

FINAL JEE-MAIN EXAMINATION - APRIL, 2023

(Held On Tuesday 11th April, 2023)

TIME: 3:00 PM to 6:00 PM

MATHEMATICS

SECTION-A

- If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8} (103x+81)$, then λ ,
 - $\frac{\lambda}{2}$ are the roots of the equation
 - (1) $4x^2 + 24x 27 = 0$ (2) $4x^2 24x + 27 = 0$
 - (3) $4x^2 + 24x + 27 = 0$ (4) $4x^2 24x 27 = 0$

Official Ans. by NTA (2)

Ans. (2)

Sol. Put x = 0

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{9^3}{8} :: \lambda = \frac{9}{2}$$

$$\therefore \frac{\lambda}{3} = \frac{3}{2}$$

$$\therefore \text{ Required equation is } : x^2 - x \left(\frac{9}{2} + \frac{3}{2} \right) x + \frac{27}{4} = 0$$

$$4x^2 - 24x + 27 = 0$$

Let the line passing through the points, P(2, -1, 2)2. and Q (5, 3, 4) meet the plane x - y + z = 4 at the point R. Then the distance of the point R from the plane x + 2y + 3z + 2 = 0 measured parallel to the

line
$$\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$$
 is equal to

- (1) $\sqrt{31}$
- (2) $\sqrt{189}$
- (3) $\sqrt{61}$
- (4) 3

Official Ans. by NTA (4)

Ans. (4)

Sol. Line:
$$\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$$

$$R(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$$

$$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$$

$$\lambda + 6 = 4$$
 $\therefore \lambda = -2$

$$\therefore R \equiv (-1, -5, 0)$$

TEST PAPER WITH SOLUTION

Line:
$$\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$$

Point
$$T = (2\mu - 1, 2\mu - 5, \mu)$$

It lies on plane

$$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$$

$$\mu = 1$$

$$T = (1, -3, 1)$$

$$\therefore RT = 3$$

- If the 1011th term from the end in the binomial **3.** expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011th term from the beginning, then |x| is equal to
 - (1) 12

(2) 8

(3) 10

(4) 15

Official Ans. by NTA (3)

Ans. (BONUS)

Sol. T_{1011} from beginning = T_{1010+1}

$$=^{2022} C_{1010} \left(\frac{4x}{5}\right)^{1012} \left(\frac{-5}{2x}\right)^{1010}$$

T₁₀₁₁ from end

$$=^{2022} C_{1010} \left(\frac{-5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010}$$

Given:
$${}^{2022}C_{1010} \left(\frac{-5}{2x}\right)^{1012} \left(\frac{4x}{5}\right)^{1010}$$

$$=2^{10}\cdot^{2022}C_{_{1010}}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$$

$$\left(\frac{-5}{2x}\right)^2 = 2^{10} \left(\frac{4x}{5}\right)^2$$

$$x^4 = \frac{5^4}{2^{16}}$$

$$\left|\mathbf{x}\right| = \frac{5}{16}$$



4. Let the function $f:[0,2] \rightarrow R$ be defined as

$$f(x) = \begin{cases} e^{\min\{x^2, x - [x]\}}, & x \in [0, 1) \\ e^{[x - \log_e x]}, & x \in [1, 2] \end{cases}$$

where [t] denotes the greatest integer less than or equal to t. Then the value of the integral $\int_{-\infty}^{2} xf(x)dx$ is

$$(1) 2e - 1$$

(2)
$$1 + \frac{3e}{2}$$

(3)
$$2e - \frac{1}{2}$$

(4)
$$(e-1)\left(e^2+\frac{1}{2}\right)$$

Official Ans. by NTA (3) Ans. (3)

