<mark>∛Saral</mark>

Held On Sunday 29th January, 2023) TIME : 9 : 00 AM to 12 : 00 NOON MATHEMATICS SUCTION-A 61. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2iw_{x-1}}-(2x+3)}, x \in R$ is (1) $\mathbb{R} - \{1-3\}$ (2) $(2, \infty) - (3)$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 = x > 2$ x + 1 > 0 = x > -1 $x + 1 \neq 1 = x \times 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ ($x - 3$) ($x + 1$) $\neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - (3)$ 62. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 1}{x^2 + 1}$. Then (1) fix is nany-one in $(1, \infty)$ (3) fix is one-one in $(1, \infty)$ (3) fix is one-one in $(1, \infty)$ 50. 50. $\int_{-1}^{2} \frac{1}{1}$ \int_{-1}^{1} \int_{-1}^{1} 50. 50. 50. $\int_{-1}^{2} \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ 50. 50. $\int_{-1}^{2} \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ 50. 51. $\int_{-1}^{2} \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ 51. 51. $\int_{-1}^{2} \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ 52. 53. 53. 53. 54. 55. 55. 55. 56. $\int_{-1}^{2} \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ 56. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57. 57.		FINAL JEE-MAIN EXAMINATION – JANUARY, 2023						
MATHEMATICSTEST PAPER WITH SOLUTIONSECTION-ASECTION-A61. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_x x} - (2x+3)}, x \in R$ is $(1) \mathbb{R} - \{1-3\}$ (2) (2, ∞) - {3} (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ (4) $\mathbb{N}(x_i) > 0$ and $\mathbb{In}(x_2) > 0$ (5) $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ $(x - 3) (x + 1) \neq 0$ $x \neq -1, 3$ (62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(-\infty, \infty)$ (4) f(x) is one-one in $(1, \infty)$ (50.Sol. $2 - x_1 + iy_1$ $2 - x_1 + x_2 = 0$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(1, \infty)$ (51.Sol. $2 - x_1 + iy_1$ $2 - x_1 + x_2 = 0$ (3) f(x) is one-one in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ (51.Sol. $2 - x_1 + iy_1$ $2 - x_1 + x_2 = 0$ (3) f(x) is one-one in $(-\infty, \infty)$ (51.Sol. $2 - \frac{1}{1}$ $1 - \frac{1}{1}$ 1 $1 - \frac{1}{1}$ 1	(He	ld On Sunday 29th January, 2023)	TIME: 9:00 AM to 12:00 NOON					
SECTION-A 61. The domain of $f(x) = \frac{\log_{\{x+1\}}(x-2)}{e^{2\log_{x}-}(-2x+3)}$, $x \in R$ is (1) $\mathbb{R} - \{1-3\}$ (2) $(2, \infty) - \{3\}$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ Official Ans. by NTA (2) Ans. (2) Sol. $x-2 > 0 \Rightarrow x > 2$ $x+1 > 0 \Rightarrow x > -1$ $x+1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^{2} - 2x - 3 \neq 0$ ($x - 3$) $(x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ (2) Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^{2} + 2x + 1}{x^{2} + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(1, \infty)$ (5). $\int_{1}^{2} \frac{1}{1}$ \int_{1}^{1} $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ Sol. $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} = 1 + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} + \frac{1}{x^{2} + 1}$ $\int_{1}^{2} \frac{1}{x^{2} + 1} + \frac{1}{x^{2} + 1} $		MATHEMATICS		TEST PAPER WITH SOLUTION				
61. The domain of $f(x) = \frac{\log_{(x^{n})}(x-2)}{e^{2\frac{\log_{(x^{n})}}(x-2-x+3)}}$, $x \in R$ is (1) $\mathbb{R} - \{1-3\}$ (2) $(2, \infty) - \{3\}$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^{2} - 2x - 3 \neq 0$ ($(x - 3)(x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^{2} + 2x + 1}{x^{2} + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(-\infty, -1)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(1, \infty)$ (5). $\int_{1}^{2} \frac{1}{\sqrt{x + 1}} = 1 + \frac{2x}{\sqrt{x + 1}}$ Sol. $\int_{1}^{2} \frac{1}{\sqrt{x + 1}} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}}$ $f(x) = \frac{(x + 1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{\sqrt{x + 1}} = 1 + 2x$		SECTION-A	63.	For two non-zero complex number z_1 and z_2 , if				
61. The domain of $f(x) = \frac{x^{1+(y)}}{x^{2}+(1-(2x+3))}, x \in R$ is (1) $\mathbb{R} - \{1-3\}$ (2) $(2, \infty) - \{3\}$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^{2} - 2x - 3 \neq 0$ ($x - 3$) ($x + 1$) $\neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^{2} + 2x + 1}{x^{2} + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(-\infty, -1)$ (3) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. 2^{-1} $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ $f(x) = \frac{(x+1)^{2}}{x^{2} + 1} = \frac{(x+1)^{2}}$		$\log_{(x+1)}(x-2)$		$\operatorname{Re}(z_1z_2) = 0$ and $\operatorname{Re}(z_1 + z_2) = 0$, then which of the				
(1) $\mathbb{R} - \{1-3\}$ (2) $(2, \infty) - \{3\}$ (3) $(-1, \infty) - \{3\}$ (4) $\mathbb{R} - \{3\}$ Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ ($x - 3$) $(x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let f: $\mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(1, \infty)$ Difficial Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \frac{1}{1}$ \int_{-1}^{1} $\int_{-1}^{$	61.	The domain of $f(x) = \frac{e^{x(x+1)}(x-1)}{e^{2\log_e x} - (2x+3)}, x \in \mathbb{R}$ is		following are possible ?				
