

FINAL JEE-MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

SECTION-A

1. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

Official Ans. by NTA (2)

Sol. (A)

p	q	$\sim q$	$p \rightarrow q$	$\sim p$	$(\sim q \wedge (p \rightarrow q))$	
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies

2. Let a, b \in R. If the mirror image of the point P(a, 6, 9) with respect to the line

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$$

is equal to :

(1) 88 (2) 86

(3) 84 (4) 90

Official Ans. by NTA (1)

Sol. P(9, 6, 9)

$$\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$$

Q = (20, b, -a - 9)

TEST PAPER WITH SOLUTION

$$\frac{20+a}{2} - 3 = \frac{b+6}{2} - 2 = \frac{-9}{2} - 1$$

$$\frac{14+a}{2} = \frac{b+6}{2} = \frac{-9}{2}$$

$\Rightarrow a = -56$ and $b = -32$

$\Rightarrow |a + b| = 88$

3. The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$, and the point (1, 0, 2) is :

(1) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

(2) $\vec{r} \cdot (3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(4) $\vec{r} \cdot (\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$

Official Ans. by NTA (3)

Sol. $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$

$\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$

point (1, 0, 2)

Eqⁿ of plane

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 + \lambda \{ \vec{r} \cdot (\hat{i} - 2\hat{j}) + 2 \} = 0$$

$$\vec{r} \cdot \left\{ \hat{i}(1+\lambda) + \hat{j}(1-2\lambda) + \hat{k}(1) \right\} - 1 + 2\lambda = 0$$

Point $\hat{i} + 0\hat{j} + 2\hat{k} = \vec{r}$

$$\therefore (\hat{i} + 2\hat{k}) \cdot \left\{ \hat{i}(1+\lambda) + \hat{j}(1-2\lambda) + \hat{k}(1) \right\} - 1 + 2\lambda = 0$$

$$1 + \lambda + 2 - 1 + 2\lambda = 0$$

$$\lambda = -\frac{2}{3}$$

$$\therefore \vec{r} \cdot \left[\hat{i} \left(\frac{1}{3} \right) + \hat{j} \left(\frac{7}{3} \right) + \hat{k} \right] = \frac{7}{3}$$

$$\vec{r} \cdot [\hat{i} + 7\hat{j} + 3\hat{k}] = 7$$

Ans. 3

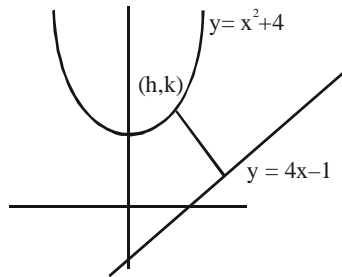
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4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

- (1) (3, 13) (2) (1, 5)
 (3) (-2, 8) (4) (2, 8)

Official Ans. by NTA (4)

Sol. Ans. (4)



P : $y = x^2 + 4$
 $k = h^2 + 4$
 L : $y = 4x - 1$
 $y - 4x + 1 = 0$

$$d = AB = \left| \frac{k - 4h + 1}{\sqrt{5}} \right| = \left| \frac{h^2 - 4 - 4h + 1}{\sqrt{5}} \right|$$

$$\frac{d(d)}{dh} = \frac{2h - 4}{\sqrt{5}} = 0$$

$h = 2$

$$\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$$

$\therefore k = 4 + 4 = 8$

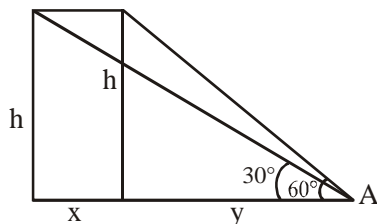
\therefore Point (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is 60° . After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30° . If the jet plane is flying at a constant height, then its height is :

- (1) $1800\sqrt{3}$ m (2) $3600\sqrt{3}$ m
 (3) $2400\sqrt{3}$ m (4) $1200\sqrt{3}$ m

Official Ans. by NTA (4)

Sol.



$$\tan 60^\circ = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y \quad \dots(1)$$

$$\tan 30^\circ = \frac{h}{x + y}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + y} \Rightarrow \sqrt{3}h = x + y \quad \dots(2)$$

