FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Tuesday 24th January, 2023) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH ANSWER**

SECTION-A

61. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to

(1) 220

(2) 210

(3) 200

(4) 105

Official Ans. by NTA (2)

Ans. (2)

Sol. $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$\mathbf{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57}$

$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$

$\Rightarrow a_1 + a_6 = 19$

$\Rightarrow 2a_1 + 5d = 19$ and $a_1 + d = 5$

$\Rightarrow a_1 = 2, d = 3$

Numbers : 2, 5, 8, 11, 14, 17

Variance = σ^2 = mean of squares − square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$8\sigma^2 = 210$

62. Let f(x) be a function such that $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(1) 6

(2) 8

(3) 7

(4) 9

Official Ans. by NTA (3)

Ans. (3)

Sol. $f(x + y) = f(x) \cdot f(y) \ \forall x, y \in N,\ f(1) = 3$

$f(2) = f^2(1) = 3^2$

$f(3) = f(1)\, f(2) = 3^3$

$f(4) = 3^4$

$f(k) = 3^k$

$\sum_{k=1}^{n} f(k) = 3279$

$f(1) + f(2) + f(3) + \ldots\ldots\ldots + f(k) = 3279$

$3 + 3^2 + 3^3 + \ldots\ldots\ldots 3^k = 3279$

$\frac{3(3^k - 1)}{3 - 1} = 3279$

$\frac{3^k - 1}{2} = 1093$

$3^k - 1 = 2186$

$3^k = 2187$

$\boxed{k = 7}$

63. The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

(1) 4 (2) 0

(3) 3 (4) 2

Official Ans. by NTA (2)

Ans. (2)

Sol. $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

Let $x + \frac{1}{x} = t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$3t(t - 1) + 1(t - 1) = 0$

$(t - 1)(3t + 1) = 0$

$t = 1, -\frac{1}{3}$

$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow$ No solution.

64. If $f(x) = \frac{2^{2x}}{2^{2x}+2}$, $x \in R$,

then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+\ldots\ldots+f\left(\frac{2022}{2023}\right)$ is equal to

(1) 2011 (2) 1010

(3) 2010 (4) 1011

Official Ans. by NTA (4)

Ans. (4)

Sol. $f(x)=\frac{4^x}{4^x+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2(4^x)}$

$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}$

$= 1$

$\Rightarrow f(x) + f(1 – x) = 1$

Now $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+\ldots\ldots+$

$\ldots\ldots\ldots+f\left(1-\frac{3}{2023}\right)+f\left(1-\frac{2}{2023}\right)+f\left(1-\frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum = 1 + 1 + 1 + ……….+ 1 (1011 times)

= 1011

65. If $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3)$, $x \in R$, then

(1) $3f(1) + f(2) = f(3)$

(2) $f(3) – f(2) = f(1)$

(3) $2f(0) – f(1) + f(3) = f(2)$

(4) $f(1) + f(2) + f(3) = f(0)$

Official Ans. by NTA (3)

Ans. (3)

Sol. $f(x)=x^3-x^2f'(1)+xf''(2)-f'''(3),\ x\in R$

Let $f'(1)=a,\ f''(2)=b,\ f'''(3)=c$

$\mathbf{f(x) = x^3 – ax^2 + bx – c}$

$f'(x)=3x^2-2ax+b$

$f''(x)=6x-2a$

$f'''(x)=6$

$c = 6, a = 3, b = 6$

$f(x) = x^3 – 3x^2 + 6x – 6$

$f(1) = –2, f(2) = 2, f(3) = 12, f(0) = –6$

$2f(0) – f(1) + f(3) = 2 = f(2)$

66. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

(1) 120

(2) 168

(3) 220

(4) 48

Official Ans. by NTA (2)

Ans. (2)

