

FINAL JEE-MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13th April, 2023)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. $\int_0^{\infty} \frac{6}{e^{3x} + 6e^{2x} + 11e^x + 6} dx$

(1) $\log_e \left(\frac{512}{81} \right)$

(2) $\log_e \left(\frac{32}{27} \right)$

(3) $\log_e \left(\frac{256}{81} \right)$

(4) $\log_e \left(\frac{64}{27} \right)$

Official Ans. by NTA (2)

Ans. (2)

Sol.

$$\begin{aligned}
 I &= \int_0^{\infty} \frac{6}{(e^x + 1)(e^x + 2)(e^x + 3)} dx \\
 &= 6 \int_0^{\infty} \left(\frac{\frac{1}{2}}{e^x + 1} + \frac{-1}{e^x + 2} + \frac{\frac{1}{2}}{e^x + 3} \right) dx \\
 &= 3 \int_0^{\infty} \frac{e^{-x}}{1 + e^{-x}} dx - 6 \int_0^{\infty} \frac{e^{-x} dx}{1 + 2e^{-x}} + 3 \int_0^{\infty} \frac{e^{-x}}{1 + 3e^{-x}} dx \\
 &= 3 \left[-\ln(1 + e^{-x}) \right]_0^{\infty} + 6 \frac{1}{2} \left[\ln(1 + 2e^{-x}) \right]_0^{\infty} \\
 &\quad - \frac{3}{3} \left[\ln(1 + 3e^{-x}) \right]_0^{\infty} \\
 &= 3 \ln 2 - 3 \ln 3 + \ln 4 \\
 &= 3 \ln \frac{2}{3} + \ln 4 \\
 &= \ln \frac{32}{27}
 \end{aligned}$$

2. $\max_{0 \leq x \leq \pi} \left\{ x - 2 \sin x \cos x + \frac{1}{3} \sin 3x \right\} =$

(1) $\frac{5\pi + 2 + 3\sqrt{3}}{6}$

(2) $\frac{\pi + 2 - 3\sqrt{3}}{6}$

(3) π

(4) 0

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$f(x) = x - \sin 2x + \frac{1}{3} \sin 3x$$

$$f'(x) = 1 - 2 \cos 2x + \cos 3x = 0$$

$$x = \frac{5\pi}{6}, \frac{\pi}{6}$$

$$\therefore f''(x) = 4 \sin 2x - 3 \sin 3x$$

$$f''\left(\frac{5\pi}{6}\right) < 0$$

$$\Rightarrow \left(\frac{5\pi}{6}\right) \text{ is point of maxima}$$

$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$$

3. The set of all $a \in \mathbb{R}$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root is :

(1) $(-6, -3)$

(2) $(-\infty, \infty)$

(3) $(-6, \infty)$

(4) $(-\infty, -3)$

Official Ans. by NTA (2)

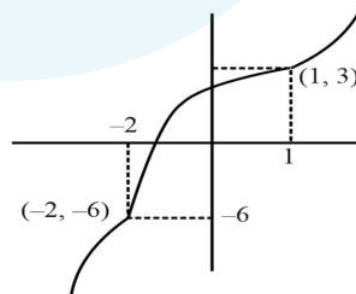
Ans. (2)

Sol.

$$f(x) = x|x-1| + |x+2|$$

$$x|x-1| + |x+2| + a = 0$$

$$x|x-1| + |x+2| = -a$$



All values are increasing.