Speed 432 km/h $\Rightarrow \frac{432 \times 20}{60 \times 60} \Rightarrow \frac{12}{5}$ km

$$\sqrt{3}h = \frac{12}{5} + y$$

$$\sqrt{3}h - \frac{12}{5} = y$$

from (1)

$$h = \sqrt{3} \left[\sqrt{3}h - \frac{12}{5} \right]$$

$$h = 3h - \frac{12\sqrt{3}}{5}$$

$$h = \frac{6\sqrt{3}}{5} \text{ km}$$

$$h = 1200\sqrt{3} \text{ m}$$

6. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$ is:

(1) $\frac{n(n-1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$

(3) $\frac{n(2n+1)(3n+1)}{6}$ (4) $\frac{n(n+1)^2(n+2)}{12}$

Official Ans. by NTA (2)

Sol. ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$

$${}^{n+1}C_2 + 2({}^3C_3 + {}^3C_2 + {}^4C_2 + \dots + {}^nC_2)$$

{use ${}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r$ }

$$= {}^{n+1}C_2 + 2({}^4C_3 + {}^4C_2 + {}^5C_3 + \dots + {}^nC_2)$$

$$\begin{aligned}
 &= {}^{n+1}C_2 + 2({}^5C_3 + {}^5C_2 + \dots + {}^nC_2) \\
 &\quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \\
 &= {}^{n+1}C_2 + 2({}^nC_3 + {}^nC_2) \\
 &= {}^{n+1}C_2 + 2 \cdot {}^{n+1}C_3 \\
 &= \frac{(n+1)n}{2} + 2 \cdot \frac{(n+1)(n)(n-1)}{2 \cdot 3} \\
 &= \frac{n(n+1)(2n+1)}{6}
 \end{aligned}$$

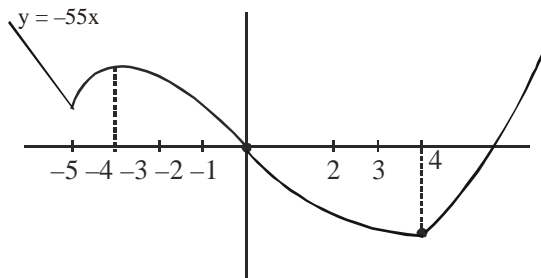
7. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined as,

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \leq x \leq 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

- (1) $(-\infty, -5) \cup (4, \infty)$
- (2) $(-5, \infty)$
- (3) $(-\infty, -5) \cup (-4, \infty)$
- (4) $(-5, -4) \cup (4, \infty)$

Official Ans. by NTA (4)



Sol.

$$f'(x) = \begin{cases} -55; & x < -5 \\ 6(x-5)(x+4); & -5 < x < 4 \\ 6(x-3)(x+2); & x > 4 \end{cases}$$

$f(x)$ is increasing in
 $x \in (-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on \mathbf{R} such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for

all $x \in \mathbf{R}$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in \mathbf{R}$, then

the value of $f(1)$ lies in the interval:

- (1) (9, 12)
- (2) (6, 9)
- (3) (0, 3)
- (4) (3, 6)

Official Ans. by NTA (2)

Sol. $f(x)f''(x) - (f'(x))^2 = 0$

$$\frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$$

$$\ln(f'(x)) = \ln f(x) + \ln c$$

$$f'(x) = cf(x)$$

$$\frac{f'(x)}{f(x)} = c$$

$$\ln f(x) = cx + k_1$$

$$f(x) = ke^{cx}$$

$$f(0) = 1 = k$$

$$f'(0) = c = 2$$

$$f(x) = e^{2x}$$

$$f(1) = e^2 \in (6, 9)$$

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point

$$\left(\frac{3\sqrt{3}}{2}, \frac{1}{2} \right) ?$$

(1) $x^2 + y^2 = 7$

(2) $y^2 = \frac{1}{6\sqrt{3}}x$

(3) $2x^2 - 18y^2 = 9$

(4) $x^2 + 9y^2 = 9$

Official Ans. by NTA (4)

