FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A = \{x \in R : |x+1| < 2\}$ and $B = \{x \in R : |x-1| \geq 2\}$. Then which one of the following statements is **NOT** true ?

(A) $A - B = (-1,1)$ (B) $B - A = R - (-3,1)$

(C) $A \cap B = (-3,-1]$ (D) $A \cup B = R - [1,3)$

Official Ans. by NTA (B)

Ans. (B)

Sol. $A : x \in (-3, 1)$ $B : x \in (-\infty, -1] \cup [3, \infty)$

$B - A = (-\infty, -3] \cup [3, \infty) = R-(-3, 3)$

2. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

(A) 37 (B) 58

(C) 68 (D) 92

Official Ans. by NTA (B)

Ans. (B)

Sol. $ax^2 - 2bx + 15 = 0$

$2\alpha = \frac{2b}{a}, \alpha^2 = \frac{15}{a}$

$\frac{\alpha}{2} = \frac{15}{2b}$

$\alpha = \frac{15}{b}$

$x^2 - 2bx + 21 = 0$

$\left(\frac{15}{b}\right)^2 - 2b\left(\frac{15}{b}\right) + 21 = 0$

$b^2 = 25$

$\alpha + \beta = 2b, \alpha\beta = 21$

$\alpha^2 + \beta^2 = 4b^2 - 42$

$= 58$

3. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1 = i\overline{z}_2$ and $\arg\left(\frac{z_1}{\overline{z}_2}\right) = \pi$. Then

(A) $\arg z_2 = \frac{\pi}{4}$ (B) $\arg z_2 = -\frac{3\pi}{4}$

(C) $\arg z_1 = \frac{\pi}{4}$ (D) $\arg z_1 = -\frac{3\pi}{4}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $\overline{z}_1 = i\overline{z}_2$

$z_1 = -iz_2$

$\arg\left(\frac{z_1}{\overline{z}_2}\right) = \pi$

$\arg\left(-i\frac{z_2}{\overline{z}_2}\right) = \pi$ $\arg(z_2) = \theta$

$-\frac{\pi}{2} + \theta + \theta = \pi$

$2\theta = \frac{3\pi}{2}$

$\arg(z_2) = \theta = \frac{3\pi}{4}$, $\arg z_1 = \frac{\pi}{4}$

4. The system of equations

$-kx + 3y - 14z = 25$

$-15x + 4y - kz = 3$

$-4x + y + 3z = 4$

is consistent for all k in the set

(A) R (B) $R - \{-11, 13\}$

(C) $R - \{13\}$ (D) $R - \{-11, 11\}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$

$\Delta \neq 0 \qquad k \in R - \{11, -11\}$ (Unique sol.)

If k = 11

$\Delta_z = \begin{vmatrix} -11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

If k = −11

$\Delta_z = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$

No solution

5. $\lim\limits_{x \to \frac{\pi}{2}} \left(\tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right) \right)$

is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$ (D) $-\frac{1}{6}$

Official Ans. by NTA (A)

Ans. (A)

Sol.

$\lim\limits_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right] =$

$\lim\limits_{x \to \frac{\pi}{2}} \frac{\tan^2 x\, [\sin^2 x - 3\sin x + 2]}{\sqrt{9} + \sqrt{9}}$

$= \lim\limits_{x \to \frac{\pi}{2}} \frac{\tan^2 x(\sin x - 1)(\sin x - 2)}{6}$

$= \frac{1}{6} \lim\limits_{x \to \frac{\pi}{2}} \tan^2 x(1 - \sin x)$

$= \frac{1}{6} \lim\limits_{x \to \frac{\pi}{2}} \frac{\sin^2 x\,(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{1}{12}$

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

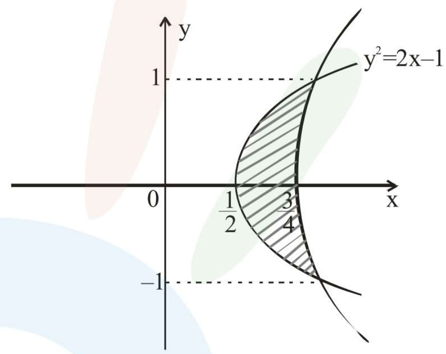
(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Ans. (A)