Sol. Minimum $\{x^2, \{x\}\} = x^2; x \in [0,1)$

$$[x - \log_e x] = 1; x \in [1, 2)$$

$$\therefore f(x) = \begin{cases} e^{x^2}; x \in [0,1) \\ e; x \in [1,2) \end{cases}$$

$$\int_{0}^{2} xf(x)dx = \int_{0}^{1} xe^{x^{2}}dx + \int_{1}^{2} ex dx$$
$$= \frac{1}{2}(e-1) + \frac{1}{2}(4-1)e$$

$$=2e-\frac{1}{2}$$

5. Let y = y(x) be the solution of the differential

equations
$$\frac{dy}{dx} + \frac{5}{x(x^5 + 1)}y = \frac{(x^5 + 1)^2}{x^7}, x > 0.$$
 If

y(1) = 2, then y(2) is equal to

$$(1) \; \frac{637}{128}$$

(2)
$$\frac{679}{128}$$

$$(3) \frac{693}{128}$$

$$(4) \frac{697}{128}$$

Official Ans. by NTA (3) Ans. (3)

Sol. I.F =
$$e^{\int \frac{5dx}{x(x^5+1)}} = e^{\int \frac{5x^{-6}dx}{(x^{-5}+1)}}$$

Put,
$$1 + x^{-5} = t \implies -5x^{-6}dx = dt$$

$$\Rightarrow e^{\int \frac{-dt}{t}} = \frac{1}{t} = \frac{x^5}{1 + x^5}$$

$$y \cdot \frac{x^{5}}{1+x^{5}} = \int \frac{x^{5}}{(1+x^{5})} \times \frac{(1+x^{5})^{2}}{x^{7}} dx$$
$$= \int x^{3} dx + \int x^{-2} dx$$

$$y \cdot \frac{x^5}{1 + x^5} = \frac{x^4}{4} - \frac{1}{x} + c$$

Given that : $x = 1 \implies y = 2$

$$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$$

$$c = \frac{7}{4}$$

$$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Now put, x = 2

$$\mathbf{y} \cdot \left(\frac{32}{33}\right) = \frac{21}{4}$$

$$y = \frac{693}{128}$$

6. If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$

and \vec{d} are coplanar; then $[\vec{a}\vec{b}\vec{c}]$ is equal to

$$(1) \left[\vec{d} \vec{c} \vec{a} \right] + \left[\vec{b} \vec{d} \vec{a} \right] + \left[\vec{c} \vec{d} \vec{b} \right]$$

(2)
$$\left[\vec{d}\vec{b}\vec{a}\right] + \left[\vec{a}\vec{c}\vec{d}\right] + \left[\vec{d}\vec{b}\vec{c}\right]$$

(3)
$$\left[\vec{a} \vec{d} \vec{b} \right] + \left[\vec{d} \vec{c} \vec{a} \right] + \left[\vec{d} \vec{b} \vec{c} \right]$$

$$(4) \left[\vec{b} \vec{c} \vec{d} \right] + \left[\vec{d} \vec{a} \vec{c} \right] + \left[\vec{d} \vec{b} \vec{a} \right]$$

Official Ans. by NTA (1)

Ans. (1)

Sol. \vec{a} , \vec{b} , \vec{c} , \vec{d} are coplanar points.

 $\vec{b} - \vec{a}, \vec{c} - \vec{a}, \vec{d} - \vec{a}$ are coplanar vectors.

So,
$$\left[\vec{b} - \vec{a} \ \vec{c} - \vec{a} \ \vec{d} - \vec{a}\right] = 0$$

$$(\vec{b} - \vec{a}) \cdot ((\vec{c} - \vec{a}) \times (\vec{d} - \vec{a})) = 0$$

$$\begin{bmatrix} \vec{b} \ \vec{c} \ \vec{d} \end{bmatrix} - \begin{bmatrix} \vec{b} \ \vec{c} \ \vec{a} \end{bmatrix} - \begin{bmatrix} \vec{b} \ \vec{a} \ \vec{d} \end{bmatrix} - \begin{bmatrix} \vec{a} \ \vec{c} \ \vec{d} \end{bmatrix} = 0$$

$$\Rightarrow \left[\vec{a}\,\vec{b}\,\vec{c}\,\right] = \left[\vec{c}\,\vec{d}\,\vec{b}\,\right] + \left[\vec{b}\,\vec{d}\,\vec{a}\,\right] + \left[\vec{d}\,\vec{c}\,\vec{a}\,\right]$$