Sol. $m = -\frac{1}{\sqrt{3}}, c = 2$

(1) $c = a\sqrt{1+m^2}$

$$c = \sqrt{7} \cdot \frac{2}{\sqrt{3}} \text{ (incorrect)}$$

(2) $c = \frac{a}{m} = \frac{1}{\frac{24\sqrt{3}}{-1}} = -\frac{1}{24}$ (incorrect)

(3) $c = \sqrt{a^2m^2 - b^2}$

$$c = \sqrt{\frac{9}{2} \cdot \frac{1}{3} - \frac{1}{2}} = 1 \text{ (incorrect)}$$

(4) $c = \sqrt{a^2m^2 + b^2}$

$$c = \sqrt{9 \cdot \frac{1}{3} + 1} = 2 \text{ (correct)}$$

10. The value of the integral, $\int_1^3 [x^2 - 2x - 2] dx$,

where $[x]$ denotes the greatest integer less than or equal to x , is :

- (1) $-\sqrt{2} - \sqrt{3} + 1$ (2) $-\sqrt{2} - \sqrt{3} - 1$
 (3) -5 (4) -4

Official Ans. by NTA (2)

Sol. $\int_1^3 ([x^2 - 2x - 2]) dx$

$$= \int_1^2 [x^2] - 3 \int_1^3 dx$$

$$= \int_1^2 0 dx + \int_2^3 1 dx + \int_3^4 2 dx + \int_4^5 3 dx - 6$$

$$= \sqrt{2} - 1 + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) - 6$$

$$= -\sqrt{2} - \sqrt{3} - 1$$

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

- (1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2} - 1$
 (3) $\sqrt{7} - 1$ (4) $\frac{1}{2\sqrt{2}}$

Official Ans. by NTA (1)

Sol. Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8} = \theta$

$$\sin 4\theta = \frac{\sqrt{63}}{8}$$

$$\cos 4\theta = \frac{1}{8}$$

$$2\cos^2 2\theta - 1 = \frac{1}{8}$$

$$\cos^2 2\theta = \frac{9}{16}$$

$$\cos 2\theta = \frac{3}{4}$$

$$2\cos^2 \theta - 1 = \frac{3}{4}$$

$$\cos^2 \theta = \frac{7}{8}$$

$$\cos \theta = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\tan \theta = \frac{1}{\sqrt{7}}$$

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

- (1) $\sim p \vee q$ (2) $p \vee \sim q$
 (3) $\sim p \wedge q$ (4) $p \wedge \sim q$

Official Ans. by NTA (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$$p \vee (\sim p \wedge \sim q)$$

$$\underbrace{(p \vee \sim p)}_t \wedge (p \vee \sim q)$$

$$p \vee \sim q$$

13. If the curve $y = ax^2 + bx + c$, $x \in \mathbb{R}$, passes through the point $(1, 2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are :

- (1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$
 (2) $a = 1, b = 0, c = 1$
 (3) $a = 1, b = 1, c = 0$
 (4) $a = -1, b = 1, c = 1$

Official Ans. by NTA (3)

Sol. $a + b + c = 2$... (1)

$$\text{and } \left. \frac{dy}{dx} \right|_{(0,0)} = 1$$

$$2ax + b \Big|_{(0,0)} = 1$$

$$b = 1$$

Curve passes through origin

$$\text{So, } c = 0$$

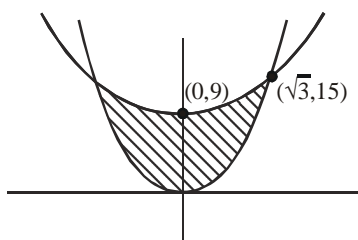
$$\text{and } a = 1$$

14. The area of the region :

$$R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$$
 is :

- (1) $11\sqrt{3}$ square units
 (2) $12\sqrt{3}$ square units
 (3) $9\sqrt{3}$ square units
 (4) $6\sqrt{3}$ square units

Official Ans. by NTA (2)



Sol.

$$\begin{aligned} \text{Required area} &= 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx \\ &= 2 \left[9x - x^3 \right]_0^{\sqrt{3}} \\ &= 2 \left[9\sqrt{3} - 3\sqrt{3} \right] = 12\sqrt{3} \end{aligned}$$

15. If a curve $y = f(x)$ passes through the point

$(1, 2)$ and satisfies $x \frac{dy}{dx} + y = bx^4$, then for what

value of b , $\int_1^2 f(x) dx = \frac{62}{5}$?