Sol. Four digit numbers greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

Five digit number = 5! = 120

Total number greater than 7000

= 120 + 48 = 168

67. If the system of equations

$x + 2y + 3z = 3$

$4x + 3y – 4z = 4$

$8x + 4y – \lambda z = 9 + \mu$

has infinitely many solutions, then the ordered pair (λ, μ) is equal to

(1) $\left(\frac{72}{5},\frac{21}{5}\right)$ (2) $\left(\frac{-72}{5},\frac{-21}{5}\right)$

(3) $\left(\frac{72}{5},\frac{-21}{5}\right)$ (4) $\left(\frac{-72}{5},\frac{21}{5}\right)$

Official Ans. by NTA (3)

Ans. (3)

Sol. $x + 2y + 3z = 3$ …….(i)

$4x + 3y – 4z = 4$ …….(ii)

$8x + 4y – \lambda z = 9 + \mu$ …….(iii)

(i) × 4 – (ii) $\Rightarrow 5y + 16z = 8$ ……. (iv)

(ii) × 2 – (iii) $\Rightarrow 2y + (\lambda–8)z = –1–\mu$ ……(v)

(iv) × 2 – (iii) × 5 $\Rightarrow (32–5(\lambda–8))z = 16–5(–1–\mu)$

For infinite solutions $\Rightarrow 72 – 5\lambda = 0 \Rightarrow \lambda=\frac{72}{5}$

$21 + 5\mu = 0 \Rightarrow \mu=\frac{-21}{5}$

$\Rightarrow (\lambda,\mu)\equiv\left(\frac{72}{5},\frac{-21}{5}\right)$

68. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(1) $\frac{-1}{2}(1-i\sqrt{3})$

(2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{-1}{2}(\sqrt{3}-i)$

(4) $\frac{1}{2}(\sqrt{3}+i)$

Official Ans. by NTA (3)

Ans. (3)

Sol. Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3=z^3$$

$$\Rightarrow\left(i\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)\right)^3$$

$$=-i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow\frac{-1}{2}(\sqrt{3}-i).$$

69. The equations of the sides AB and AC of a triangle ABC are

$(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is

(1) $\sqrt{6}$

(2) $2\sqrt{2}$

(3) 2

(4) 4

Official Ans. by NTA (2)

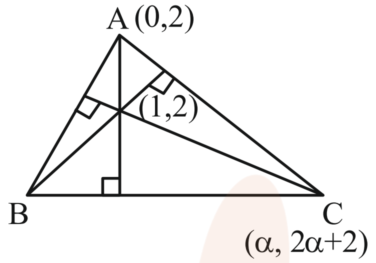
Ans. (2)

Sol. $AB:(\lambda+1)x+\lambda y=4$

$AC:\lambda x+(1-\lambda)y+\lambda=0$

Vertex A is on y-axis

$\Rightarrow x=0$



So $y=\frac{4}{\lambda},y=\frac{\lambda}{\lambda-1}$

$\Rightarrow\frac{4}{\lambda}=\frac{\lambda}{\lambda-1}$

$\Rightarrow\lambda=2$

AB : $3x+2y=4$

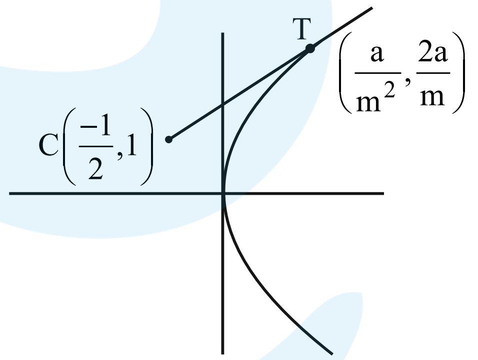
AC : $2x-y+2=0$

$\Rightarrow$ A(0,2) Let C $(\alpha, 2\alpha+2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right)=-1$

$\left(\frac{2\alpha}{\alpha-1}\right)\left(-\frac{3}{2}\right)=-1\ \Rightarrow\alpha=-\frac{1}{2}$