(b) $(1 - x) - \{3\}$ (c) $\mathbb{R} - \{3\}$ (c) $(1 - x) - \{3\}$ (d) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{3\}$ (e) Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ ($x - 3$) ($x + 1 \neq 0$ $x \neq -1, 3$ So Ans (2, ∞) - {3} 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is one-one in $(-\infty, \infty)$ (4) f(x) is one-one in $(1, \infty)$ 30 f(x) is one-one in $(-\infty, \infty)$ (51. $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_3$ $Re(x_1 + x_2) = x_1 + x_2 = 0$ $x_1 \& x_2$ are of opposite sign 91. Let $A \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $15x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation : (1) $7x^2 + 245x - 250 = 0$ (2) $7x^2 - 245x + 250 = 0$		(1) $\mathbb{R} - \{1-3\}$ (2) (2, ∞) - $\{3\}$		(A) $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$				
(c) $[1, \infty] > 0$ and $Im(z_2) < 0$ (c) $Im(z_1) > 0$ and $Im(z_2) < 0$ (d) $Im(z_1) < 0$ and $Im(z_2) < 0$ (e) $Im(z_1) < 0$ and $Im(z_2) < 0$ (f) $Im(z_1) < 0$ and $Im(z_2) < 0$ (h) $Im(z_1) < 0$ and $Im(z_2)$ (h)		$(1, 1, 2) \qquad (1, 2)$		(B) $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$				
Official Ans. by NTA (2) Ans. (2) Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ $(x - 3) (x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: R \to R$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. 2^{-1} -1 -1 1 $f(x) = \frac{(x + 1)^2}{r^2 + 1} = 1 + \frac{2x}{r^2 + 1}$ $f(x) = \frac{(x + 1)^2}{r^2 + 1} = 1 + \frac{2x}{r^2 + 1}$ (D) $Im(z_1) < 0$ and $Im(z_2) < 0$ Choose the correct answer from the options given below : (1) B and D (2) B and C (3) A and B (4) A and C Official Ans. by NTA (2) Ans. (2) Sol. $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$ $Re(z_1 + z_2) = x_1 + x_2 = 0$ $x_1 \& x_2$ are of opposite sign $y_1 \& y_2$ are the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $12x^2 - 245x + 250 = 0$ (2) $7x^2 - 245x + 250 = 0$ (3) $49x^2 - 245x + 250 = 0$		$(3) (-1, \infty) - \{3\} \qquad (4) \mathbb{R} - \{3\}$		(C) $Im(z_1) > 0$ and $Im(z_2) < 0$				
Ans. (2) Sol. $x-2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ $(x - 3) (x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbf{R} \to \mathbf{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{1}^{2} \int$		Official Ans. by NTA (2)		(D) $Im(z_1) < 0$ and $Im(z_2) < 0$				
Sol. $x - 2 > 0 \Rightarrow x > 2$ $x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0$ and $x > 0$ Denominator $x^2 - 2x - 3 \neq 0$ $(x - 3) (x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ (5) Sol. $\int_{-1}^{2} \frac{1}{1}$ $\int_{-1}^{1} \frac{1}{1}$ $\int_{-1}^{1} \frac{1}{1}$ $f(x) = \frac{(x + 1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$	C 1	Ans. (2)		Choose the correct answer from the options given				
$x + 1 > 0 \Rightarrow x > -1$ $x + 1 \neq 1 \Rightarrow x \neq 0 \text{ and } x > 0$ Denominator $x^{2} - 2x - 3 \neq 0$ (1) B and D (2) B and C (3) A and B (4) A and C Official Ans. by NTA (2) (3) A ans. (2) (4) f(x) is many-one in (-\infty, -1) (2) f(x) is many-one in (-\infty, -1) (2) f(x) is many-one in (-\infty, -1) (3) f(x) is one-one in (-\infty, -1) (4) f(x) is one-one in (-\infty, -1) (5) f(x) is one-one in (-\infty, -1) (6) f(x) is one-one in (-\infty, -1) (7) f(x) is one-one in (-\infty, -1) (9) f(x) is one-one in (-\infty, -1) (1) f(x)	501.	$x - 2 \ge 0 \Rightarrow x \ge 2$		below :				
Sol. $x + 1 \neq 1 \rightarrow X \neq 0$ and $X > 0$ $y = x^{2} + 2x + 1 \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ (3) A and B (4) A and C Official Ans. by NTA (2) Ans. (2) Sol. $z_{1} = x_{1} + iy_{1}$ $z_{2} = x_{2} + iy_{2}$ Re $(z_{1}z_{2}) = x_{1}x_{2} - y_{1}y_{2} = 0$ Re $(z_{1} + z_{2}) = x_{1} + x_{2} = 0$ (1) f(x) is many-one in $(-\infty, \infty)$ (3) f(x) is one-one in $(1, \infty)$ (3) f(x) is one-one in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $y_{1} = \frac{x^{2} + 1^{2}}{1} = 1 + \frac{2x}{x^{2} + 1} = 1 + \frac{2x}{x^{2} + 1}$ (3) A and B (4) A and C Official Ans. by NTA (2) Ans. (2) Sol. $z_{1} = x_{1} + iy_{1}$ $z_{2} = x_{2} + iy_{2}$ Re $(z_{1}z_{2}) = x_{1}x_{2} - y_{1}y_{2} = 0$ Re $(z_{1} + z_{2}) = x_{1} + x_{2} = 0$ (3) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $y_{1} \ll y_{2}$ are of opposite sign (4) A and C Official Ans. by NTA (2) Ans. (2) Sol. $z_{1} = x_{1} + iy_{1}$ Re $(z_{1}+z_{2}) = x_{1}x_{2} = 0$ Re $(z_{1}+z_{2}) = x_{1} + x_{2} = 0$ (3) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $y_{1} \ll y_{2}$ are of opposite sign (4) A and C Official Ans. by NTA (2) Ans. (2) Sol. $z_{1} = x_{1} + iy_{1}$ Re $(z_{1}+z_{2}) = x_{1}x_{2} = 0$ Re $(z_{1}+z_{2}) = x_{1} + x_{2} = 0$ (3) $f(x) = (x+1)^{2}$ f		$X + 1 > 0 \Rightarrow X > -1$ $y + 1 \neq 1 \Rightarrow y \neq 0 \text{ and } y > 0$		(1) B and D (2) B and C				
below matrix $x^2 - 2x - 3 \neq 0$ $(x - 3) (x + 1) \neq 0$ $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \frac{1}{1}$ $\int_{-1}^{1} \frac{1}{1}$ $\int_{-1}^{2} \frac{1}{1}$ $f(x) = \frac{(x+1)^2}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$		$x + 1 \neq 1 \implies x \neq 0$ and $x > 0$ Denominator		(3) A and B (4) A and C				
$f(x) = \frac{(x + 1)^2}{x^2 + 1}$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$ Then (1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ but not in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{-1}^$		$x^2 - 2x - 3 \neq 0$		Official An <mark>s. by</mark> NTA (2)				
Sol. $x \neq -1, 3$ So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$ Then (1) $f(x)$ is many-one in $(-\infty, -1)$ (2) $f(x)$ is many-one in $(1, \infty)$ (3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) $f(x)$ is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{$		$(x-3)(x+1) \neq 0$		Ans. (2)				