7. If $f: \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$\int\limits_{0}^{\pi/2} f\left(\sin 2x\right) \cdot \sin x dx + \alpha \int\limits_{0}^{\pi/4} f\left(\cos 2x\right) \cdot \cos x \, dx = 0 \,,$$

then α is equal to

$$(1) - \sqrt{3}$$

(2)
$$\sqrt{2}$$

(3)
$$\sqrt{3}$$

$$(4) - \sqrt{2}$$

Official Ans. by NTA (4)

Ans. (4)

Sol.
$$I = \int_{0}^{\frac{\pi}{4}} f(\sin 2x) \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(\sin 2x) \sin x \, dx$$

$$+\alpha \int_{0}^{\frac{\pi}{4}} f(\cos 2x) \cos x \, dx = 0$$

Apply king in first part and put $x - \frac{\pi}{4} = t$ in second part.



$$I = \int_{0}^{\frac{\pi}{4}} f(\cos 2x) \sin\left(\frac{\pi}{4} - x\right) dx + \int_{0}^{\frac{\pi}{4}} f(\cos 2t) \sin\left(\frac{\pi}{4} + t\right) dt$$

$$+ \alpha \int_{0}^{\frac{\pi}{4}} f(\cos 2x) \cos x \, dx = 0$$

$$I = \int_{0}^{\frac{\pi}{4}} f(\cos 2x) \left[2\sin \frac{\pi}{4} \cdot \cos x + \alpha \cos x \right] dx = 0$$

$$I = \left(\alpha + \sqrt{2}\right) \int_{0}^{\frac{\pi}{4}} f(\cos 2x) \cos x \, dx = 0$$

$$\therefore \alpha = -\sqrt{2}$$

8. If the system of linear equations

$$7x + 11y + \alpha z = 13$$

$$5x + 4y + 7z = \beta$$

$$175x + 194y + 57z = 361$$

has infinitely many solutions, then $\alpha + \beta + 2$ is equal to

(1)4

(2) 3

(3)5

(4) 6

Official Ans. by NTA (1) Ans. (1)

Sol.
$$7x + 11y + \alpha z = 13$$
 (i)

$$5x + 4y + 7z = \beta \qquad \dots (ii)$$

$$175x + 194y + 57z = 361$$
 (iii)

$$(i) \times 10 + (ii) \times 21 - (iii)$$

$$z(10\alpha + 147 - 57) = 130 + 21\beta - 361$$

$$\therefore 10\alpha + 90 = 0$$

$$\alpha = -9$$

$$130 - 361 + 21\beta = 0$$

$$\beta = 11$$

$$\alpha + \beta + 2 = 4$$

9. The domain of the function

$$f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$$
 is (where [x] denotes the

greatest integer less than or equal to x)

- $(1) \left(-\infty, -2\right) \cup \left(5, \infty\right) \qquad (2) \left(-\infty, -3\right] \cup \left[6, \infty\right)$
- $(3) \left(-\infty, -2\right) \cup [6, \infty) \qquad (4) \left(-\infty, -3\right] \cup \left(5, \infty\right)$

Official Ans. by NTA (3)

Ans. (3)

Sol.
$$[x]^2 - 3[x] - 10 > 0$$

$$[x] < -2or[x] > 5$$

- Let P be the plane passing through the points (5, 3, 10. 0), (13, 3, -2) and (1, 6, 2). For $\alpha \in \mathbb{N}$, if the distances of the points A $(3, 4, \alpha)$ and B $(2, \alpha, a)$ from the plane P are 2 and 3 respectively, then the positive value of a is
 - (1)6

(2) 4

(3) 3

(4)5

Official Ans. by NTA (2)

Ans. (2)

Sol.
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

Normal of the plane = $3\hat{i} - 4\hat{j} + 12\hat{k}$

Plane: 3x - 4y + 12z = 3

Distance from A $(3, 4, \alpha)$

$$\left| \frac{9-16+12\alpha-3}{13} \right| = 2$$

 $\alpha = 3$

 $\alpha = -8$ (rejected)

Distance from B (2, 3, a)

$$\left| \frac{6-12+12a-3}{13} \right| = 3$$

11. The converse of the statement $((\sim p) \land q) \Rightarrow r$ is

$$(1) (\sim r) \Rightarrow p \land c$$

$$(1) (\sim r) \Rightarrow p \land q \qquad (2) (\sim r) \Rightarrow ((\sim p) \land q)$$

