AP . AQ

$\Rightarrow 1.12 = \lambda . \lambda$

$\Rightarrow \lambda = 2\sqrt{3}$



Area $\Delta PQB = \frac{1}{2} \times 2\lambda \times AB$

$$\Delta = \frac{1}{2}.4\sqrt{3} \times 12$$

$$= 24\sqrt{3}$$

7. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Official Ans. by NTA (1)

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \dots$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \dots$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \dots + \text{up to infinite terms}$$

$$\Rightarrow S = \frac{13}{4}$$

8. The value of

$$\lim_{h \to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}\right\}$$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Official Ans. by NTA (1)

Sol. $L = \lim_{h \to 0} 2\left(\frac{\sqrt{3}\left(\frac{1}{2}\cosh + \frac{\sqrt{3}}{2}\sinh\right) - \left(\frac{\sqrt{3}}{2}\cosh - \frac{\sinh}{2}\right)}{\left(\sqrt{3}h\right)\left(\sqrt{3}\right)}\right)$

$$L = \lim_{h \to 0} \frac{4\sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exect same magnitude but opposite sign powers of t in the binomial expression given

$$\text{so } T_6 = {}^{10}C_5\left(tx^25\right)^5\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^5$$

$T_6 = f(x) = {}^{10}C_5\left(x\sqrt{1-x}\right)$; for maximum

$$f'(x) = 0 \Rightarrow x = \frac{2}{3} \ \& \ f''\left(\frac{2}{3}\right) < 0$$

$$\text{so } f(x)_{max.} = {}^{10}C_5\left(\frac{2}{3}\right).\frac{1}{\sqrt{3}}$$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

Official Ans. by NTA (1)

Sol. $$\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200}\frac{dB}{B} = \lambda\int_0^2 dt \Rightarrow \lambda = \frac{1}{2}\ell n\left(\frac{6}{5}\right)$$

$$\int_{1000}^{2000}\frac{dB}{B} = \frac{1}{2}\ell n\left(\frac{6}{5}\right)\int_0^T dt \Rightarrow \ T = \frac{2\ell n2}{\ell n\left(\frac{6}{5}\right)}$$

$\Rightarrow k = 2\ell n2$

11. If $(1,5,35)$, $(7,5,5)$, $(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Official Ans. by NTA (3)

Sol. A(1, 5, 35), B(7, 5, 5), C(1, λ, 7), D(2λ, 1, 2)

$\overline{AB} = 6\hat{i} - 30\hat{k}$, $\overline{BC} = -6\hat{i}(\lambda-5)\hat{j} + 2\hat{k}$,

$\overline{CD} = (2\lambda-1)\hat{i} + (1-\lambda)\hat{j} - 5\hat{k}$

Points are coplanar

$$\Rightarrow 0 = \begin{vmatrix} 6 & 0 & -30 \\ -6 & \lambda-5 & 2 \\ 2\lambda-1 & 1-\lambda & -5 \end{vmatrix}$$

$= 6(-5\lambda + 25 - 2 + 2\lambda)$

$-30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$

$= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$

$= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$

$= 6(10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$

$\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

$$\lambda_1 + \lambda_2 = \frac{44}{5}$$

12. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}; 0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1 - y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1}x}{r} = a$, $\frac{\cos^{-1}x}{r} = b$, $\frac{\tan^{-1}y}{r} = c$

So, $a + b = \frac{\pi}{2r}$

$$\cos\left(\frac{\pi c}{a+b}\right)=\cos\left(\frac{\pi\tan^{-1}y}{\frac{\pi}{2r}r}\right)$$

$= \cos(2\tan^{-1}y)$, let $\tan^{-1}y = \theta$

$= \cos(2\theta)$

$$=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-y^2}{1+y^2}$$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

Official Ans. by NTA (3)

Sol. **(I)** First possiblity is 1, 1, 1, 1, 1, 2, 3

required number $= \frac{7!}{5!} = 7 \times 6 = 42$

(II) Second possiblity is 1, 1, 1, 1, 2, 2, 2

required number $= \frac{7!}{4!\ 3!}=\frac{7\times6\times5}{6}=35$

Total = 42 + 35 = 77

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|, \forall (x,y) \in R$

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0, \forall x \in R$

(3) $f(x) = 0, \forall x \in R$

(4) $f(x) > 0, \forall x \in R$

Official Ans. by NTA (4)