So Ans $(2, \infty) - \{3\}$ 62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) $f(x)$ is many-one in $(-\infty, -1)$ (2) $f(x)$ is many-one in $(1, \infty)$ (3) $f(x)$ is one-one in $(1, \infty)$ but not in $(-\infty, \infty)$ (4) $f(x)$ is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{-1}$		$x \neq -1, 3$						
62. Let $f: \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) $f(x)$ is many-one in $(-\infty, -1)$ (2) $f(x)$ is many-one in $(1, \infty)$ (3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) $f(x)$ is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{$		So Ans $(2, \infty) - \{3\}$	Sol.	$\mathbf{z}_1 = \mathbf{x}_1 + \mathbf{i}\mathbf{y}_1$				
$f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$. Then (1) $f(x)$ is many-one in $(-\infty, -1)$ (2) $f(x)$ is many-one in $(1, \infty)$ (3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) $f(x)$ is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{1}^{$	62.	Let $f: R \to R$ be a function such that		$z_{1} = x_{2} + iy_{2}$				
$r(x) = \frac{x^2 + 1}{x^2 + 1} = 1 + \frac{2x}{x^2 + 1}$ $Re(z_1 z_2) = x_1 x_2 - y_1 y_2 = 0$ $Re(z_1 + z_2) = x_1 + x_2 = 0$ $Re(z_1 + z_2) = x_1 + x_2 = 0$ $Re(z_1 + z_2) = x_1 + x_2 = 0$ $Re(z_1 + z_2) = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_2 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_1 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_2 = x_1 + x_2 = 0$ $r_$		$f(x) = \frac{x^2 + 2x + 1}{2x + 1}$ Then						
(1) f(x) is many-one in $(-\infty, -1)$ (2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $(1, \infty)$ but not in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_$		$x^{2} + 1$		$Re(z_1z_2) = x_1x_2 - y_1y_2 = 0$				
(2) f(x) is many-one in $(1, \infty)$ (3) f(x) is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{1}^{1} \int_{1}^{1}$		(1) $f(x)$ is many-one in $(-\infty, -1)$						
(3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$ (4) $f(x)$ is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{1}^{1$		(2) $f(x)$ is many-one in $(1, \infty)$		$\operatorname{Re}(z_1 + z_2) = x_1 + x_2 = 0$				
(4) f(x) is one-one in $(-\infty, \infty)$ Official Ans. by NTA (3) Ans. (3) Sol. $\int_{-1}^{2} \int_{-1}^{1} \int_{1}^{1} \int_{1}^$		(3) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$		x & x are of opposite sign				
Sol. $f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ $y_1 \& y_2 \text{ are of opposite sign}$ $f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ $y_1 \& y_2 \text{ are of opposite sign}$ $y_1 \& y_2 \text{ are of opposite sign}$ $f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ $(1) 7x^2 + 245x - 250 = 0$ $(2) 7x^2 - 245x + 250 = 0$ $(3) 49x^2 - 245x + 250 = 0$		(4) $f(x)$ is one-one in $(-\infty, \infty)$		$x_1 \propto x_2$ are of opposite sign				
Sol. $f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ $64. \text{Let } \lambda \neq 0 \text{ be a real number. Let } \alpha, \beta \text{ be the roots of the equation } 14x^2 - 31x + 3\lambda = 0 \text{ and } \alpha, \gamma \text{ be the roots of the equation } 35x^2 - 53x + 4\lambda = 0. \text{ Then } \frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation :} (1) 7x^2 + 245x - 250 = 0 (2) 7x^2 - 245x + 250 = 0 (3) 49x^2 - 245x + 250 = 0$		Official Ans. by NTA (3) Ans. (3)		$y_1 \& y_2$ are of opposite sign				
Sol. $f(x) = \frac{(x+1)^2}{r^2+1} = 1 + \frac{2x}{r^2+1}$ $64. \text{Let } \lambda \neq 0 \text{ be a real number. Let } \alpha, \beta \text{ be the roots of the equation } 14x^2 - 31x + 3\lambda = 0 \text{ and } \alpha, \gamma \text{ be the roots of the equation } 35x^2 - 53x + 4\lambda = 0. \text{ Then } \frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation :} (1) 7x^2 + 245x - 250 = 0 (2) 7x^2 - 245x + 250 = 0 (3) 49x^2 - 245x + 250 = 0$		Ans. (3)						
of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation :}$ $(1) 7x^2 + 245x - 250 = 0$ $(2) 7x^2 - 245x + 250 = 0$ $(3) 49x^2 - 245x + 250 = 0$	Sol.		64.	Let $\lambda \neq 0$ be a real number. Let α , β be the roots				
$\frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation } 35x^2 - 53x + 4\lambda = 0. \text{ Then}$ $\frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation :}$ $(1) 7x^2 + 245x - 250 = 0$ $(2) 7x^2 - 245x + 250 = 0$ $(3) 49x^2 - 245x + 250 = 0$				of the equation $14x^2 - 3lx + 3\lambda = 0$ and α , γ be the				
$\frac{3\alpha}{\beta} \text{ and } \frac{4\alpha}{\gamma} \text{ are the roots of the equation :}$ $f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ $(1) 7x^2 + 245x - 250 = 0$ $(2) 7x^2 - 245x + 250 = 0$ $(3) 49x^2 - 245x + 250 = 0$				roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then				
$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$		-1 1		3α and 4α are the roots of the equation i				
$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ (1) $7x^2 + 245x - 250 = 0$ (2) $7x^2 - 245x + 250 = 0$ (3) $49x^2 - 245x + 250 = 0$				$\frac{\beta}{\beta} = \frac{\beta}{\gamma}$ are the roots of the equation .				
$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$ (2) $7x^2 - 245x + 250 = 0$ (3) $49x^2 - 245x + 250 = 0$				$(1) 7x^2 + 245x - 250 = 0$				
$f(x) = \frac{(x+1)}{x^2+1} = 1 + \frac{2x}{x^2+1} $ (3) $49x^2 - 245x + 250 = 0$		$(r+1)^2$ 2r		$(2) 7x^2 - 245x + 250 = 0$				
		$f(x) = \frac{(x+1)}{x^2+1} = 1 + \frac{2x}{x^2+1}$		$(3) 49x^2 - 245x + 250 = 0$				
$(4) 49x^2 + 245x + 250 = 0$				$(4) 49x^2 + 245x + 250 = 0$				
$f(x) = 1 + \frac{1}{x +$		$f(x) = 1 + \frac{1}{x + 1}$		Official Ans. by NTA (3)				
$x + -\frac{x}{x}$ Ans. (3)		$\begin{array}{c} x + - \\ x \end{array}$		Ans. (3)				