4. The negation of the statement $((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$ is

- (1) equivalent to $\sim A$
- (2) equivalent to $\sim C$
- (3) equivalent to $B \vee \sim C$
- (4) a fallacy

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$p : ((A \wedge (B \vee C)) \Rightarrow (A \vee B)) \Rightarrow A$$

$$[\sim(A \wedge (B \vee C)) \vee (A \vee B)] \Rightarrow A$$

$$[(A \wedge (B \vee C)) \wedge \sim(A \vee B)] \vee A$$

$$(f \vee A) = A$$

$$\sim p \equiv \sim A$$

5. The distance of the point $(-1, 2, 3)$ from the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 10$ parallel to the line of the shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is :

- (1) $3\sqrt{6}$
- (2) $3\sqrt{5}$
- (3) $2\sqrt{6}$
- (4) $2\sqrt{5}$

Official Ans. by NTA (3)

Ans. (3)

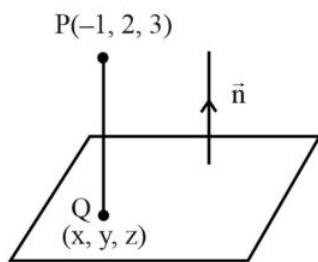
Sol.

$$\text{Let } L_1 : \vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$$

$$L_2 : \vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{n} = \hat{i} - \hat{j} - 2\hat{k}$$



Equation of line along shortest distance of L_1 and L_2

$$\frac{x+1}{1} = \frac{y-2}{-1} = \frac{z-3}{-2} = r$$

$$\Rightarrow (x, y, z) \equiv (r-1, 2-r, 3-2r)$$

$$\Rightarrow (r-1) - 2(2-r) + 3(3-2r) = 10$$

$$\Rightarrow r = -2$$

$$\Rightarrow Q(x, y, z) \equiv (-3, 4, 7)$$

$$\Rightarrow PQ = \sqrt{4+4+16} = 2\sqrt{6}$$

6. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

- (1) $\frac{21}{16}$
- (2) $\frac{81}{64}$
- (3) $\frac{15}{16}$
- (4) $\frac{37}{16}$

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$P(H) = \frac{3}{4}$$

$$P(T) = \frac{1}{4}$$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4} \times \frac{3}{4}$	$\left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^2 \times \frac{3}{4}$

$$\text{Mean } \bar{X} = \frac{3}{4} + \frac{3}{8} + 3\left(\frac{1}{64} + \frac{3}{64}\right)$$

$$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$= 3\left(\frac{7}{16}\right)$$

$$= \frac{21}{16}$$

7. For the system of linear equations

$$2x + 4y + 2az = b$$

$$x + 2y + 3z = 4$$

$$2x - 5y + 2z = 8$$

which of the following is **NOT** correct ?

- (1) It has infinitely many solutions if $a = 3, b = 6$
- (2) It has unique solution if $a = b = 6$
- (3) It has unique solution if $a = b = 8$
- (4) It has infinitely many solution if $a = 3, b = 8$

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$\Delta = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix} = 18(3-a)$$

$$\Delta_x = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix} = (64 + 19b - 72a)$$

For unique solution $\Delta = 0$

$$\Rightarrow \boxed{a \neq 3} \text{ and } \boxed{b \in \mathbb{R}}$$

For Infinitely many solution ;

$$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$$

$$\Rightarrow a = 3 \quad \because \Delta = 0$$

$$\text{and } b = 8 \quad \because \Delta_x = 0$$

8. For the differentiable function

$$f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}, \text{ let } 3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10, \text{ then}$$

$$\left| f(3) + f'\left(\frac{1}{4}\right) \right| \text{ is equal to}$$

(1) 7

(2) $\frac{33}{5}$

(3) $\frac{29}{5}$

(4) 13

Official Ans. by NTA (4)

Ans. (4)

Sol.

$$\left[3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10 \right] \times 3$$

$$\left[2f(x) + 3f\left(\frac{1}{x}\right) = x - 10 \right] \times 2$$

$$5f(x) = \frac{3}{x} - 2x - 10$$

$$f(x) = \frac{1}{5} \left(\frac{3}{x} - 2x - 10 \right)$$

$$f'(x) = \frac{1}{5} \left(-\frac{3}{x^2} - 2 \right)$$

$$\left| f(3) + f'\left(\frac{1}{4}\right) \right| = \left| \frac{1}{5}(1 - 6 - 10) + \frac{1}{5}(-48 - 2) \right|$$

$$= |-3 - 10| = 13$$

9. Let the tangent and normal at the point $(3\sqrt{3}, 1)$

on the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$ meet the y-axis at the

points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x = 2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β) , then $\alpha^2 - \beta^2$ is equal to

(1) $\frac{314}{5}$

(2) $\frac{304}{5}$

(3) 60

(4) 61

Official Ans. by NTA (2)

Ans. (2)

Sol.

