FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Saturday 08th April, 2023) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS	TEST PAPER WITH ANSWER

SECTION-A

1. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are co-prime, then m+n is equal to

(1) 316

(2) 314

(3) 317

(4) 315

Official Ans. by NTA (3)

Ans. (3)

2. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + … and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(1) 11310

(2) 11280

(3) 11290

(4) 11260

Official Ans. by NTA (3)

Ans. (3)

3. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 12

(2) 11

(3) 9

(4) 10

Official Ans. by NTA (4)

Ans. (4)

4. Let A = $\left\{\theta \in (0, 2\pi) : \frac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is

(1) π

(2) 2π

(3) 4π

(4) 3π

Official Ans. by NTA (3)

Ans. (3)

5. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(1) $12^3 - 12$

(2) $11^3 - 11$

(3) $10^3 - 10$

(4) $13^3 - 13$

Official Ans. by NTA (1)

Ans. (1)

6. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, then k is equal to

(1) 1890

(2) 945

(3) 2835

(4) 5670

Official Ans. by NTA (4)

Ans. (4)

7. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$x + y + \sqrt{3}z = 0$

$-x + (\tan\theta)y + \sqrt{7}z = 0$

$x + y + (\tan\theta)z = 0$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S}\theta$ is equal to

(1) 40

(2) 10

(3) 20

(4) 30

Official Ans. by NTA (3)

Ans. (3)

8. If the probability that the random variable X takes values x is given by $P(X = x) = k(x + 1)3^{-x}$, x = 0, 1, 2, 3… , where k is a constant, then $P(X \geq 2)$ is equal to

(1) $\frac{7}{27}$

(2) $\frac{11}{18}$

(3) $\frac{7}{18}$

(4) $\frac{20}{27}$

Official Ans. by NTA (1)

Ans. (1)

9. The value of $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is

(1) 54

(2) 18

(3) 27

(4) 36

Official Ans. by NTA (4)

Ans. (4)

10. The integral $\int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to

(1) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$

(2) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

(3) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$

(4) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$

Official Ans. by NTA (2)

Ans. (Bonus)

11. The area of the quadrilateral ABCD with vertices A(2, 1, 1), B(1, 2, 5), C (–2, –3, 5) and D (1, –6, –7) is equal to

(1) 48

(2) $8\sqrt{38}$

(3) 54

(4) $9\sqrt{38}$

Official Ans. by NTA (2)

Ans. (2)

12. For $a, b \in \mathbb{Z}$ and $|a - b| \leq 10$, let the angle between the plane P: $ax + y - z = b$ and the line $l : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(1) 25

(2) 85

(3) 48

(4) 32

Official Ans. by NTA (4)

Ans. (4)

13. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(1) 34 but not by 14

(2) both 14 and 34

(3) neither 14 nor 34

(4) 14 but not by 34

Official Ans. by NTA (1)

Ans. (1)

14. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a + b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b + c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c + a)\hat{k}$ are also coplanar, then 6 (a + b + c) is equal to

(1) 0

(2) 6

(3) 12

(4) 4

Official Ans. by NTA (3)

Ans. (3)

15. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the point P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is

(1) $\frac{3}{2}$

(2) $\frac{5}{2}$

(3) 1

(4) $-\frac{1}{2}$

Official Ans. by NTA (2)

Ans. (2)

16. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(1) $p \wedge q$

(2) $p \wedge (\sim q)$

(3) $p \wedge (q \wedge (\sim p))$

(4) $p \vee (q \vee (\sim p))$

Official Ans. by NTA (1)

Ans. (1)

17. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x \to \frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2} \right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$,then k is equal to

(1) 2β

(2) 2α

(3) α

(4) β

Official Ans. by NTA (2)

Ans. (2)

18. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to:

(1) 12

(2) 10

(3) 19

(4) 14

Official Ans. by NTA (4)

Ans. (4)

19. Let A(0,1), B(1, 1) and C(1, 0) be the mid – points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(1) 6

(2) 8

(3) 12

(4) $\frac{9}{2}$

Official Ans. by NTA (2)

Ans. (2)

20. Let A = {1, 2, 3, 4, 5, 6, 7}.Then the relation R = $\{(x,y) \in A \times A : x + y = 7\}$is

(1) transitive but neither symmetric nor reflexive

(2) reflexive but neither symmetric nor transitive

(3) an equivalence relation

(4) symmetric but neither reflexive nor transitive

Official Ans. by NTA (4)

Ans. (4)

SECTION-B

21. Let [t] denote the greatest integer function. If $\int_0^{2.4} [x^2]dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to ____

Official Ans. by NTA (6)

Ans. (Bonus)

22. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ____

Official Ans. by NTA (309)

Ans. (309)

23. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and x, $\sqrt{2}y$, z are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to____

Official Ans. by NTA (150)

Ans. (150)

24. If domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to _____

Official Ans. by NTA (20)

Ans. (20)

25. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer $\leq$ x. Then $m^2 + mn + n^2$ is equal to _____.

Official Ans. by NTA (9)

Ans. (9)

26. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix x = –3 are in the ratio 3 : 1. If R(α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to____:

Official Ans. by NTA (16)

Ans. (16)

27. Let the solution curve x = x(y), $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2\cos y\, dx - (1 + 3x \log_e(\cos y))\sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right)=\frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right)=\frac{1}{\log_e m-\log_e n}$, where m and n are co-prime, then mn is equal to

Official Ans. by NTA (12)

Ans. (12)

28. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1)on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then α + β + γ is equal to ____.

Official Ans. by NTA (11)

Ans. (11)

29. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto function $f : R \to S$ such that $f(a) \neq 1$, is equal to _____.

Official Ans. by NTA (384)

Ans. (180)

30. Let the area enclosed by the lines x + y = 2, y = 0, x = 0 and the curve $f(x) = \min\left\{x^2+\frac{3}{4}, 1+[x]\right\}$ where [x] denotes the greatest integer $\leq$ x, be A. Then the value of 12A is _____

Official Ans. by NTA (17)

Ans. (17)