Sol. Required area = $2\int_0^1 \left(\frac{y^2 + 3}{4} - \frac{y^2 + 1}{2} \right) dy$

$= 2\int_0^1 \frac{1 - y^2}{4} dy = \frac{1}{2} \left| y - \frac{y^3}{3} \right|_0^1 = \frac{1}{3}$



7. The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ...x^{500}$, $x > 0$, is

(A) ${}^{501}C_{101}(5)^{399}$ (B) ${}^{501}C_{101}(5)^{400}$

(C) ${}^{501}C_{100}(5)^{400}$ (D) ${}^{500}C_{101}(5)^{399}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

$= \frac{(5+x)^{501} - x^{501}}{(5+x) - x} = \frac{(5+x)^{501} - x^{501}}{5}$

$\Rightarrow$ coefficient x^{101} in given expression

$= \frac{{}^{501}C_{101} 5^{400}}{5} = {}^{501}C_{101} 5^{399}$

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to

(A) $\frac{2\cdot3^{12}+10}{4}$ (B) $\frac{19\cdot3^{10}+1}{4}$

(C) $5\cdot3^{10}-2$ (D) $\frac{9\cdot3^{10}+1}{2}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $S=1\cdot3^0+2 3^1+3\cdot3^2+.....+10.3^9$

$3S=1\cdot3^1+2.3^2.....................+9\times3^9+10\times3^{10}$

$-2S=(1\cdot3^0+3^1+3^2.....3^9)-10.3^{10}$

$S=5\times3^{10}-\left(\frac{3^{10}-1}{4}\right)$

$S=\frac{20.3^{10}-3^{10}+1}{4}=\frac{19.3^{10}+1}{4}$

9. Let P be the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y – X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X – Y are on the same side of P

Official Ans. by NTA (C)

Ans. (C)

Sol. $P_1+\lambda P_2=0$

$\Rightarrow(x+3y-z-5)+\lambda(2x-y+z-3)=0$

$(2,1,-2)$ lies on this plane

$\therefore\lambda=1\Rightarrow$ plane is $3x+2y-8=0$

10. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is

(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$

(C) $y^2-x^2=2xy$ (D) $x^2-y^2=2xy$

Official Ans. by NTA (D)

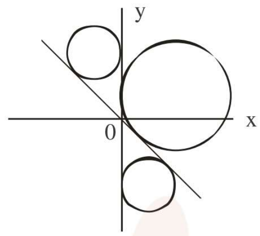
Ans. (D)

Sol. Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right|=|h|$

$k^2-h^2+2hk=0$

$\therefore$ Equation of locus is $y^2-x^2+2xy=0$



11. Water is being filled at the rate of $1\,cm^3/sec$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is

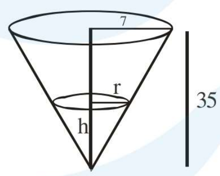
(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Official Ans. by NTA (C)

Ans. (C)

Sol. From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$

$=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx, n \in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference -2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

Official Ans. by NTA (D)

Ans. (D)

Sol. $b_n = \int_0^{\pi/2} \frac{1 + \cos 2nx}{\sin x} dx$

$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2(n+1)x - \cos^2 nx}{\sin x} dx$

$= \int_0^{\pi/2} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$

$= \left(\frac{\cos(2n+1)x}{2n+1}\right)_0^{\pi/2} = \frac{-1}{2n+1}$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d.$= -2$

13. If y = y(x) is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Official Ans. by NTA (B)

Ans. (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2}\left(\frac{y}{x}\right)^2$ $\quad y = vx$

$\frac{dv}{v^2} = -\frac{3dx}{2x}$

$-\frac{1}{v} = -\frac{3}{2}\ell n|x| + C$

$-\frac{x}{y} = \frac{-3}{2}\ell n|x| + C$

$x = e, \ y = \frac{e}{3}$

$C = -\frac{3}{2}$

When x = 1, y = $\frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve x = 12(t + sin t cos t), $y = 12(1 + \sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3 + 2\sqrt{2}\right)$ (B) $3\left(7 + 4\sqrt{3}\right)$

(C) 27 (D) 48

Official Ans. by NTA (C)

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{2(1 + \sin t) \times \cos t}{1 + \cos 2t}$

