FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let f: R → R be a continuous function such that f (3x) – f (x) = x. If f (8) = 7, then f (14) is equal to :

(A) 4 (B) 10

(C) 11 (D) 16

Official Ans. by NTA (B)

Ans. (B)

Sol. $f(x) - f(x/3) = x/3$

$f(x/3) - f(x/3^2) = x/3^2$

.... on adding

$$f(x) - \lim_{n\to\infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} ....\infty\right)$$

$$f(x) - f(0) = \frac{x}{2}$$

$f(8) = 7$; $f(0) = 3$

$f(x) = x/2 + 3$

$f(14) = 10$

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $\text{Re}(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

(A) $\arg z_2 = \pi - \tan^{-1} 3$

(B) $\arg (z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Official Ans. by NTA (D)

Ans. (D)

Sol. $AB = AO.\, z^{-i\pi/2} = -2 + i$

So $OB = (-2 + i) + (1 + 2i)$

$z_2 = -1 + 3i$

$\therefore |2z_1 - z_2| = \sqrt{10}$

3. If the system of linear equations.

$8x + y + 4z = -2$

$x + y + z = 0$

$\lambda x - 3y = \mu$

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane $8x + y + 4z + 2 = 0$ is :

(A) $3\sqrt{5}$ (B) 4

(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $$D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

Also $D_1 = D_2 = D_3 = 0$

So $\mu = -2$

Point $\left(4, -2, -\frac{1}{2}\right)$

Distance from plane $= \frac{10}{3}$

4. Let A be a 2 × 2 matrix with det (A) = –1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be :

(A) –1 (B) 2

(C) 1 (D) $-\sqrt{2}$

Official Ans. by NTA (B)

Ans. (B)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc = -1$

$|A + I||\text{adj } A + I| = 4$

$\Rightarrow ad - bc + a + d + 1 = 2$ or -2

$a + d = 2$ or -2

5. The odd natural number a, such that the area of the region bounded by y= 1, y = 3, x = 0, x = y^a is $\frac{364}{3}$, equal to :

(A) 3 (B) 5

(C) 7 (D) 9

Official Ans. by NTA (B)

Ans. (B)

Sol. $A=\int_1^3 y^a.dy=\left.\frac{y^{a+1}}{a+1}\right|_1^3=\frac{364}{3}$

$\Rightarrow a=5$

6. Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3,.... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560 (B) 1540

(C) 1330 (D) 2600

Official Ans. by NTA (C)

Ans. (C)

Sol. $\left(\left(2^1 2^2.....2^{60}\right)\left(4^1.4^2......4^n\right)\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\left(2^{30\times61}\,4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$2^{1830+n^2+n}=2^{\frac{(225)(60+n)}{8}}$

$=8n^2-217n+1140=0$

$n=20,\ \frac{57}{8}$

$\sum_{k=1}^{n} nk-k^2=\frac{n^2(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}$

= 1330

7. If the function

$$f(x)=\begin{cases}\dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)-\{0\}\\ k & ,x=0\end{cases}$$

is continuous at x = 0, then k is equal to :

(A) 1 (B) –1

(C) e (D) 0

Official Ans. by NTA (A)

Ans. (A)

Sol. $\lim_{x\to0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$$\lim_{x\to0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$$

$\therefore k=1$

8. If f(x) = $\begin{cases}x+a & , x\le0\\ |x-4|, & x>0\end{cases}$ and

g(x) = $\begin{cases}x+1 & , x<0\\ (x-4)^2+b, & x\ge0\end{cases}$

are continuous on R, then (gof) (2) + (fog) (–2) is equal to :

(A) –10 (B) 10

(C) 8 (D) –8

Official Ans. by NTA (D)

Ans. (D)

Sol. $f(x)=\begin{cases}x+a & ; x\le0\\ |x-4|; & x>0\end{cases}$; $g(x)=\begin{cases}x+1 & ; x<0\\ (x-4)^2+b; & x\ge0\end{cases}$

For continuity a = 4 and b = –15

$g(f(2))+f(g(-2))$

$=g(2)+f(-1)=-8$

9. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7 & , x \le 1 \\ -2x + \log_2(b^2-4), & x > 1 \end{cases}$

Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

(A) (–6, –2)

(B) (2, 6)

(C) [–6, –2) ∪ (2, 6]

(D) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Official Ans. by NTA (C)

Ans. (C)

Sol. $f(1) = 3$

For $x < 1$, $f'(x) = 3x^2 - 2x + 10 > 0$

$\Rightarrow f(x)$ is increasing

For $x > 1$, $f'(x) < 0$

$\Rightarrow$ function is decreasing.