(1) 5 (2) 10

(3) $\frac{62}{5}$ (4) $\frac{31}{5}$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F. = $e^{\int \frac{1}{x} dx} = x$

So, solution of D.E. is given by

$$y \cdot x = \int b \cdot x^3 \cdot x dx + c$$

$$y = \frac{c}{x} + \frac{bx^4}{5}$$

Passes through $(1, 2)$

$$2 = c + \frac{b}{5} \quad \dots(1)$$

$$\int_1^2 f(x) dx = \frac{62}{5}$$

$$\left[c \ln x + \frac{bx^5}{25} \right]_1^2 = \frac{62}{5}$$

$$c \ln 2 + \frac{31b}{25} = \frac{62}{5} \quad \dots(2)$$

By equation (1) & (2)

$$c = 0 \text{ and } b = 10$$

16. Let $f(x)$ be a differentiable function defined on $[0, 2]$ such that $f'(x) = f'(2-x)$ for all $x \in (0, 2)$,

$f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x) dx$

is :

- (1) $1 - e^2$ (2) $1 + e^2$
(3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Official Ans. by NTA (2)

Sol. $f'(x) = f'(2-x)$

$$f(x) = -f(2-x) + c$$

put $x = 0$

$$f'(0) = -f'(2) + c$$

$$c = f(0) + f(2) = 1 + e^2$$

$$\text{so, } f(x) + f(2-x) = 1 + e^2$$

$$I = \int_0^2 f(x) dx$$

$$I = \int_0^2 f(2-x) dx$$

$$2I = \int_0^2 (f(x) + f(2-x)) dx$$

$$2I = (1 + e^2) \int_0^2 dx$$

$$I = 1 + e^2$$

17. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

- (1) no solution
(2) exactly two solutions
(3) infinitely many solutions
(4) a unique solution

Official Ans. by NTA (3)

Sol. Let $A^T = A$ and $B^T = -B$

$$C = A^2B^2 - B^2A^2$$

$$\begin{aligned} C^T &= (A^2B^2)^T - (B^2A^2)^T \\ &= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T \\ &= B^2A^2 - A^2B^2 \end{aligned}$$

$$C^T = -C$$

C is skew symmetric.

$$\text{So } \det(C) = 0$$

so system have infinite solutions.

18. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c),

(2, b) and (a, b) be $(\frac{10}{3}, \frac{7}{3})$. If α, β are the roots

of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

- (1) $\frac{71}{256}$ (2) $\frac{69}{256}$
 (3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

Official Ans. by NTA (4)

Sol. $\frac{a+2+a}{3} = \frac{10}{3}$

$a = 4$

and $\frac{c+b+b}{3} = \frac{7}{3}$

$c + 2b = 7$

also $2b = a + c$

$2b - a + 2b = 7$

$b = \frac{11}{4}$

now $4x^2 + \frac{11}{4}x + 1 = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\frac{-11}{4}\right)^2 - 3\left(\frac{1}{4}\right)$

$= \frac{121}{16} - \frac{3}{4} = \frac{-71}{16}$

19. For the system of linear equations :

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$, consider the following statements :

- (A) The system has unique solution if $k \neq 2, k \neq -2$.
 (B) The system has unique solution if $k = -2$.
 (C) The system has unique solution if $k = 2$.
 (D) The system has no-solution if $k = 2$.
 (E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

- (1) (C) and (D) only
 (2) (B) and (E) only
 (3) (A) and (E) only
 (4) (A) and (D) only

Official Ans. by NTA (4)

Sol. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

so, A is correct and B, C, E are incorrect. If $k = 2$

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$

So no solution

D is correct.

20. The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is :

- (1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$
 (3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

Official Ans. by NTA (3)

Sol. Total subsets = $2^5 = 32$

Probability = $\frac{{}^5C_2 \times 3^3}{32 \times 32} = \frac{10 \times 27}{12^{10}} = \frac{135}{2^9}$

SECTION-B

1. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$ exists, is

equal to _____.