<mark>∛Saral</mark>

Final JEE-Main Exam January, 2023/29-01-2023/Morning Session

Sol.	$14x^2 - 31x + 3\lambda = 0$	6
	$\alpha + \beta = \frac{31}{14}$ (1) and $\alpha\beta = \frac{3\lambda}{14}$ (2)	
	$35x^2 - 53x + 4\lambda = 0$	
	$\alpha + \gamma = \frac{53}{35} \dots (3) \text{ and } \alpha \gamma = \frac{4\lambda}{35} \dots (4)$	
	$\frac{(2)}{(4)} \Rightarrow \frac{\beta}{\gamma} = \frac{3 \times 35}{4 \times 14} = \frac{15}{8} \Rightarrow \beta = \frac{15}{8}\gamma$	
	$(1)-(3) \Rightarrow \beta - \gamma = \frac{31}{14} - \frac{53}{35} = \frac{155 - 106}{70} = \frac{7}{10}$	
	$\frac{15}{8}\gamma - \gamma = \frac{7}{10} \Longrightarrow \gamma = \frac{4}{5}$	
	$\Rightarrow \beta = \frac{15}{8} \times \frac{4}{5} = \frac{3}{2}$	s
	$\Rightarrow \alpha = \frac{31}{14} - \beta = \frac{31}{14} - \frac{3}{2} = \frac{5}{7}$	
	$\Rightarrow \lambda = \frac{14}{3} \alpha \beta = \frac{14}{3} \times \frac{5}{7} \times \frac{3}{2} = 5$	
	so, sum of roots $\frac{3\alpha}{\beta} + \frac{4\alpha}{\gamma} = \left(\frac{3\alpha\gamma + 4\alpha\beta}{\beta\gamma}\right)$	
	$=\frac{\left(3\times\frac{4\lambda}{35}+4\times\frac{3\lambda}{14}\right)}{\beta\gamma}=\frac{12\lambda(14+35)}{14\times35\beta\gamma}$	
	$=\frac{49\times12\times5}{490\times\frac{3}{2}\times\frac{4}{5}}=5$	
	Product of roots	6

$$=\frac{3\alpha}{\beta}\times\frac{4\alpha}{\gamma}=\frac{12\alpha^2}{\beta\gamma}=\frac{12\times\frac{25}{49}}{\frac{3}{2}\times\frac{4}{5}}=\frac{250}{49}$$

So, required equation is $x^2 - 5x + \frac{250}{49} = 0$

$$\Rightarrow 49x^2 - 245x + 250 = 0$$

Consider the following system of questions 5. $\alpha x + 2y + z = 1$ $2\alpha x + 3y + z = 1$ $3x + \alpha y + 2z = \beta$ For some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct. (1) It has no solution if $\alpha = -1$ and $\beta \neq 2$ (2) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$ (3) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$ (4) It has a solution for all $\alpha \neq -1$ and $\beta = 2$ Official Ans. by NTA (2) Ans. (2) Sol. $D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1,3$ $D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Longrightarrow \beta = 2$ $D_{y} = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$ $D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$ $\beta = 2, \alpha = -1$ $\alpha = -1, \beta = 2$ Infinite solution

66. Let α and β be real numbers. Consider a 3 × 3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

(3)
$$\beta = 8$$
 (4) $\beta = -8$

Official Ans. by NTA (4) Ans. (4) Sol. $A^{2} = 3A + \alpha I$ $A^{3} = 3A^{2} + \alpha A$ $A^{3} = 3(3A + \alpha I) + \alpha A$ $A^{3} = 9A + \alpha A + 3\alpha I$ $A^{4} = (9 + \alpha)A^{2} + 3\alpha A$ $= (9 + \alpha)(3A + \alpha I) + 3\alpha A$ $= A(27 + 6\alpha) + \alpha(9 + \alpha)$ $\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$ $\Rightarrow \beta = \alpha(9 + \alpha) = -8$