$\lim_{x \to 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at x = 1

$3 \ge -2 + \log_2(b^2 - 4)$

$32 \ge b^2 - 4 > 0$

$b \in [-6, -2) \cup (2, 6]$

10. If $a = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2n}{n^2+k^2}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0,1)$, then :

(A) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(B) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(D) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Official Ans. by NTA (C)

Ans. (C)

Sol. $a = \frac{1}{n} \sum_{k=1}^{n} \frac{2}{1+\left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1+x^2} dx = \frac{\pi}{2}$

$f(x) = \tan\left(\frac{x}{2}\right); x \in (0, 1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2}+1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

11. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of y(x) is

(A) $\frac{1}{8}$

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $\frac{dy}{dx} + 2y \tan x = \sin x$

I.F = $e^{\int 2\tan x\, dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$y(\sec^2 x) = \int \sin x \sec^2 x\, dx + C$

$y.\sec^2 x = \sec x + C$

Put $x = \frac{\pi}{3}, y = 0$

$y = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore y_{max} = \frac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let f(x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D . Then the area of the quadrilateral ACBD is equal to

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$ (D) 9

Official Ans. by NTA (B)

Ans. (B)

Sol. $(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow x^2+y^2+x-3y-2=0$

Put x = 0

$\Rightarrow y^2-3y-2=0$

$\Rightarrow y=\frac{3\pm\sqrt{17}}{2}$

Put y = 0

$\Rightarrow x^2+x-2=0$

$(x+2)(x-1)=0$

$\therefore A(-2,0), B(1,0), C\left(0,\frac{3+\sqrt{17}}{2}\right), D\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area = $\frac{1}{2}.3.\sqrt{17}=\frac{3\sqrt{17}}{2}$

13. Let the tangent drawn to the parabola $y^2=24x$ at the point (α,β) is perpendicular to the line 2x +2y = 5. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha+4,\beta+4)$ does NOT pass through the point :

(A) (25, 10) (B) (20, 12)

(C) (30, 8) (D) (15, 13)

Official Ans. by NTA (D)

Ans. (D)

Sol. Tangent at (α,β) has slope 1

$\beta^2=24\alpha$

Equation of tangent $y\beta=12(x+\alpha)$, $\frac{12}{\beta}=1$

$\Rightarrow \alpha=6, \beta=12$

$\therefore(\alpha+4, \beta+4)=(10,16)$

Normal at (10, 16) to $\frac{x^2}{36}-\frac{y^2}{144}=1$ is

2x + 5y = 100

14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line x + y – z = 0 = x – 2y + 3z – 5 is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D) 1

Official Ans. by NTA (A)

Ans. (A)

Sol. d.r's of the line = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 1 & -2 & 3\end{vmatrix}=\hat{i}-4\hat{j}-3\hat{k}$

$\therefore$ equation of line is

$\vec{r}=\hat{i}+2\hat{j}+4\hat{k}+\lambda(\hat{i}-4\hat{j}-3\hat{k})$

Let A(1, 2, 4) and P be $(1+\lambda, 2-4\lambda, 4-3\lambda)$

$\therefore \overrightarrow{PA}.(\hat{i}-4\hat{j}-3\hat{k})=0$

$\lambda=\frac{1}{2}$

$\Rightarrow P\left(\frac{1}{2},2,\frac{-5}{2}\right)$

$|AP|=\sqrt{\frac{21}{2}}$

15. Let $\vec{a}=\alpha\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k},\alpha>0$. If the projection of $\vec{a}\times\vec{b}$ on the vector $-\hat{i}+2\hat{j}-2\hat{k}$ is 30, then α is equal to

(A) $\frac{15}{2}$ (B) 8

(C) $\frac{13}{2}$ (D) 7

Official Ans. by NTA (D)

Ans. (D)

Sol. $\vec{a}\times\vec{b}=(1-\alpha)\hat{i}+(\alpha^2-2)\hat{j}+(\alpha-2)\hat{k}$

Projection of $\vec{a}\times\vec{b}$ on $-\hat{i}+2\hat{j}-2\hat{k}$

$=\frac{(\vec{a}\times\vec{b}).(-\hat{i}+2\hat{j}-2\hat{k})}{3}=30$

$\Rightarrow 2\alpha^2-\alpha-91=0$

$\Rightarrow \alpha=7,-\frac{13}{2}$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1)=\frac{4}{243}$, then P(X = 4 or 5) is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$

(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $np=\alpha$ .....(1)

$npq=\alpha/3$ .....(2)

From (1) & (2)

$q = 1/3$ & $p = 2/3$

${}^nC_1\, q^{n-1}p^1=\frac{4}{243}$

$\frac{n}{3^n}=\frac{2}{243}$

$n = 6$

P (4 or 5) = ${}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^6C_5\left(\frac{2}{3}\right)^5.\left(\frac{1}{3}\right)^0$

$=\frac{16}{27}$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1)=\frac{2+3p}{6}$, $P(E_2)=\frac{2-p}{8}$ and $P(E_3)=\frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then (p_1+p_2) is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) 1

Official Ans. by NTA (D)

Ans. (D)

Sol. $0\le P(E_i)\le 1$ for i = 1, 2, 3

$\Rightarrow -2/3\le p\le 1$

E_1 & E_2 & E_3 are mutually exclusive

$P(E_1)+P(E_2)+P(E_3)\le 1$

$\Rightarrow 2/3\le p\le 1$

$p_1=1$, $p_2=2/3$

$p_1+p_2=5/3$

18. Let

$S=\{\theta\in[0,2\pi]:8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\}$. Then