(3)
$$((\sim p) \lor q) \Rightarrow r$$
 (4) $(p \lor (\sim q)) \Rightarrow (\sim r)$

Official Ans. by NTA (4) Ans. (4)

Sol. Converse of
$$((\sim p) \land q) \Rightarrow r$$

$$\equiv r \Rightarrow (\sim p \land q)$$

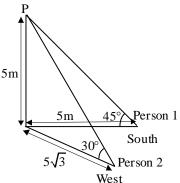
$$\equiv \sim r \vee (\sim p \wedge q)$$

$$\equiv \sim r \lor (p \lor \sim q) \equiv (p \lor \sim q) \Longrightarrow \sim r$$

- **12.** The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due west of the tower is 30°. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to
 - $(1)\ 10$
- (3) $5\sqrt{5}$
- (4) $\frac{5}{2}\sqrt{5}$

Official Ans. by NTA (1) Ans. (1)





Sol. West Distance = 10 (By Pythagoras theorem)

- 13. Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5b^3c^2d$ is 3750 β , then the value of β is
 - (1)90
- (2) 110
- (3)55
- (4) 108

Official Ans. by NTA (1) Ans. (1)

Sol.
$$\frac{5\left(\frac{a}{5}\right) + 3\left(\frac{b}{3}\right) + 2\left(\frac{c}{2}\right) + d}{11} \ge \left(\frac{a^5b^3c^2d}{5^53^32^2}\right)^{1/11}$$

$$1 \ge \left(\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}\right)^{1/1}$$

 $\beta = 90$

- 14. If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to
 - (1)72
- (2)115

- (3)92
- (4)69

Official Ans. by NTA (3)

Ans. (3)

Sol.
$$(x-2)^2 + y^2 = r^2$$

Solving with ellipse, we get

$$(x-2)^2 + \frac{36-x^2}{4} = r^2$$

$$3x^2 - 16x + 52 - 4r^2 = 0$$

$$D = 0 \Longrightarrow 4r^2 = \frac{92}{3}$$

- **15.** Let the mean of 6 observation 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to
 - (1) $\frac{10}{3}$
- (2) $\frac{7}{3}$

(3) 3

 $(4) \frac{8}{3}$

Official Ans. by NTA (4)

Ans. (4)

Sol.
$$x + y = 18 \{ \because mean = 5 \} \dots (i)$$

$$10 = \frac{1+4+16+25+x^2+y^2}{6} - 25$$

$$x^2 + y^2 = 164 \dots$$
 (ii)

By solving (i) and (ii)

$$x = 8, y = 10$$

$$M.D.(\overline{x}) = \frac{\sum |x_i - \overline{x}|}{6} = \frac{8}{3}$$

- 16. The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1:3:5, is equal to
 - (1)25
- (2)63
- (3)41
- (4)92

Official Ans. by NTA (2)

Ans. (2)

Sol.
$$^{n+2}C_{r-1}: ^{n+2}C_r: ^{n+2}C_{r+1}=1:3:5$$

$$\frac{{}^{n+2}C_{r-1}}{{}^{n+2}C_r} = \frac{1}{3}$$

$$n = 4r - 3 \dots (i)$$

$$\frac{{}^{n+2}C_{r}}{{}^{n+2}C_{r+1}} = \frac{3}{5}$$

$$8r - 1 = 3n \dots (ii)$$

From, (i) and (ii)

r = 2 and n = 5

Required sum = 63

- 17. If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is
 - (1) 103
- (2) 104
- (3) 101
- (4) 102

Official Ans. by NTA (1)

Ans. (1)

Sol. $4 \times 4! + 1 \times 3! + 1 = 103$

18. For $a \in C$, let $A = \{z \in C : Re(a + \overline{z}) > Im(\overline{a} + z)\}$

and
$$B = \{z \in C : Re(a + \overline{z}) < Im(\overline{a} + z)\}$$
. Then

among the two statements:

- (S1): If Re (A), Im (A) > 0, then the set A contains all the real numbers
- (S2): If Re (A), Im (A) < 0, then the set B contains all the real numbers,
- (1) Only (S1) is true
- (2) both are false
- (3) Only (S2) is true
- (4) Both are true

Official Ans. by NTA (2)

Ans. (2)



Sol. Let $a = x_1 + iy_1 \ z = x + iy$

Now $\operatorname{Re}(a+\overline{z}) > \operatorname{Im}(\overline{a}+z)$

$$\therefore x_1 + x > -y_1 + y$$

$$x_1 = 2$$
, $y_1 = 10$, $x = -12$, $y = 0$

Given inequality is not valid for these values.