So $C\left(-\frac{1}{2},1\right)$



Let Equation of tangent be $y=mx+\frac{3}{2m}$

$m^2+2m-3=0$

$\Rightarrow m=1,-3$

So tangent which touches in first quadrant at T is

$T\equiv\left(\frac{a}{m^2},\frac{2a}{m}\right)$

$\equiv\left(\frac{3}{2},3\right)$

$\Rightarrow CT=\sqrt{4+4}=2\sqrt{2}$

70. The set of all values of a for which $\lim_{x\to a}([x-5]-[2x+2]) = 0$, where $[\alpha]$ denotes the greater integer less than or equal to α is equal to

(1) $(-7.5, -6.5)$ (2) $(-7.5, -6.5]$

(3) $[-7.5, -6.5]$ (4) $[-7.5, -6.5)$

Official Ans. by NTA (1)

Ans. (1)

Sol. $\lim_{x\to a}([x-5]-[2x+2])=0$

$\lim_{x\to a}([x]-5-[2x]-2)=0$

$\lim_{x\to a}([x]-[2x])=7$

$[a]-[2a]=7$

$a\in I,\quad a=-7$

$a\notin I,\quad a=I+f$

Now, $[a]-[2a]=7$

$-I-[2f]=7$

Case-I: $f\in\left(0,\frac{1}{2}\right)$

$2f\in(0,1)$

$-I=7$

$I=-7\Rightarrow a\in(-7,-6.5)$

Case-II: $f\in\left(\frac{1}{2},1\right)$

$2f\in(1,2)$

$-I-1=7$

$I=-8\Rightarrow a\in(-7.5,-7)$

Hence, $a\in(-7.5,-6.5)$

71. If $({}^{30}C_1)^2+2({}^{30}C_2)^2+3({}^{30}C_3)^2+\ldots\ldots+30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

(1) 30 (2) 60

(3) 15 (4) 10

Official Ans. by NTA (3)

Ans. (3)

Sol. $S=0.({}^{30}C_0)^2+1.({}^{30}C_1)^2+2.({}^{30}C_2)^2+\ldots\ldots+30.({}^{30}C_{30})^2$

$S=30.({}^{30}C_0)^2+29.({}^{30}C_1)^2+28.({}^{30}C_2)^2+\ldots\ldots+0.({}^{30}C_0)^2$

$2S=30.({}^{30}C_0{}^2+{}^{30}C_1{}^2+\ldots\ldots+{}^{30}C_{30}{}^2)$

$S=15.{}^{60}C_{30}=15\cdot\frac{60!}{(30!)^2}$

$\frac{15\cdot 10!}{(30!)^2}=\frac{\alpha\cdot 60!}{(30!)^2}$

$\Rightarrow \alpha=15$

72. Let the plane containing the line of intersection of the planes

P1: $x+(\lambda+4)y+z=1$ and

P2: $2x+y+z=2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is

(1) $5\sqrt{6}$

(2) $4\sqrt{6}$

(3) $2\sqrt{6}$

(4) $3\sqrt{6}$

Official Ans. by NTA (4)

Ans. (4)

Sol. Equation of plane passing through point of intersection of P1 and P2

$P=P1+kP2$

$(x+(\lambda+4)y+z-1)+k(2x+y+z-2)=0$

Passing through (0, 1, 0) and (1, 0, 1)

$(\lambda+4-1)+k(1-2)=0$

$(\lambda+3)-k=0$ ....(1)

Also passing (1, 0, 1)

$(1+1-1)+k(2+1-2)=0$

$1+k=0$

$k=-1$

put in (1)

$\lambda+3+1=0$

$\lambda=-4$

Then point $(2\lambda, \lambda, -\lambda)$

$(-8, -4, 4)$

$d=\left|\frac{-16-4+4-2}{\sqrt{6}}\right|$

$d=\frac{18}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=3\sqrt{6}$

73. Let $\vec{\alpha}=4\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{\beta}=\hat{i}+2\hat{j}-4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, then the value of $5\vec{\beta}_2\cdot(\hat{i}+\hat{j}+\hat{k})$ is

(1) 6

(2) 11

(3) 7

(4) 9

Official Ans. by NTA (3)

Ans. (3)