$n(S)+\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to :

(A) 0 (B) –2

(C) –4 (D) 12

Official Ans. by NTA (C)

Ans. (C)

Sol. $8^{2\sin^2\theta}+8^{2-2\sin^2\theta}=16$

$y+\frac{64}{y}=16$

$\Rightarrow y=8$

$\Rightarrow \sin^2\theta=½$

$n(S)+\sum_{\theta\in S}\frac{1}{\cos(\pi/4+2\theta)\sin(\pi/4+2\theta)}$

$= 4 + (-2)\times 4 = -4$

19. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\tan\left(2(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8})+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(A) a tautology

(B) a contradiction

(C) equivalent to $(p \Rightarrow q) \wedge q$

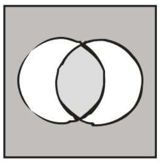
(D) equivalent to $(p \Rightarrow q) \wedge p$

Official Ans. by NTA (D)

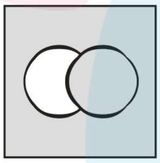
Ans. (D)

Sol. $(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



$p \Leftrightarrow q$



$p \rightarrow q$

SECTION-B

1. If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to-

Official Ans. by NTA (272)

Ans. (272)

Sol. $(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2+r^2}{p^2} = 272$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is

Official Ans. by NTA (180)

Ans. (180)

Sol. $3\times\frac{5!}{2!2!}+\frac{5!}{3!\times 2!}+\frac{5!}{2!}+\frac{5!}{3!}=180$

3. The series of positive multiples of 3 is divided into sets : {3}, {6, 9,12}, {15, 18, 21, 24, 27},... Then the sum of the elements in the 11^{th} set is equal to___________,

Official Ans. by NTA (6993)

Ans. (6993)

Sol. $S_{11} = 3[101+102+......+121]$

$=\frac{3}{2}(222)\times 21 = 6993$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is

Official Ans. by NTA (3)

Ans. (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow (x-1)^2 (x + 1) (x^5 + 3x - 1) = 0$

Let $f(x) = x^5 + 3x - 1$

$f'(x) > 0 \ \forall x \in R$

Hence 3 real distinct roots.

5. If the coefficients of x and x^2 in the expansion of $(1 + x)^p(1 - x)^q$, p, q $\le$ 15, are –3 and–5 respectively, then the coefficient of x^3 is equal to_____________.

Official Ans. by NTA (23)

Ans. (23)

Sol. Since coefficient of x is –3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow p - q = -3$ ....(1)

Comparing coefficients of x^2

$-{}^pC_1{}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq+\frac{p(p-1)}{2}+\frac{q(q-1)}{2} = -5$ ....(2)

Solving (1) and (2)

p = 8, q = 11

Coefficient of x^3 is

$-{}^{q}C_3 + {}^{p}C_3 + {}^{p}C_1\,{}^{q}C_2 - {}^{p}C_2\,{}^{q}C_1$

$= -{}^{11}C_3 + {}^{8}C_3 + {}^{8}C_1\,{}^{11}C_2 - {}^{8}C_2\,{}^{11}C_1$

= 23

6. If

$n(2n+1)\int_0^1 (1-x^n)^{2n} dx = 1177\int_0^1 (1-x^n)^{2n+1} dx$, then $n \in N$ is equal to _______

Official Ans. by NTA (24)

Ans. (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n} dx$, $I_2 = \int_0^1 (1-x^n)^{2n+1} dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1} . 1\, dx$

$= (1-x^n)^{2n+1}.x\Big|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nx^{n-1})x\, dx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$

$(2n^2+n+1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(>3) be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then α is equal to _______

Official Ans. by NTA (6)

Ans. (6)

Sol. $x^4 = 3yx.y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put (3, 3) , C = –2

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

8. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 15a and x – y = 3 respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to

Official Ans. by NTA (3)

Ans. (3)

Sol. Coordinates of A(1, –2), $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre H(2, a)

Slope of AH = p

a + 2 = p .....(1)

Slope of BH = –1

31a – 2ab = 15a + 4p–2 .....(2)

From (1) and (2)

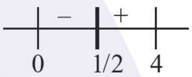
a = 1 & p = 3

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a – 1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to-

Official Ans. by NTA (45)

Ans. (45)

Sol. $f'(x) = 4x - \frac{1}{x}$



$a = \frac{1}{2}$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$\frac{1}{y_1} = \frac{3 - y_1}{4 - x_1} \Rightarrow y_1^2 - 6y_1 + 8 = 0$

$y_1 = 2, 4$

$\Rightarrow$ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$\frac{x}{9} + \frac{y}{36} = 1$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

10. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is______________.

Official Ans. by NTA (153)

Ans. (153)

Sol. Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1, 3$

Q (1, 1, 3) ; R (5, 7, 7) ; P (4, 2, 7)

Area of triangle PQR = ½ $\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$= \sqrt{153}$