S1 is false.

Now Re $(a + \overline{z}) < Im(\overline{a} + z)$

$$x_1 + x < -y_1 + y$$

$$x_1 = -2$$
, $y_1 = -10$, $x = 12$, $y = 0$

Given inequality is not valid for these values.

S2 is false.

- 19. Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \le b_2 \text{ and } b_1 \le a_2\}$. Then the number of elements in the set R is
 - (1) 26
- (2) 160
- (3) 180
- (4) 52

Official Ans. by NTA (2)

Ans. (2)

Sol. Let $a_1 = 1 \Rightarrow 5$ choices of b_2

- $a_1 = 3 \Rightarrow 4$ choices of b_2
- $a_1 = 4 \Rightarrow 4$ choices of b_2
- $a_1 = 6 \Rightarrow 2$ choices of b_2
- $a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2) 16 ways.

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

 $b_1 = 4 \Rightarrow 3$ choices of a_2

 $b_1 = 5 \Longrightarrow 2$ choices of a_2

 $b_1 = 8 \Rightarrow 1$ choices of a_2

Required elements in R = 160

20. Let f and g be two functions defined by

$$f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}$$

Then (gof)(x) is

- (1) Differentiable everywhere
- (2) Continuous everywhere but not differentiable exactly at one point
- (3) Not continuous at x = -1
- (4) Continuous everywhere but not differentiable at x = 1

Official Ans. by NTA (2)

Ans. (2)

Sol.
$$f(x) = \begin{cases} x+1, & x < 0 \\ 1-x, & 0 \le x < 1 \\ x-1, & 1 \le x \end{cases}$$

$$g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \ge 0 \end{cases}$$

$$g(f(x)) = \begin{cases} x+2, & x < -1 \\ 1, & x \ge -1 \end{cases}$$

g(f(x)) is continuous everywhere

g(f(x)) is not differentiable at x = -1

Differentiable everywhere else

SECTION-B

21. The number of points, where the curve

$$f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in \mathbb{R}$$
 cuts x-axis, is equal to

Official Ans. by NTA (2)

Ans. (2)

Sol. Let $e^{2x} = t$

$$\Rightarrow$$
 t⁴ - t³ - 3t² - t + 1 = 0

$$\Rightarrow t^2 + \frac{1}{t^2} - \left(t + \frac{1}{t}\right) - 3 = 0$$

$$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$$

$$\Rightarrow$$
 t + $\frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

Two real values of t.

22. Let the probability of getting head for a biased coin

be $\frac{1}{4}$. It is tossed repeatedly until a head appears.

Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$

has no real root is $\frac{p}{q}$, where p and q are co-prime,

then q - p is equal to

Official Ans. by NTA (27)

Ans. (27)



Sol.
$$64x^2 + 5Nx + 1 = 0$$

$$D = 25N^2 - 256 < 0$$

$$\Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$$

$$N = 1, 2, 3$$

:. Probability =
$$\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64}$$

$$\therefore q - p = 27$$

23. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If \vec{c} is a vector such that $\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3} |\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to

Official Ans. by NTA (285)

Ans. (285)