Given ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$

$$\frac{x}{4\sqrt{3}} + \frac{y}{4} = 1$$

$$y = 4$$

$$\frac{x}{4} - \frac{4}{4\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$y = -8$$

$$x^2 + y^2 + 4y - 32 = 0$$

$$hx + ky + 2(y + k) - 32 = 0$$

$$k = -2$$

$$hx + 2k - 32 = 0$$

$$hx = 36$$

$$\alpha = h = \frac{36}{2\sqrt{5}}$$

$$\beta = k = -2$$

$$\alpha^2 - \beta^2 = \frac{304}{5}$$

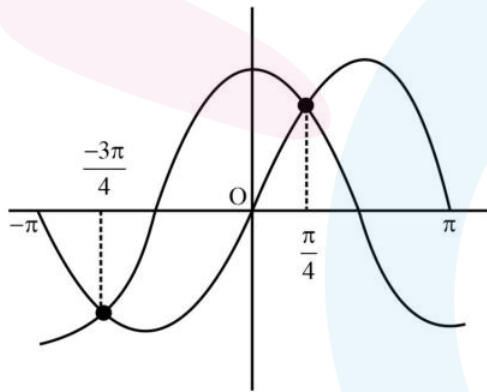
10. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}$, $-\pi \leq x \leq \pi$ and the x-axis is

- (1) $2(\sqrt{2} + 1)$
- (2) $2\sqrt{2}(\sqrt{2} + 1)$
- (3) $4(\sqrt{2})$
- (4) 4

Official Ans. by NTA (4)

Ans. (4)

Sol.



Area =

$$\left| \int_{-\pi}^{-\frac{3\pi}{4}} \sin x \, dx \right| + \left| \int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} \cos x \, dx \right| + \left| \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x \, dx \right| + \left| \int_{\frac{\pi}{4}}^{\pi} \sin x \, dx \right| = 4$$

11. The number of symmetric matrices of order 3, with all the entries from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, is :

- (1) 6^{10}
- (2) 9^{10}
- (3) 10^9
- (4) 10^6

Official Ans. by NTA (4)

Ans. (4)

Sol.

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \{0, 1, 2, \dots, 9\}$$

Number of matrices = 10^6

12. Among :

$$(S1) : \lim_{n \rightarrow \infty} \frac{1}{n^2} (2 + 4 + 6 + \dots + 2n) = 1$$

$$(S2) : \lim_{n \rightarrow \infty} \frac{1}{n^{16}} (1^{15} + 2^{15} + 3^{15} + \dots + n^{15}) = \frac{1}{16}$$

- (1) Both (S1) and (S2) are true
- (2) Both (S1) and (S2) are false
- (3) Only (S2) is true
- (4) Only (S1) is true

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$S_1 : \lim_{n \rightarrow \infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow \text{True}$$

$$S_2 : \lim_{n \rightarrow \infty} \frac{1}{n^{16}} \left(\sum r^{15} \right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum \left(\frac{r}{n} \right)^{15} = \int_0^1 x^{15} \, dx = \frac{1}{16} \Rightarrow \text{True}$$

13. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that $PM:MQ=3:1$. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

- (1) $(-3, 43)$
- (2) $(-6, 45)$
- (3) $(3, 33)$
- (4) $(6, 29)$

Official Ans. by NTA (1)

Ans. (1)

Sol.