Official Ans. by NTA (12)

Ans. by ALLEN (BONUS)

Sol. Bonus

$\sum_{i=0}^k \binom{10}{i} \binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i} \binom{13}{k+1-i}$

${}^{25}C_k + {}^{25}C_{k+1}$

${}^{26}C_{k+1}$

as nC_r is defined for all values of n as will as r so ${}^{26}C_{k+1}$ always exists

Now k is unbounded so maximum value is not defined.

2. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of

$|\lambda|$ is _____.

Official Ans. by NTA (1)

Sol.
$$\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z-0}{-\frac{1}{2}}$$

$$\frac{x-0}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$$

$$\text{Shortest distance} = \frac{(a_2 - a_1) \cdot (b_1 \times b_2)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i} \left(\frac{1}{2} + \frac{1}{2} \right) - \hat{j} \left(1 + \frac{1}{2} \right) + \hat{k} \left(1 - \frac{1}{2} \right)$$

$$= \hat{i} - \frac{3}{2}\hat{j} + \frac{\hat{k}}{2} = \frac{2\hat{i} - 3\hat{j} + \hat{k}}{2}$$

$$\frac{b_1 \times b_2}{|b_1 \times b_2|} = \frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}}$$

$$\frac{(a_2 - a_1) \cdot (b_1 \times b_2)}{|b_1 \times b_2|} = \left(-\lambda\hat{i} + \left(-2\lambda + \frac{1}{2} \right) + \lambda\hat{k} \right)$$

$$\left(\frac{2\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{14}} \right)$$

$$= \left| \frac{-2\lambda + 6\lambda - \frac{3}{2} + \lambda}{\sqrt{14}} \right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\left| 5\lambda - \frac{3}{2} \right| = \frac{7}{2}$$

$$5\lambda = \frac{3}{2} \pm \frac{7}{2}$$

$$5\lambda = 5, -2$$

$$\lambda = 1, -\frac{2}{5}$$

3. If $a + \alpha = 1$, $b + \beta = 2$ and

$$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, \quad x \neq 0, \text{ then the value}$$

$$\text{of expression } \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} \text{ is } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (2)

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \quad \dots(1)$

replace x by $\frac{1}{x}$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \quad \dots(2)$$

(1) + (2)

$$(a + \alpha)f(x) + (a + \alpha)f\left(\frac{1}{x}\right) = x(b + \beta) + (b + \beta)\frac{1}{x}$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

4. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (-5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to _____.

Official Ans. by NTA (56)

Sol. Let point is (h, k)

$$\text{So, } \sqrt{(h-5)^2 + k^2} = 3\sqrt{(h+5)^2 + k^2}$$

$$8x^2 + 8y^2 + 100x + 200 = 0$$

$$x^2 + y^2 + \frac{25}{2}x + 25 = 0$$

$$r^2 = \frac{(25)^2}{4^2} - 25$$

$$4r^2 = \frac{25^2}{4} - 100$$

$$4r^2 = 156.25 - 100$$

$$4r^2 = 56.25$$

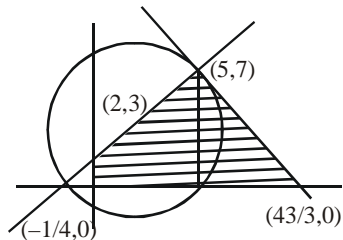
$$\text{After round of } 4r^2 = 56$$

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point $(5, 7)$ is A, then $24A$ is equal to _____.

Official Ans. by NTA (1225)

Ans. by ALLEN (1225 / BONUS)

Sol.



Equation of normal

$$4x - 3y + 1 = 0$$

and equation of tangents

$$3x + 4y - 43 = 0$$

$$\text{Area of triangle} = \frac{1}{2} \left(\frac{43}{3} + \frac{1}{4} \right) \times (7)$$

$$= \frac{1}{2} \left(\frac{172 + 3}{12} \right) \times 7$$

$$A = \frac{1225}{24}$$

$$24A = 1225$$

* as positive x-axis is given in the question so question should be bonus.

6. If the variance of 10 natural numbers $1, 1, 1, \dots, 1, k$ is less than 10, then the maximum possible value of k is _____.

Official Ans. by NTA (11)

Sol.