Sol.
$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \ \vec{b} = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{b} \cdot (\vec{a} \times \vec{c}) = 27, \ \vec{a} \cdot \vec{b} = 0$$

$$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}$$

Let θ be angle between \vec{b} , $\vec{a} \times \vec{c}$

Then
$$|\vec{b}| \cdot |\vec{a} \times \vec{c}| \sin \theta = 3\sqrt{14}$$

$$|\vec{\mathbf{b}}| \cdot |\vec{\mathbf{a}} \times \vec{\mathbf{c}}| \cos \theta = 27$$

$$\Rightarrow \sin \theta = \frac{\sqrt{14}}{\sqrt{95}}$$

$$\therefore \left| \vec{b} \right| \times \left| \vec{a} \times \vec{c} \right| = 3\sqrt{95}$$

$$\Rightarrow |\vec{a} \times \vec{c}| = \sqrt{3} \times \sqrt{95}$$

24. Let $S = \left\{ z \in C - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R \right\}$. If $\alpha - \frac{13}{11}i \in S, \alpha \in \mathbb{R} - \{0\}$, then $242\alpha^2$ is equal to

Official Ans. by NTA (1680)

Ans. (1680)

Sol.
$$\left(\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2}\right) \in \mathbb{R}$$

$$\Rightarrow 1 + \frac{\left(1 \operatorname{liz} - 13\right)}{\left(z^2 - 3\operatorname{iz} - 2\right)} \in \mathbb{R}$$

Put
$$z = \alpha - \frac{13}{11}i$$

$$\Rightarrow$$
 $(z^2 - 3iz - 2)$ is imaginary

Put
$$z = x + iy$$

$$\Rightarrow (x^2 - y^2 + 2xyi - 3ix + 3y - 2) \in \text{Imaginary}$$

$$\Rightarrow \text{Re}(x^2 - y^2 + 3y - 2 + (2xy - 3x)i) = 0$$

$$\Rightarrow$$
 x² - y² + 3y - 2 = 0

$$x^2 = y^2 - 3y + 2$$

$$x^{2} = (y-1)(y-2)$$
 : $z = \alpha - \frac{13}{11}i$

Put
$$x = \alpha$$
, $y = \frac{-13}{11}$

$$\alpha^2 = \left(\frac{-13}{11} - 1\right) \left(\frac{-13}{11} - 2\right)$$

$$\alpha^2 = \frac{\left(24 \times 35\right)}{121}$$

$$242\alpha^2 = 48 \times 35 = 1680$$

25. For $k \in \mathbb{N}$, if the sum of the series

$$1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots$$
 is 10, then the value of k

is

Official Ans. by NTA (2)

Ans. (2)



Sol.
$$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \text{upto} \infty$$

$$9 = \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \text{upto } \infty$$

$$\frac{9}{k} = \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \dots \text{upto} \infty$$

$$S = 9\left(1 - \frac{1}{k}\right) = \frac{4}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \dots \text{upto } \infty$$

$$\frac{S}{k} = \frac{4}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \dots upto \infty$$

$$\left(1 - \frac{1}{k}\right)S = \frac{4}{k} + \frac{1}{k^3} + \frac{1}{k^4} + \frac{1}{k^5} + \dots \infty$$

$$9\left(1 - \frac{1}{k}\right)^2 = \frac{4}{k} + \frac{\frac{1}{k^3}}{\left(1 - \frac{1}{k}\right)}$$

$$9(k-1)^3 = 4k(k-1)+1$$

$$k = 2$$

26. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Then the number of functions $f : A \rightarrow B$ satisfying f(1) + f(2) = f(4) - 1 is equal to

Official Ans. by NTA (360)

Ans. (360)

Sol.
$$f(1)+f(2)+1=f(4) \le 6$$

$$f(1)+f(2) \le 5$$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow 4$ mappings

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow 3$ mappings

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow 2$ mappings

Case (iv) $f(1) = 4 \Rightarrow f(2) = 1 \Rightarrow 1$ mapping

f(5) & f(6) both have 6 mappings each

Number of functions = $(4+3+2+1)\times 6\times 6 = 360$

27. Let the tangent to the parabola $y^2 = 12x$ at the point (3, α) be perpendicular to the line 2x + 2y = 3. Then the square of distance of the point (6, -4) from the normal to the hyperbola $\alpha^2 x^2 - 9y^2 = 9\alpha^2$ at its point ($\alpha - 1$, $\alpha + 2$) is equal to

Official Ans. by NTA (116)

Ans. (116)

Sol. : $P(3,\alpha)$ lies on $y^2 = 12 x$

$$\Rightarrow \alpha = \pm 6$$

But,
$$\frac{dy}{dx}\Big|_{(3,\alpha)} = \frac{6}{\alpha} = 1 \Rightarrow \alpha = 6(\alpha = -6 \text{ reject})$$