$$9 \left(t + \frac{1}{t} \right)^2 = 100$$

$$t = 3$$

$$\Rightarrow P(81, 54) \text{ \& } Q(1, -6)$$

$$M(21, 9)$$

$$\Rightarrow L \text{ is } (y-9) = \frac{-4}{3}(x-21)$$

$$3y - 27 = -4x + 84$$

$$4x + 3y = 111$$

14. For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $f \circ g(x) = x + 3 - \sqrt{x}$.

Then $f(0)$ is equal to

- (1) 1
- (2) -3
- (3) 5
- (4) 0

Official Ans. by NTA (3)

Ans. (3) or Bonus

Sol.

$$g(x) = \sqrt{x} + 1$$

$$f \circ g(x) = x + 3 - \sqrt{x}$$

$$\begin{aligned} &= (\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 5 \\ &= g^2(x) - 3g(x) + 5 \\ \Rightarrow f(x) &= x^2 - 3x + 5 \\ \therefore f(0) &= 5 \end{aligned}$$

But, if we consider the domain of the composite function $f \circ g(x)$ then in that case $f(0)$ will be not defined as $g(x)$ cannot be equal to zero.

15. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to

- (1) $\frac{4}{15}$
- (2) $\frac{1}{15}$
- (3) $\frac{14}{15}$
- (4) $\frac{8}{15}$

Official Ans. by NTA (2)

Ans. (2)

Sol.

$$\begin{aligned} \left\{ \frac{4^{2022}}{15} \right\} &= \left\{ \frac{2^{4044}}{15} \right\} \\ &= \left\{ \frac{(1+15)^{1011}}{15} \right\} \\ &= \frac{1}{15} \end{aligned}$$

16. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector \vec{d} satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to

- (1) 413
- (2) 423
- (3) 323
- (4) 313

Official Ans. by NTA (1)

Ans. (1)

Sol.:

$$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$$

$$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$$

$$\Rightarrow \vec{d} = \vec{c} + \lambda \vec{b}$$

$$\text{Also } \vec{d} \cdot \vec{a} = 24$$

$$\Rightarrow (\vec{c} + \lambda \vec{b}) \cdot \vec{a} = 24$$

$$\lambda = \frac{24 - \vec{a} \cdot \vec{c}}{\vec{b} \cdot \vec{a}} = \frac{24 - 6}{9} = 2$$

$$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$$

$$= 8\hat{i} - 5\hat{j} + 18\hat{k}$$

$$\Rightarrow |\vec{d}|^2 = 64 + 25 + 324 = 413$$

17. Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a

matrix A and $|A| = 2$, then $[\alpha \ -2\alpha \ \alpha]B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$ is

equal to :-

- (1) 16
- (2) 32
- (3) -16
- (4) 0

Official Ans. by NTA (3)

Ans. (3)

Sol.

$$\text{Given, } B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$$

$$|B| = 4$$

$$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$$

$$-\alpha^2 + 6\alpha - 8 = 0$$

$$\alpha = 2, 4$$

Given, $\alpha > 2$

So, $\alpha = 2$ is rejected

$$[4 \ -8 \ 4] \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]_{1 \times 1}$$

18. Let $s_1, s_2, s_3, \dots, s_{10}$ respectively be the sum to 12 terms of 10 A.P.s whose first terms are $1, 2, 3, \dots, 10$ and the common differences are $1, 3, 5, \dots, 19$ respectively. Then $\sum_{i=1}^{10} s_i$ is equal to

- (1) 7380
- (2) 7220
- (3) 7360
- (4) 7260

Official Ans. by NTA (4)

Ans. (4)

Sol.

$$S_k = 6(2k + (11)(2k - 1))$$

$$S_k = 6(2k + 22k - 11)$$

$$S_k = 144k - 66$$

$$\begin{aligned} \sum_{k=1}^{10} S_k &= 144 \sum_{k=1}^{10} k - 66 \times 10 \\ &= 144 \times \frac{10 \times 11}{2} - 660 \\ &= 7920 - 660 \\ &= 7260 \end{aligned}$$

19. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0, y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at

- (1) Two points
- (2) no point
- (3) infinite number of points
- (4) one point

Official Ans. by NTA (2)

Ans. (2)

Sol.

$$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$$

$$\text{I.F.} = e^{-x}$$

$$ye^{-x} = \int 7e^{-x} dx$$

$$\Rightarrow ye^{-x} = -7e^{-x} + c$$

$$\Rightarrow y = -7 + ce^x$$

$$-7 + 7e^x = -7 + 8e^x$$

$$\Rightarrow e^x = 0$$

No solution

20. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

- (1) -2
- (2) 2
- (3) -4
- (4) 4

Official Ans. by NTA (3)

Ans. (3)

Sol.

$$(x + 2y + az - 2) + \lambda(x - y + z - 3) = 0$$

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$\lambda = -\frac{7}{2}, a = 3, b = 1$$

$$\frac{2}{\sqrt{a}} = \frac{|5a + 11c + bc - 6a + 1|}{\sqrt{25 + 121 + 1}}$$

$$c = -1$$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

SECTION-B

21. Let α be the constant term in the binomial

expansion of $\left(\sqrt{x} - \frac{6}{3x^2}\right)^n$, $n \leq 15$. If the sum of

the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____.

Official Ans. by NTA (36)

Ans. (36)

Sol.

$$T_{k+1} = {}^n C_k (x)^{\frac{n-k}{2}} (-6)^k (x)^{\frac{-3k}{2}}$$

$$\frac{n-k}{2} - \frac{3k}{2} = 0$$

$$n - 4k = 0$$

$$(-5)^n - \left({}^n C_{\frac{n}{4}} (-6)^{\frac{n}{4}} \right) = 649$$

By observation $(625 + 24 = 649)$, we get $n = 4$

$$\therefore n = 4 \text{ \& } k = 1$$

Required is coefficient of x^{-4} is $\left(\sqrt{4} - \frac{6}{3} \right)^4$

$${}^4 C_1 (-6)^3$$

By calculating we will get $\lambda = 36$

22. If

$$S = \left\{ x \in \mathbb{R} : \sin^{-1} \left(\frac{x+1}{\sqrt{x^2+2x+2}} \right) - \sin^{-1} \left(\frac{x}{\sqrt{x^2+1}} \right) = \frac{\pi}{4} \right\},$$

then

$$\sum_{x \in \mathbb{R}} \left(\sin \left((x^2+x+5) \frac{\pi}{2} \right) - \cos \left((x^2+x+5) \pi \right) \right) \text{ is}$$

equal to _____.

Official Ans. by NTA (4)

Ans. (4)

Sol.

$$\sin^{-1} \left(\frac{(x+1)}{\sqrt{(x+1)^2+1}} \right) - \sin^{-1} \left(\frac{x}{\sqrt{x^2+1}} \right) = \frac{\pi}{4}$$

$$\therefore \frac{t}{\sqrt{t^2+1}} \in (-1,1)$$

$$\sin^{-1} \left(\frac{(x+1)}{\sqrt{(x+1)^2+1}} \right) = \sin^{-1} \left(\frac{x}{\sqrt{x^2+1}} \right) + \frac{\pi}{4}$$

$$\begin{aligned} \frac{(x+1)}{\sqrt{(x+1)^2+1}} &= \left(\frac{1}{\sqrt{2}} \right) \cos \left(\sin^{-1} \left(\frac{x}{\sqrt{x^2+1}} \right) \right) + \frac{1}{\sqrt{2}} \left(\frac{x}{\sqrt{x^2+1}} \right) \\ &= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{x^2+1}} + \frac{x}{\sqrt{x^2+1}} \right) \end{aligned}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{2}} \left(\frac{1+x}{\sqrt{x^2+1}} \right)$$