67. Let x = 2 be a root of the equation $x^2 + px + q = 0$ $(1 - cos(x^2 - 4px + q^2 + 8q + 16))$

and
$$f(x) = \begin{cases} \frac{1-\cos(x-4px+q^2+6q+16)}{(x-2p)^4}, & x \neq 2p \\ 0 & , & x = 2p \end{cases}$$

Then
$$\lim_{x \to 2^{n^+}} [f(x)]$$

where [.] denotes greatest integer function, is (1) 2 (2) 1 (3) 0 (4) -1

Official Ans. by NTA (3)

Ans. (3)

Sol.

$$\lim_{x \to 2p^{+}} \left(\frac{1 - \cos(x^{2} - 4px + q^{2} + 8q + 16)}{(x^{2} - 4px + q^{2} + 8q + 16)^{2}} \right) \left(\frac{(x^{2} - 4px + q^{2} + 8q + 16)^{2}}{(x - 2p)^{2}} \right)$$
$$\lim_{h \to 0} \frac{1}{2} \left(\frac{(2p + h)^{2} - 4p(2p + h) + q^{2} + 82 + 16}{h^{2}} \right)^{2} = \frac{1}{2}$$

Using L'Hospital's

 $\lim_{x\to 2p^+} [f(x)] = 0$

68. L

Let
$$f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x$$
,
 $x \in \mathbb{R}$ be a function which satisfies
 $f(x) = x + \int_{0}^{\pi/2} \sin(x + y) f(y) dy$. Then (a + b)

is equal to

(1) $-\pi(\pi+2)$ (2) $-2\pi(\pi+2)$ (3) $-2\pi(\pi-2)$ (4) $-\pi(\pi-2)$

Official Ans. by NTA (2)

Ans. (2)

Sol.
$$f(x) = x + \int_{0}^{\pi/2} (\sin x \cos y + \cos x \sin y) f(y) dy$$

 $f(x) = x + \int_{0}^{\pi/2} ((\cos y f(y) dy) \sin x + (\sin y f(y) dy) \cos x) \dots (1)$

∛Saral

On comparing with

$$F(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x, \ x \in \mathbb{R} \text{ then}$$
$$\Rightarrow \frac{a}{\pi^2 - 4} = \int_0^{\pi/2} \cos y f(y) dy \qquad \dots (2)$$
$$\Rightarrow \frac{b}{\pi^2 - 4} = \int_0^{\pi/2} \sin y f(y) dy \qquad \dots (3)$$

Add (2) and (3)

$$\frac{a+b}{\pi^2-4} = \int_0^{\pi/2} (\sin y + \cos y) f(y) dy \dots (4)$$

$$\frac{a+b}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) f\left(\frac{\pi}{2} - y\right) dy \dots (5)$$

Add (4) and (5)

$$\frac{2(a+b)}{\pi^2 - 4} = \int_{0}^{\pi/2} (\sin y + \cos y) \left(\frac{\pi}{2} + \frac{(a+b)}{\pi^2 - 4} (\sin y + \cos y)\right) dy$$

$$= \pi + \frac{a+b}{\pi^2 - 4} \left(\frac{\pi}{2} + 1\right)$$

$$(a+b) = -2\pi (\pi + 2)$$

69. Let
$$A = \{(x, y) \in \mathbb{R}^2 : y \ge 0, 2x \le y \le \sqrt{4 - (x - 1)^2} \}$$

and
$$\mathbf{B} = \left\{ (\mathbf{x}, \mathbf{y}) \in \mathbb{R} \times \mathbb{R} : 0 \le \mathbf{y} \le \min\left\{ 2\mathbf{x}, \sqrt{4 - (\mathbf{x} - 1)^2} \right\} \right\}$$

Then the ratio of the area of A to the area of B is

(1)
$$\frac{\pi - 1}{\pi + 1}$$
 (2) $\frac{\pi}{\pi - 1}$

(3)
$$\frac{\pi}{\pi + 1}$$
 (4) $\frac{\pi + 1}{\pi - 1}$

Official Ans. by NTA (1)

Ans. (1)

Sol. $y^2 + (x-1)^2 = 4$



shaded portion = circular (OABC)

 $-Ar(\Delta OAB)$

$$=\frac{\pi(4)}{4}-\frac{1}{2}(2)(1)$$

$$\mathbf{A} = (\pi - 1)$$



Area B = Ar (ΔAOB) + Area of arc of circle (ABC)

$$=\frac{1}{2}(1)(2) + \frac{\pi(2)^2}{4} = \pi + 1$$
$$\frac{A}{B} = \frac{\pi - 1}{\pi + 1}$$

70. Let Δ be the area of the region

$$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \le 21, y^2 \le 4x, x \ge 1\}.$$
 Then

$$\frac{1}{2} \left(\Delta - 21 \sin^{-1} \frac{2}{\sqrt{7}} \right) \text{ is equal to}$$
(1) $2\sqrt{3} - \frac{1}{3}$ (2) $\sqrt{3} - \frac{2}{3}$
(3) $2\sqrt{3} - \frac{2}{3}$ (4) $\sqrt{3} - \frac{4}{3}$

Official Ans. by NTA (4)

Ans. (4)



71. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line x + y = 1, if this ray intersects x-axis at Q, then the abscissa of Q is

(1)
$$\frac{2}{(\sqrt{3}-1)}$$
 (2) $\frac{2}{3+\sqrt{3}}$

(3)
$$\frac{2}{3-\sqrt{3}}$$
 (4) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$

Official Ans. by NTA (2)

Ans. (2)

Sol. Suppose of reflected ray = $\tan 60^\circ = \sqrt{3}$

Line
$$y = \frac{x}{\sqrt{3}}$$
 intersect $y + x = 1$ at $\left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$