Now, hyperbola $\frac{x^2}{9} - \frac{y^2}{36} = 1$, normal at

$$Q(\alpha - 1, \alpha + 2)$$
 is $\frac{9x}{5} + \frac{36y}{8} = 45$

$$\Rightarrow$$
 2x + 5y - 50 = 0

Now, distance of (6, -4) from 2x + 5y - 50 = 0 is equal to

$$\left| \frac{2(6)-5(4)-50}{\sqrt{2^2+5^2}} \right| = \frac{58}{\sqrt{29}}$$

 \Rightarrow Square of distance = 116

28. Let the line $\ell: x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane P: x + 2y + 3z = 4 at the point (α, β, γ) . If the angle between the line ℓ and the plane P is

$$\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$$
, then $\alpha + 2\beta + 6\gamma$ is equal to

Official Ans. by NTA (11) Ans. (11)



Sol.
$$\ell: \mathbf{x} = \frac{\mathbf{y} - 1}{2} = \frac{\mathbf{z} - 3}{\lambda}, \lambda \in \mathbb{R}$$

DR's of line ℓ (1, 2, λ)

DR's of normal vector of plane P: x + 2y + 3z = 4 are (1, 2, 3)

Now, angle between line ℓ and plane P is given by

$$\sin \theta = \left| \frac{1 + 4 + 3\lambda}{\sqrt{5 + \lambda^2} \cdot \sqrt{14}} \right| = \frac{3}{\sqrt{14}} \left(\text{given} \cos \theta = \sqrt{\frac{5}{14}} \right)$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$

lies on plane P.

$$\Rightarrow$$
 t = -1

$$\Rightarrow \left(-1,-1,\frac{7}{3}\right) \equiv \left(\alpha,\beta,\gamma\right)$$

$$\Rightarrow \alpha + 2\beta + 6\gamma = 11$$

29. If the line $\ell_1: 3y - 2x = 3$ is the angular bisector of

the lines
$$\ell_2: x - y + 1 = 0$$
 and $\ell_3: \alpha x + \beta y + 17 = 0$,

then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to

Official Ans. by NTA (348)

Ans. (348)

Sol. Point of intersection of ℓ_1 : 3y - 2x = 3

$$\ell_2: x - y + 1 = 0 \text{ is } P \equiv (0,1)$$

Which lies on ℓ_3 : $\alpha x + \beta y + 17 = 0$,

$$\Rightarrow \beta = -17$$

Consider a random point Q = (-1,0)

on $\ell_2: x - y + 1 = 0$, image of Q about

$$\ell_2: x-y+1=0$$
 is $Q' \equiv \left(\frac{-17}{13}, \frac{6}{13}\right)$ which is

calculated by formulae

$$\frac{x-(-1)}{2} = \frac{y-0}{-3} = -2\left(\frac{-2+3}{13}\right)$$

Now, Q' lies on ℓ_3 : $\alpha x + \beta y + 17 = 0$

$$\Rightarrow \boxed{\alpha = 7}$$

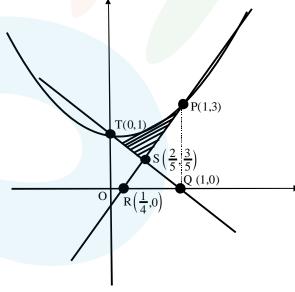
Now,
$$\alpha^2 + \beta^2 - \alpha - \beta = 348$$

30. If A is the area in the first quadrant enclosed by the curve $C: 2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line x + y = 1, then the value of 60A is

Official Ans. by NTA (16)

Ans. (16)

Sol.



$$y = 2x^2 + 1$$

Tangent at (1, 3)

$$y = 4x - 1$$

$$A = \int_{0}^{1} (2x^{2} + 1) dx - \text{area of } (\Delta QOT) - \text{area of}$$

 (ΔPQR) + area of (ΔQRS)

$$A = \left(\frac{2}{3} + 1\right) - \frac{1}{2} - \frac{9}{8} + \frac{9}{40} = \frac{16}{60}$$