$\Rightarrow \frac{2(1 + \sin t)\cos t}{2\cos^2 t} = \sqrt{3}$

$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$

15. The value of 2sin(12°) – sin(72°) is :

(A) $\frac{\sqrt{5}\left(1 - \sqrt{3}\right)}{4}$ (B) $\frac{1 - \sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1 - \sqrt{5}\right)}{2}$ (D) $\frac{\sqrt{3}\left(1 - \sqrt{5}\right)}{4}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $\sin 12° + \sin 12° - \sin 72°$

$= \sin 12° - 2\cos 42° \sin 30°$

$= \sin 12° - \sin 48°$

$= -2\cos 30° \sin 18°$

$= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{3}}{4}(1-\sqrt{5})$

16. A biased die is marked with numbers 2,4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Official Ans. by NTA (D)

Ans. (D)

Sol. $P(n) = \frac{1}{n}$

$P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$

$P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$

$P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability = $\frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim(p \Rightarrow q)$ (D) $\sim(q \Rightarrow p)$

Official Ans. by NTA (C)

Ans. (C)

Sol. $\sim p \vee q \equiv p \rightarrow q$

$\sim q \wedge p \equiv \sim (p \rightarrow q)$

Negation of $\sim(p \rightarrow q) \rightarrow (p \rightarrow q)$

is $\sim(p \rightarrow q) \wedge (\sim(p \rightarrow q))$ i.e. $\sim(p \rightarrow q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

Official Ans. by NTA (C)

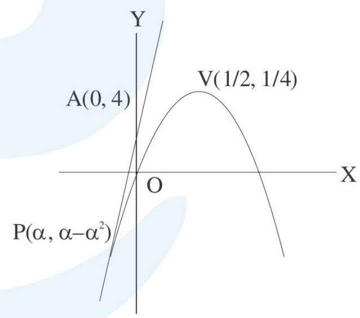
Ans. (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\tan^{-1}\left[\dfrac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right] \Rightarrow \tan^{-1}\left(\dfrac{\cos\frac{\pi}{4}-1}{\sin\frac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

20. The line $y=x+1$ meets the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

(A) 20 (B) 12

(C) 11 (D) 8

Official Ans. by NTA (A)

Ans. (A)

Sol. Ellipse $x^2+2y^2=4$

Line y = x + 1

Point of intersection

$x^2+2(x+1)^2=4$

$3x^2+4x-2=0$

$|x_1-x_2|=\dfrac{\sqrt{40}}{3}$

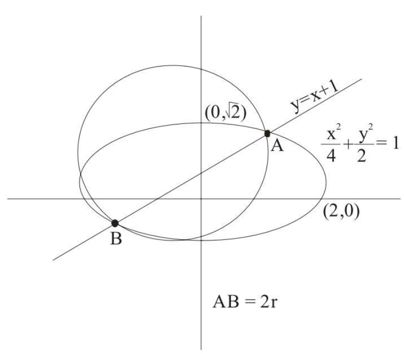
$AB=2r=|x_1-x_2|\sqrt{1+m^2}$,

m is slope of given line

$AB=\dfrac{\sqrt{40}}{3}\sqrt{1+1}$

$2r=\dfrac{\sqrt{80}}{3}\Rightarrow r=\dfrac{\sqrt{80}}{6}$

$(3r)^2=\left(3\times\dfrac{\sqrt{80}}{6}\right)^2=\dfrac{80}{4}=20$



SECTION-B

1. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set

$\{(n,m): n,m\in\{1,2,.....,10\}$ and $nA^n+mB^m=I\}$ is ____

Official Ans. by NTA (1)

Ans. (1)

Sol. $A^2=A$ and $B^2=B$

Therefore equation $nA^n+mB^m=I$ becomes

nA + mB = I, which gives m = n = 1

Only one set possible

2. Let $f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq 0\end{cases}$, where [t] is the greatest integer $\leq t$. Then, in the open interval (–1, 1), the number of points where fog is discontinuous is equal to _____

Official Ans. by NTA (62)

Ans. (62)

Sol. $f(g(x))=\left[2g^2(x)\right]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1; x<0\\ \left[2(2x+3)^2\right]+1; x\geq 0\end{cases}$

$\therefore$ fog is discontinuous whenever $2(2x-3)^2$ or $2(2x+3)^2$ belongs to integer except x = 0.