Sol. $$\left|\frac{f(x)-f(y)}{(x-y)}\right|\le|(x-y)|$$

$x - y = h$ let $\Rightarrow x = y + h$

$$\lim_{x\to0}\left|\frac{f(y+h)-f(y)}{h}\right|\le0$$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = k$ (constant)

and $f(0) = 1$ given

So, $f(y) = 1 \Rightarrow f(x) = 1$

15. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

(1) (2,2) (2) (0,0)

(3) (2,9) (4) $\left(3,\frac{21}{2}\right)$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Since, slope is maximum so,

$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$ | $\frac{d^3y}{dx^3}=12x-30$

$x = 2, 3$ | at $x = 2$, $\frac{d^3y}{dx^3}<0$

So, maxima

at x = 2

$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$

$= 8 - 40 + 72 - 38 = 80 - 78 = 2$

point (2, 2)

16. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a

(1) Right angled triangle

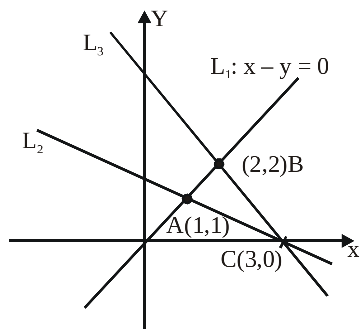
(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

Official Ans. by NTA (3)

Sol.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : x + y = 6$

on solving L_1 and L_2 :

y = L and x = 1

L_1 and L_3 :

x = 2

y = 2

L_2 and L_3 :

x + y = 3

2x + y = 6

x = 3

y = 0

$$AC = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{4+1} = \sqrt{5}$$

$$AB = \sqrt{1+1} = \sqrt{2}$$

so its an isosceles triangle

17. Consider the three planes

$P_1 : 3x + 15y + 21z = 9$,

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1, P_2 and P_3 all are parallel

Official Ans. by NTA (2)

Sol. $P_1 : x + 5y + 7z = 3$,

$P_2 : x - 3y - z = 5$

$P_3 : x + 5y + 7z = \frac{5}{2}$

so P_1 and P_3 are parallel.

18. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) (a + 2) (a + 3) (a + 4)

(2) –2

(3) (a + 1) (a + 2) (a + 3)

(4) 0

Official Ans. by NTA (2)

Sol. $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

= 4(a + 2) – 4a – 10

= 4a + 8 – 4a – 10 = –2

19. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Official Ans. by NTA (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ (using king)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x)\cos^2 x}{1+3^x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos^2 x dx = 2\int_0^{\pi/2} \cos^2 x \, dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos^2 x \, dx = \frac{\pi}{4}$$

20. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :

(1) $S = \{(x,y) \mid x^2 + y^2 = 4\}$

(2) $S = \{(x,y) \mid x^2 + y^2 = 1\}$

(3) $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$

(4) $S = \{(x,y) \mid x^2 + y^2 = 2\}$

Official Ans. by NTA (4)

Sol. Equivalence class of (1, –1) is a circle with centre at (0,0) and radius = $\sqrt{2}$

$\Rightarrow x^2 + y^2 = 2$

$S = \{(x,y) \mid x^2 + y^2 = 2\}$

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

Official Ans. by NTA (2)

Sol. $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) = ax + \frac{a^{3/2}}{2}$ ...(1)

$\Rightarrow 2yy' = a$

put in equation (1)

$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$

$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$

squaring

$(y^2 - 2xyy')^2 = \frac{y^3(y')^3}{2}$

$\therefore$ order = 1

degree = 3

Degree – order = 3 – 1 = 2

2. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, k $\in$ R is

Official Ans. by NTA (11)

Sol. 3 sinx + 4 cosx = k + 1

$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$

$\Rightarrow k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values of k = 11

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

Official Ans. by NTA (1)

Sol. $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1)^{1/2} = \log_2(x-3)$

$\Rightarrow (x-1)^{1/2} = x - 3$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2,5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x - 3) \rightarrow$ its domain $x > 3$

finally x is 5

$\therefore$ No. of solutions = 1.

4. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

x = 1 satisfying the equation

$\therefore$ x– 1 is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$x = 1, \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2}$

$x = 1, -\omega^2, -\omega$

sum of 162^{th} power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

5. Let m,n $\in$ N and gcd (2,n) = 1. If

$$30\binom{30}{0}+29\binom{30}{1}+\ldots+2\binom{30}{28}+1\binom{30}{29}=n.2^m,$$

then n + m is equal to

(Here $\binom{n}{k}={}^nC_k$)

Official Ans. by NTA (45)

Sol. $30({}^{30}C_0) + 29({}^{30}C_1) + \ldots + 2({}^{30}C_{28}) + 1({}^{30}C_{29})$

$= 30({}^{30}C_{30}) + 29({}^{30}C_{29})+\ldots\ldots + 2({}^{30}C_2) + 1({}^{30}C_1)$

$$=\sum_{r=1}^{30} r\left({}^{30}C_r\right)$$

$$=\sum_{r=1}^{30} r\left(\frac{30}{r}\right)\left({}^{29}C_{r-1}\right)$$

$$=30\sum_{r=1}^{30}{}^{29}C_{r-1}$$

$$=30\left({}^{29}C_0+{}^{29}C_1+{}^{29}C_2+\ldots+{}^{29}C_{29}\right)$$

$$=30\left(2^{29}\right)=15(2)^{30}=n(2)^m$$

$\therefore$ n = 15, m = 30

n + m = 45

6. If y = y(x) is the solution of the equaiton

$$e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x, y(0)=0;$$

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \text{D.E is } \frac{dt}{dx}+t\cos x=\cos x$$

$$\text{I.F.} = e^{\int\cos x\,dx}=e^{\sin x}$$

$$\Rightarrow \text{solution is } t.e^{\sin x}=\int\cos x e^{\sin x}$$

$$\Rightarrow e^{\sin y}\,e^{\sin x}=e^{\sin x}+c$$

$\because$ x = 0, y = 0 $\Rightarrow$ c = 0

$$\Rightarrow e^{\sin y}=1$$

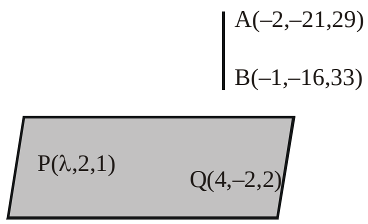
$$\Rightarrow y = 0$$

$$\Rightarrow 1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)=1$$

7. Let (λ,2,1) be a point on the plane which passes through the ponit (4,–2,2). If the plane is perpendicular to the line joining the points (–2,–21,29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to

Official Ans. by NTA (8)

Sol.



$$\overline{AB}.\,\overline{PQ}=0$$

$$\Rightarrow\left(\hat{i}+5\hat{j}-6\hat{k}\right).\left((4-\lambda)\hat{i}-4\hat{j}+\hat{k}\right)=0$$

$$\Rightarrow 4-\lambda-20-6=0$$

$$\Rightarrow \lambda=-22$$

$$\Rightarrow\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=4+8-4=8$$

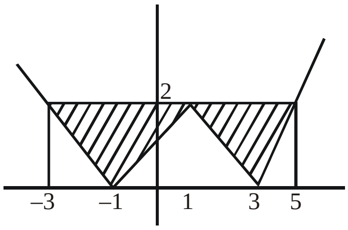
8. The area bounded by the lines y = ||x – 1| –2| is

Official Ans. by NTA (8)

Ans. By ALLEN (BONUS)

Sol. Remark :

Question is incomplete it should be area bounded by y = |x – 1| – 2| and y = 2



Area $= 2\left(\frac{1}{2}.4.2\right)$

9. The value of the integral $\int_0^{\pi} |\sin 2x|\,dx$ is

Official Ans. by NTA (2)

Sol. Put 2x = t ⇒ 2dx = dt

$$\Rightarrow\ I = \frac{1}{2}\int_0^{2\pi} |\sin t|\,dt$$

$$= \int_0^{\pi} |\sin t|\,dt$$

= 2

10. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

Official Ans. by NTA (1)

Sol. $\sqrt{3}\left(\cos x\right)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$

$$\Rightarrow \left(\sqrt{3}\cos x + 1\right)\left(\cos x - 1\right) = 0$$

$$\Rightarrow \cos x = 1 \ \text{ or } \ \cos x = -\frac{1}{\sqrt{3}} \ \text{(reject)}$$

$$\Rightarrow x = 0 \ \text{only}$$