After solving this equation, we get

$$x = -1 \text{ or } x = 0$$

$$S = \{-1, 0\}$$

$$\sum_{x \in \mathbb{R}} \left(\sin \left((x^2+x+5) \frac{\pi}{2} \right) - \cos \left((x^2+x+5) \pi \right) \right)$$

$$\begin{aligned} &= \left[\sin \left(\frac{5\pi}{2} \right) - \cos(5\pi) \right] + \left[\sin \left(\frac{5\pi}{2} \right) - \cos(5\pi) \right] \\ &= 4 \end{aligned}$$

23. Let $\omega = z\bar{z} + k_1z + k_2iz + \lambda(1+i)$, $k_1, k_2 \in \mathbb{R}$. Let

$\text{Re}(\omega) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y -axis. If

the curve $\text{Im}(\omega) = 0$ intersects C at A and B , then $30(AB)^2$ is equal to _____.

Official Ans. by NTA (24)

Ans. (24)

Sol.

$$\omega = z\bar{z} + k_1z + k_2iz + \lambda(1+i)$$

$$\text{Re}(w) = x^2 + y^2 + k_1x - k_2y + \lambda = 0$$

$$\text{Centre} \equiv \left(\frac{-k_1}{2}, \frac{k_2}{2} \right) \equiv (1, 2)$$

$$\Rightarrow k_1 = -2, k_2 = 4$$

$$\text{radius} = 1 \Rightarrow \lambda = 4$$

$$\text{Im} = k_1y + k_2x + \lambda = 0$$

$$\therefore 2x - y + 2 = 0$$

$$d = \frac{2}{\sqrt{5}}$$

$$\frac{l^2}{4} = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\therefore 30l^2 = 24$$

24. Let for $x \in \mathbb{R}, S_0(x) = x,$

$$S_k(x) = C_k x + k \int_0^x S_{k-1}(t) dt, \text{ where}$$

$$C_0 = 1, C_k = 1 - \int_0^1 S_{k-1}(x) dx, \quad k = 1, 2, 3, \dots \text{ Then}$$

$S_2(3) + 6C_3$ is equal to _____.

Official Ans. by NTA (18)

Ans. (18)

Sol.

Given,

$$S_k(x) = C_k x + k \int_0^x S_{k-1}(t) dt,$$

Put $k = 2$ and $x = 3$

$$S_2(3) = C_2(3) + 2 \int_0^3 S_1(t) dt \quad \dots(1)$$

Also,

$$S_1(x) = C_1(x) + \int_0^x S_0(t) dt$$

$$= C_1 x + \frac{x^2}{2}$$

$$S_2(3) = 3C_2 + 2 \int_0^3 \left(C_1 t + \frac{t^2}{2} \right) dt$$

$$= 3C_2 + 9C_1 + 9$$

Also,

$$C_1 = 1 - \int_0^1 S_0(x) dx = \frac{1}{2}$$

$$C_2 = 1 - \int_0^1 S_1(x) dx = 0$$

$$\begin{aligned} C_3 &= 1 - \int_0^1 S_2(x) dx \\ &= 1 - \int_0^1 \left(C_2 x + C_1 x^2 + \frac{x^3}{3} \right) dx \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} S_2(x) &= C_2 x + 2 \int_0^x S_1(t) dt \\ &= C_2 x + C_1 x^2 + \frac{x^3}{3} \end{aligned}$$

$$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9 = 18$$

25. The sum to 20 terms of the series

$$2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - \dots \text{ is equal to } \underline{\hspace{2cm}}.$$

Official Ans. by NTA (1310)

Ans. (1310)

Sol.

$$\begin{aligned} &(2^2 - 3^2 + 4^2 - 5^2 + \dots + 20 \text{ terms}) + \\ &\qquad\qquad\qquad (2^2 + 4^2 + \dots + 10 \text{ terms}) \\ &= -(2 + 3 + 4 + 5 + \dots + 11) + 4[1 + 2^2 + \dots + 10^2] \\ &= -\left[\frac{21 \times 22}{2} - 1 \right] + 4 \times \frac{10 \times 11 \times 21}{6} \\ &= 1 - 231 + 14 \times 11 \times 10 \\ &= 1540 + 1 - 231 \\ &= 1310 \end{aligned}$$

26. The number of seven digit positive integers formed using the digits 1,2,3 and 4 only and sum of the digits equal to 12 is _____.