$\therefore$ 62 points of discontinuity.

3. The value of b > 3 for which

$12\int_3^b \dfrac{1}{(x^2-1)(x^2-4)}dx=\log_e\left(\dfrac{49}{40}\right)$, is equal to

Official Ans. by NTA (6)

Ans. (6)

Sol. $\frac{12}{3}\left[\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right]=\log\frac{49}{40}$

$\frac{12}{3}\cdot\left[\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right|\right]_3^b=\log\frac{49}{40}$

$\ell n\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2}=\ell n\frac{49}{50}$

b = 6

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot 3^9$, then β is equal to _____

Official Ans. by NTA (83)

Ans. (83)

Sol. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$= {}^{10}C_r 2^{10-r}3^r x^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get β = 83

5. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____

Official Ans. by NTA (21)

Ans. (21)

Sol. Mean deviation about mean of first n natural numbers is $\frac{n^2-1}{4n}$

$\therefore n=21$

6. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $(\vec{b}-\vec{a})\cdot(\hat{k}-\hat{j})+(\vec{b}+\vec{a})\cdot(\hat{i}-\hat{k})$ is equal to

Official Ans. by NTA (14)

Ans. (14)

Sol. $(\vec{a}\times\vec{b}).\vec{b}=0$

$\Rightarrow 13-1-4\lambda=0\Rightarrow\lambda=3$

$\Rightarrow\vec{b}=\hat{i}+\hat{j}+3\hat{k}\ \Rightarrow\ \vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$

$\Rightarrow(\vec{a}\times\vec{b})\times\vec{b}=(13\hat{i}-\hat{j}-4\hat{k})\times(\hat{i}+\hat{j}+3\hat{k})$

$\Rightarrow -21\vec{b}-11\vec{a}=\hat{i}-43\hat{j}+14\hat{k}$

$\Rightarrow\ \vec{a}=-2\hat{i}+2\hat{j}-7\hat{k}$

Now $(\vec{b}-\vec{a}).(\hat{k}-\hat{j})+(\vec{b}+\vec{a}).(\hat{i}-\hat{k})=14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Official Ans. by NTA (243)

Ans. (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times 2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1.3=216$

Total = 243

8. Let $f(x)=\left|(x-1)(x^2-2x-3)\right|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

Official Ans. by NTA (3)

Ans. (3)

Sol. $f(x)=\begin{cases}(x^2-1)(x-3)+(x-3), x\in(0,1]\cup[3,4)\\ -(x^2-1)(x-3)+(x-3), x\in[1,3]\end{cases}$

$\Rightarrow f'(x)=\begin{cases}3x^2-6x,\ x\in(0,1)\cup(3,4)\\ -3x^2+6x+2,\ x\in(1,3)\end{cases}$

f(x) is non-derivable at x = 1 and x = 3

also f'(x) = 0 at $x=1+\sqrt{\frac{5}{3}}$ $\Rightarrow$ m + M = 3

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to

Official Ans. by NTA (85)

Ans. (85)

Sol. $e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$ **......(1)**

$A\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\frac{64}{5a^2}-\frac{144}{25b^2}=1$ **......(2)**

Solving (1) & (2) $b=\frac{6}{5}$ $a=\frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8}+\frac{5b^2y}{12}=a^2+b^2$

Comparing it $8\sqrt{5}x+\beta y=\lambda$

Gives $\lambda=100,\ \beta=15$

$\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Official Ans. by NTA (51)

Ans. (51)

Sol. $8x+4\sqrt{2}y=1,\ z=0$

$$\Rightarrow\frac{x-\frac{1}{8}}{1}=\frac{y-0}{-\sqrt{2}}=\frac{z-0}{0}=\lambda$$

$-8x-6\sqrt{3}z=1, y=0$

$$\Rightarrow\frac{x+\frac{1}{8}}{3\sqrt{3}}=\frac{y-0}{0}=\frac{z-0}{-4}$$

$$\begin{vmatrix}\frac{1}{4} & 0 & 0\\ 1 & -\sqrt{2} & 0\\ 3\sqrt{3} & 0 & -4\end{vmatrix}=\sqrt{2}$$

$d=\frac{1}{\sqrt{51}}$

$\frac{1}{d^2}=51$