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$$

$$= \frac{9 + k^2}{10} - \left(\frac{9 + k}{10} \right)^2 < 10$$

$$90 + 10k^2 - 81 - k^2 - 18k < 1000$$

$$9k^2 - 18k - 991 < 0$$

$$k^2 - 2k < \frac{991}{9}$$

$$(k - 1)^2 < \frac{1000}{9}$$

$$\frac{-10\sqrt{10}}{3} < k - 1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

$$k \leq 11$$

Maximum value of k is 11.

7. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α , then 2α is _____.

Official Ans. by NTA (3)

Sol. Let number are a, ar, ar^2, ar^3

$$a \frac{(r^4 - 1)}{r - 1} = \frac{65}{12} \quad \dots(1)$$

$$\frac{1}{a} \left(\frac{1}{r^4} - 1 \right) = \frac{65}{18}$$

$$\frac{1}{ar^3} \left(\frac{1 - r^3}{1 - r} \right) = \frac{65}{18} \quad \dots(2)$$

$$\frac{(1)}{(2)} \Rightarrow a^2 r^3 = \frac{3}{2}$$

$$\text{and } a^3 \cdot r^3 = 1$$

$$ar = 1$$

$$(ar)^2 \cdot r = \frac{3}{2}$$

$$r = \frac{3}{2}, a = \frac{2}{3}$$

$$\text{So, third term} = ar^2 = \frac{2}{3} \times \frac{9}{4}$$

$$\alpha = \frac{3}{2}$$

$$2\alpha = 3$$

8. The students S_1, S_2, \dots, S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Official Ans. by NTA (31650)

Sol. If group C has one student then number of groups

$${}^{10}C_1 [2^9 - 2] = 5100$$

If group C has two students then number of groups

$${}^{10}C_2 [2^8 - 2] = 11430$$

If group C has three students then number of groups

$$= {}^{10}C_3 \times [2^7 - 2] = 15120$$

$$\text{So total groups} = 31650$$

9. Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$, and $n = [k]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to _____.

Official Ans. by NTA (310)

Sol.
$$K = \frac{1}{2^9} \left[\frac{\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^{24}} + \frac{\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{24}} \right]$$

$$K = \frac{1}{512} \left[\frac{\left(e^{i\frac{2\pi}{3}}\right)^{21}}{\left(e^{-\frac{i\pi}{4}}\right)^{24}} + \frac{\left(e^{i\frac{\pi}{3}}\right)^{21}}{\left(e^{\frac{i\pi}{4}}\right)^{24}} \right]$$

$$K = \frac{1}{512} \left[e^{i(14\pi+6\pi)} + e^{i(7\pi-6\pi)} \right]$$

$$K = \frac{1}{512} \left[e^{20\pi i} + e^{\pi i} \right]$$

$$K = \frac{1}{512} [1 + (-1)] = 0$$

$$n = [k] = 0$$

$$\sum_{j=0}^5 (j+5)^2 - \sum_{j=0}^5 (j+5)$$

$$\sum_{j=0}^5 (j^2 + 25 + 10j - j - 5)$$

$$\sum_{j=0}^5 (j^2 + 9j + 20)$$

$$\sum_{j=0}^5 j^2 + 9 \sum_{j=0}^5 j + 20 \sum_{j=0}^5 1$$

$$\frac{5 \times 6 \times 11}{6} + 9 \left(\frac{5 \times 6}{2} \right) + 20 \times 6$$

$$= 55 + 135 + 120$$

$$= 310$$

10. The number of the real roots of the equation

$$(x+1)^2 + |x-5| = \frac{27}{4} \text{ is } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (2)

Sol. Case-I

$$x \leq 5$$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$(x+1)^2 - (x+1) - \frac{3}{4} = 0$$

$$x+1 = \frac{3}{2}, -\frac{1}{2}$$

$$x = \frac{1}{2}, -\frac{3}{2}$$

Case-II

$$x > 5$$

$$(x+1) + (x-5) = \frac{27}{4}$$

$$(x+1)^2 + (x+1) - \frac{51}{4} = 0$$

$$x = \frac{-1 \pm \sqrt{52}}{2} \text{ (rejected as } x > 5)$$

So, the equation have two real root.