Official Ans. by NTA (413)

Ans. (413)

Sol.

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 &= 12, \quad x_i \in \{1, 2, 3, 4\} \\ \text{No. of solutions} &= {}^{5+7-1} C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!} = 413 \end{aligned}$$

27. Let m_1 and m_2 be the slopes of the tangents drawn from the point $P(4,1)$ to the hyperbola $H: \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x -axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____.

Official Ans. by NTA (8)

Ans. (8)

Sol.

Equation of tangent to the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$$y = mx \pm \sqrt{a^2 - b^2m^2}$$

passing through (4, 1)

$$1 = 4m \pm \sqrt{25 - 16m^2} \Rightarrow 4m^2 - m - 3 = 0$$

$$\Rightarrow m = 1, \frac{-3}{4}$$

Equation of tangent with positive slopes 1 & $\frac{3}{4}$.

$$\left. \begin{aligned} 4y &= 3x - 16 \\ y &= x - 3 \end{aligned} \right\} \text{with positive intercept on x-axis.}$$

$$\alpha = \frac{16}{3}, \beta = 3$$

Intersection points:

$$Q : (-4, -7)$$

$$P : (4, 1)$$

$$PQ^2 = 128$$

$$\frac{PQ^2}{\alpha\beta} = \frac{128}{16} = 8$$

28. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13, then α is equal to _____.

Official Ans. by NTA (15)

Ans. (15)

Sol.

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$$\frac{x - \frac{5}{3}}{1} = \frac{y - \frac{5}{3}}{-2} = \frac{z - \frac{8}{3}}{1} = \frac{-2\left(1 \times \frac{5}{3} + (-2) \times \frac{8}{3} + 1 \times \frac{8}{3} - 2\right)}{1^2 + 2^2 + 1^2} = \frac{1}{3}$$

$$\therefore x = 2, y = 1, z = 3$$

$$13^2 = (6 - 2)^2 + (-2 - 1)^2 + (\alpha - 3)^2$$

$$\Rightarrow (\alpha - 3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$$

29. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If \vec{b} is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to _____.

Official Ans. by NTA (66)

Ans. (66)

Sol.

$$|\vec{a}| = \sqrt{11}, |\vec{c}| = \sqrt{22}$$

$$|\vec{a}| = |\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}|\sin\theta$$

$$\sqrt{11} = \sqrt{50}\sqrt{22}\sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{10}$$

$$|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c}$$

$$= |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta$$

$$= 50 + 22 + 2 \times \sqrt{50} \times \sqrt{22} \times \frac{\sqrt{99}}{10}$$

$$= 72 + 66$$

$$|72 - |\vec{b} + \vec{c}|^2| = 66$$

30. Let the mean of the data

x	1	3	5	7	9
Frequency(f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m + \sigma^2}$ is equal to _____.

Official Ans. by NTA (8)

Ans. (8)

Sol.

$$5 = \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4 + 72 + 140 + 7\alpha + 72}{64 + \alpha}$$

$$\Rightarrow 320 + 5\alpha = 288 + 7\alpha \Rightarrow 2\alpha = 32 \Rightarrow \alpha = 16$$

$$M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} \text{ where } \sum f_i = 64 + 16 = 80$$

$$M.D.(\bar{x}) = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80}$$

$$= \frac{8}{5}$$

$$\text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$= \frac{4 \times 16 + 24 \times 4 + 0 + 16 \times 4 + 8 \times 16}{80} = \frac{352}{80}$$

$$\therefore \frac{3\alpha}{m + \sigma^2} = \frac{3 \times 16}{\frac{128}{80} + \frac{352}{80}} = 8$$