Equation of reflected ray is

$$y - \frac{1}{\sqrt{3} + 1} = \sqrt{3} \left(x - \frac{\sqrt{3}}{\sqrt{3} + 1} \right)$$

Put
$$y = 0 \Rightarrow x = \frac{2}{3 + \sqrt{3}}$$

72. Let B and C be the two points on the line y + x = 0such that B and C are symmetric with respect to the origin. Suppose A is a point on y - 2x = 2 such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is

(1)
$$3\sqrt{3}$$
 (2) $2\sqrt{3}$
(3) $\frac{8}{\sqrt{3}}$ (4) $\frac{10}{\sqrt{3}}$

Official Ans. by NTA (3)

Sol.

$$y = 2$$

Ans. (3)

 $y = y = 0$

 $y = 0$

 y

At A x = y

- Y 2x = 2
- (-2, -2)

Height from line x + y = 0

$$h = \frac{4}{\sqrt{2}}$$

Area of
$$\Delta = \frac{\sqrt{3}}{4} \frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$$

73. Let the tangents at the points A (4, -11) and B(8, -5)on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle, whose centre is C and the line joining A and B is its tangent, is equal to

(1)
$$\frac{3\sqrt{3}}{4}$$
 (2) $2\sqrt{13}$

(3)
$$\sqrt{13}$$
 (4) $\frac{2\sqrt{13}}{3}$

Ans. (4)

Sol. Equation of tangent at A (4, -11) on circle is

$$\Rightarrow 4x - 11y - 3\left(\frac{x+4}{2}\right) + 10\left(\frac{y-11}{2}\right) - 15 = 0$$

Saral

$$\Rightarrow 5x - 12y - 152 = 0 \dots (1)$$

Equation of tangent at B (8, -5) on circle is

$$\Rightarrow 8x - 5y - 3\left(\frac{x+8}{2}\right) + 10\left(\frac{y-5}{2}\right) - 15 = 0$$
$$\Rightarrow 13 x - 104 = 0 \Rightarrow x = 8$$
$$put \text{ in } (1) \Rightarrow y = \frac{28}{3}$$
$$r = \left|\frac{3.8 + \frac{2.28}{3} - 34}{\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$$

74. Let [x] denote the greatest integer $\leq x$. Consider the function $f(x) = \max \{x^2, 1+[x]\}$. Then the value of the integral $\int_{-1}^{2} f(x) dx$ is :

(1)
$$\frac{5+4\sqrt{2}}{3}$$
 (2) $\frac{8+4\sqrt{2}}{3}$
(3) $\frac{1+5\sqrt{2}}{3}$ (4) $\frac{4+5\sqrt{2}}{3}$
Official App. by NTA (1)

Official Ans. by NTA (1) Ans. (1)



*<u>Saral</u>

Final JEE-Main Exam January, 2023/29-01-2023/Morning Session

75.	If the vectors $\vec{a} = \lambda \hat{i} + \mu \hat{j} + 4k$, $b = -2\hat{i} + 4\hat{j} - 2k$	5
	and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the	e
	projection of \vec{a} on the vector \vec{b} is $\sqrt{54}$ units, the	n
	the sum of all possible values of $\lambda + \mu$ is equal to	
	(1) 0 (2) 6	
	(3) 24 (4) 18	
	Official Ans. by NTA (3)	
	Ans. (3)	
	$ \lambda \mu 4 $	
Sol.	$\begin{vmatrix} -2 & 4 & -2 \end{vmatrix} = 0$	
	2 3 1	
	$\lambda(10) - \mu(2) + 4(-14) = 0$	
	$10\lambda - 2\mu = 56$	
	$5\lambda - \mu = 28 \qquad \dots \dots (1)$	
	$\frac{\vec{a} \cdot \vec{b}}{\left \vec{b}\right } = \sqrt{54}$	
	$\frac{-2\lambda+4\mu-8}{\sqrt{24}} = \sqrt{54}$	
	$-2\lambda + 4\mu - 8 = \sqrt{54 \times 24} \qquad \dots (2)$	
	By solving equation (1) & (2)	
	$\Rightarrow \lambda + \mu = 24$	

76. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

(1) $\frac{5}{24}$	(2)	$\frac{2}{15}$
(3) $\frac{1}{6}$	(4)	$\frac{5}{36}$

Official Ans. by NTA (DROP)

Sol.

Required probability =1-
$$\frac{D_{(15)} + {}^{15} C_{1} \cdot D_{(14)} + {}^{15} C_{2} D_{(13)}}{15!}$$

Taking D₍₁₅₎ as $\frac{15!}{e}$
D₍₁₄₎ as $\frac{14!}{e}$
D₍₁₃₎ as $\frac{13!}{e}$
We get, 1- $\left(\frac{\frac{15!}{e} + 15 \cdot \frac{14!}{e} + \frac{15 \times 14}{2} \times \frac{13!}{e}}{15!}\right)$

$$=1 - \left(\frac{1}{e} + \frac{1}{e} + \frac{1}{2e}\right) = 1 - \frac{5}{2e} \approx .08$$

77. Let
$$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4\left(3\pi + \theta\right)\right) - 2\left(1 - \sin^2 2\theta\right)$$
 and

$$S = \left\{ \theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2} \right\}. \text{ If } 4\beta = \sum_{\theta \in S} \theta,$$

then $f(\beta)$ is equal to

(1)
$$\frac{11}{8}$$
 (2) $\frac{5}{4}$
(3) $\frac{9}{8}$ (4) $\frac{3}{2}$

Official Ans. by NTA (2)

Sol.

$$f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3x + \theta)\right) - 2(1 - \sin^2 2\theta)$$
$$S = \left\{\theta \in [0, \pi]: f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$$
$$\Rightarrow f(\theta) = 3(\cos^4\theta + \sin^4\theta) - 2\cos^2 2\theta$$

$$\Rightarrow f(\theta) = 3\left(1 - \frac{1}{2}\sin^2 2\theta\right) - 2\cos^2 2\theta$$

$$\Rightarrow$$
 f(θ) = 3 - $\frac{3}{2}$ sin² 2 θ - 2cos² θ

$$= \frac{3}{2} - \frac{1}{2}\cos^{2}2\theta = \frac{3}{2} - \frac{1}{2}\left(\frac{1 + \cos 4\theta}{2}\right)$$

$$f(\theta) = \frac{5}{4} - \frac{\cos 4\theta}{4}$$

$$f'(\theta) = \sin 4\theta$$

$$\Rightarrow$$
 f'(θ) = sin 4 θ = $-\frac{\sqrt{3}}{2}$

$$\Rightarrow 4\theta = n\pi + (-1)^n \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4} + (-1)^n \frac{\pi}{12}$$

<mark>∛Saral</mark>

$$\Rightarrow \theta = \frac{\pi}{12}, \left(\frac{\pi}{4} - \frac{\pi}{12}\right), \left(\frac{\pi}{2} + \frac{\pi}{12}\right), \left(\frac{3\pi}{4} - \frac{\pi}{12}\right)$$
$$\Rightarrow 4\beta = \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4} = \frac{3\pi}{2}$$
$$\Rightarrow \beta = \frac{3\pi}{8} \Rightarrow f(\beta) = \frac{5}{4} - \frac{\cos\frac{3\pi}{2}}{4} = \frac{5}{4}$$

78. If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \lor q) \land ((\sim p) \lor r)\} \rightarrow ((\sim q) \lor r)$ false ? (1) p = T, q = F, r = T (2) p = T, q = T, r = F (3) p = F, q = T, r = F (4) p = T, q = F, r = F Official Ans. by NTA (3)

Sol.

	p	q	r	$(p \lor q) \land ((\sim p) \lor r)$	$\sim q \lor r$
(1)	Т	F	Т	Т	Т
(2)	Т	Т	F	F	F
(3)	F	Т	F	Т	F
(4)	Т	F	F	F	Т
Option (3) $(p \lor q) \land (\neg q \lor r) \rightarrow (\neg p \lor r)$ will be					

False.

Ans. (3)

79. There rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then 10 ($\mu^2 + \sigma^2$) is equal to

(1) 20

(2) 250

- (3) 25
- (4) 30

Official Ans. by NTA (1)

Ans. (1)

Sol.

Х	P(x)	XP(X)	X ² P(X)
0	1/6	0	0
1	1/2	1/2	1/2
2	3/10	6/10	12/10
3	1/30	1/10	9/30
	6		

$$\sum x P(x) = \frac{6}{2} = \mu$$

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$

$$\sigma^2 + \mu^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$$

$$10(\sigma^2 + \mu^2) = 20$$
 Ans.

80. Let y = f(x) be the solution of the differential equation $y(x + 1) dx - x^2 dy = 0$, y(1) = e. Then $\lim_{x \to 0^+} f(x)$ is equal to

(1) 0
(2)
$$\frac{1}{e}$$

(3) e^2
(4) $\frac{1}{e^2}$

Official Ans. by NTA (1) Ans. (1)

Sol.
$$\frac{x+1}{x^2}dx = \frac{dy}{x}$$

$$\ln x - \frac{1}{x} = \ln y + c$$

(1, e)c = -2

$$\ln x - \frac{1}{x} = \ln y - 2$$

$$y = e^{\ln x} - \frac{1}{x} + 2$$

lim $e^{\ln x - 1} - \frac{1}{x} + 2$

$$\lim_{x \to 0^+} e - - + \frac{1}{x}$$
$$= e^{-\infty}$$
$$= 0$$

<mark>∛Saral</mark> Final JEE-Main Exam January, 2023/29-01-2023/Morning Session

So

SECTION-B

81. Let the co-ordinates of one vertex of $\triangle ABC$ be $A(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of \triangle ABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____.

Official Ans. by NTA (9)

Ans. (9)

Sol. A. $(O_1 2, \alpha)$

$$(-\alpha_1 1, -4)$$
 B C $(5i+2j+3k)$

$$\left|\frac{1}{2} \cdot 2\sqrt{21} \cdot \begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha + 4 \\ 5 & 2 & 3 \end{vmatrix} \left|\frac{1}{\sqrt{25 + 4 + 9}}\right| = 21\sqrt{21}$$

$$\sqrt{(2\alpha+5)^2 + (2\alpha+20)^2 + (2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$
$$\Rightarrow 12\alpha^2 + 80\alpha + 450 = 798$$
$$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$$
$$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$$

82. Let the equation of the plane P containing the line $x+10 = \frac{8-y}{2} = z$ be ax + by + 3z = 2(a+b) and the distance of the plane P from the point (1, 27, 7) be c. Then $a^2 + b^2 + c^2$ is equal to

Official Ans. by NTA (355)

Ans. (355)

Sol. The line
$$\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$$
 have a point (-10, 8, 0)
with d. r. (1, -2, 1)
 \therefore the plane ax + by + 3z = 2 (a + b)
 \Rightarrow b = 2a
& dot product of d.r.'s is zero
 \therefore a - 2b + 3 = 0

$$\therefore a = 1 \& b = 2$$

Distance from (1, 27, 7) is
 $c = \frac{1+54+21-6}{\sqrt{14}} = \frac{70}{\sqrt{14}} = 5\sqrt{14}$
 $\therefore a^2 + b^2 + c^2 = 1 + 4 + 350$
 $= 355$
83. Suppose f is a function satisfying $f(x + y) = f(x) + f(y)$
for all $x, y \in \mathbb{N}$ and $f(1) = \frac{1}{5}$. If
 $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to......
Official Ans. by NTA (10)
Ans. (10)
Sol. $\because f(1) = \frac{1}{5} \because f(2) = f(1) + f(1) = \frac{2}{5}$
 $f(2) = \frac{2}{5} f(3) = f(2) + f(1) = \frac{3}{5}$
 $f(3) = \frac{3}{5}$
 $\therefore \sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}$
 $= \frac{1}{5} \sum_{n=1}^{m} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$
 $= \frac{1}{5} \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{m+1} - \frac{1}{m+2}\right)$
 $= \frac{1}{5} \left(\frac{1}{2} - \frac{1}{m+2}\right) = \frac{m}{10(m+2)} = \frac{1}{12}$
 $\therefore m = 10$

84. Let a₁, a₂, a₃, be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to _____.

Official Ans. by NTA (60)

Ans. (60)

Sol.
$$a_4 \cdot a_6 = 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3$$

& $a_5 + a_7 = 24 \Rightarrow a_5 + a_5 r^2 = 24 \Rightarrow (1 + r^2) = 8 \Rightarrow r = \sqrt{7}$
 $\Rightarrow a = \frac{3}{49}$
 $\Rightarrow a_1 a_9 + a_2 a_4 a_9 + a_5 + a_7 = 9 + 27 + 3 + 21 = 60$

85. Let \vec{a} , \vec{b} and \vec{c} be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}$, $\lambda \vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If \overrightarrow{AB} , \overrightarrow{AC} and \overrightarrow{AD} are coplanar, then λ is : Official Ans. by NTA (2) Ans. (2) Sol. $\overrightarrow{AB} = (\lambda - 1)\vec{a} - 2\vec{b} + 3\vec{c}$ $\overrightarrow{AC} = 2\vec{a} + 3\vec{b} - 4\vec{c}$

$$AC = 2a + 3b - 4c$$

$$\overline{AD} = \overline{a} - 3\overline{b} + 5\overline{c}$$

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$$

$$\Rightarrow (\lambda - 1) = 1 \Rightarrow \lambda = 2$$

86. If all the six digit numbers $x_1 x_2 x_3 x_4 x_5 x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is

Official Ans. by NTA (32)



71 words

- $2 4 5 6 7 8 \rightarrow 72^{\text{th}} \text{ word}$ 2 + 4 + 5 + 6 + 7 + 8 = 32
- 87. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation f(x + y) = f(x) + f(y) - 1, $\forall x$, $y \in \mathbb{R}$. If f'(0) = 2, then |f(-2)| is equal to _____.

Official Ans. by NTA (3)

Ans. (3)

Sol.
$$f(x + y) = f(x) + f(y) - 1$$

 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
 $f'(x) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = f'(0) = 2$
 $f'(x) = 2 \Longrightarrow dy = 2dx$
 $y = 2x + C$
 $x = 0, y = 1, c = 1$
 $y = 2x + 1$
 $|f(-2)| = |-4+1| = |-3| = 3$

88. If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the

co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then

 $(\alpha\beta)^2$ is equal to _____.

Official Ans. by NTA (1)

Ans. (1)

Sol. Coefficient of
$$x^9$$
 in $\left(\alpha x^3 + \frac{1}{\beta x}\right) = {}^{11}C_6 \cdot \frac{\alpha^5}{\beta^6}$

: Both are equal

$$\therefore \frac{11}{C_6} \cdot \frac{\alpha^5}{\beta^6} = -\frac{11}{C_5} \cdot \frac{\alpha^6}{\beta^5}$$
$$\Rightarrow \frac{1}{\beta} = -\alpha$$
$$\Rightarrow \alpha\beta = -1$$
$$\Rightarrow (\alpha\beta)^2 = 1$$

89. Let the coefficients of three consecutive terms in the binomial expansion of (1 + 2x)ⁿ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is .

Official Ans. by NTA (1120)

Ans. (1120)



	Jului
Sol.	$\mathbf{t}_{r+1} = {}^{n}\mathbf{C}_{r}\left(2\mathbf{x}\right)^{r}$
	$\Rightarrow \frac{{}^{n}C_{r-1}(2)^{r-1}}{{}^{n}C_{r}(2)^{r}} = \frac{2}{5}$
	$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$
	$\Rightarrow \frac{r}{n-r+1} = \frac{4}{5} \Rightarrow 5r = 4n - 4r + 4$
	$\Rightarrow 9r = 4(n+1) \qquad \dots (1)$
	$\Rightarrow \frac{{}^{n}C_{r}(2)^{r}}{{}^{n}C_{r+1}(2)^{r+1}} = \frac{5}{8}$
	$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4} \Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$
	$\Rightarrow 4r + 4 = 5n - 5r \Rightarrow 5n - 4 = 9r \dots (2)$
	From (1) and (2)
	$\Rightarrow 4n + 4 = 5n - 4 \Rightarrow n = 8$
	$(1) \Rightarrow r = 4$
	so, coefficient of middle term is

$${}^{8}C_{4}2^{4} = 16 \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 16 \times 70 = 1120$$

90. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____.

Official Ans. by NTA (1436)

Ans. (1436)

Sol. No of 5 digit numbers starting with digit 1 $= 5 \times 5 \times 5 \times 5 = 625$ No of 5 digit numbers starting with digit 2 $= 5 \times 5 \times 5 \times 5 = 625$ No of 5 digit numbers starting with 31 $= 5 \times 5 \times 5 = 125$ No of 5 digit numbers starting with 32 $= 5 \times 5 \times 5 = 125$ No of 5 digit numbers starting with 33 $= 5 \times 5 \times 5 = 125$ No of 5 digit numbers starting with 351 $= 5 \times 5 = 25$ No of 5 digit numbers starting with 352 $= 5 \times 5 = 25$ No of 5 digit numbers starting with 3531 = 5 No of 5 digit numbers starting with 3532 = 5 Before 35337 will be 4 numbers, So rank of 35337 will be 1690

So, in descending order serial number will be 3125 - 1690 + 1 = 1436