Sol. **Let** $\vec{\beta}_1 = \lambda\vec{\alpha}$

Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$

$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$

$\vec{\beta}_2.\vec{\alpha} = 0$

$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$

$\Rightarrow 4 - 16\alpha + 6 - 9\lambda - 25\lambda - 20 = 0$

$\Rightarrow 50\lambda = -10$

$\Rightarrow \boxed{\lambda = \frac{-1}{5}}$

$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$

$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$

$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

$5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

74. The locus of the mid points of the chords of the circle C_1: $(x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

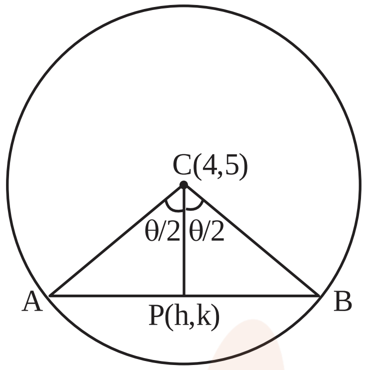
(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

Ans. (4)

Sol. **In ΔCPB**



$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$

$\Rightarrow (h - 4)^2 + (k - 5)^2 = 4\cos^2\frac{\theta}{2}$

Now $(x - 4)^2 + (y - 5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$

$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$r_2 = 2\cos\frac{\theta_2}{2}$

$r_3 = 2\cos\frac{\pi}{3} = 1$

$\Rightarrow r_1^2 = r_2^2 + r_3^2$

$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$

$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$

$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$

$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2}}$

75. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) –1

(2) 3

(3) 1

(4) 5

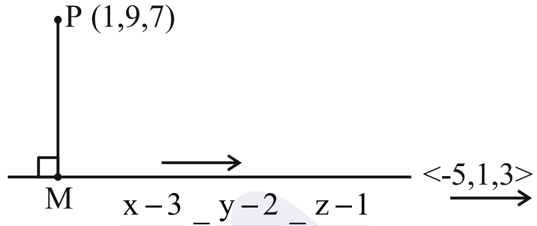
Official Ans. by NTA (4)

Ans. (4)

Sol. Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$



$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3}$$

$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

76. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy\, dy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $3e^2$

(2) e^2

(3) $2e^2$

(4) $\frac{3e^2}{2}$

Official Ans. by NTA (3)

Ans. (3)

Sol. $(x^2 - 3y^2)\, dx + 3xy\, dy = 0$

$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y}$ (1)

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$

$\Rightarrow v dv = \frac{-1}{3x}$

Integrating both side

$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$

$y(1) = 1$

$\Rightarrow \boxed{\frac{1}{2} = c}$

$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$

$\Rightarrow y^2 = -\frac{2}{3}x^2 \ln x + x^2$

$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$

$\Rightarrow \boxed{6y^2(e) = 2e^2}$

77. Let p and q be two statements.

Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge (\sim q))$

(2) $p \vee ((\sim p) \wedge q)$

(3) $(\sim p) \vee q$

(4) $p \vee (p \wedge q)$

Official Ans. by NTA (3)

Ans. (3)

Sol. $\sim (p \wedge (p \rightarrow \sim q))$

$\equiv \sim p \vee \sim(\sim p \vee \sim q)$

$\equiv \sim p \vee (p \wedge q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$

$\equiv t \wedge (\sim p \vee q)$

$\equiv \sim p \vee q$

78. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

(3) 150

(4) 125

Official Ans. by NTA (2)

Ans. (2)

Sol.
$$\begin{bmatrix} - - - - - - - - - - - - - - \\ - - - - - - - - - - - - - - \\ - - - - - - - - - - - - - - \\ - - - - - - - - - - - - - - \\ - - - - - - - - - - - - - - \end{bmatrix}$$

In each row and each column exactly one is to be placed –

$\therefore$ No. of such matrices = 5 × 4 × 3 × 2 × 1 = 120

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to 5 \text{ ways} \\ \to 4 \text{ ways} \\ \to 3 \text{ ways} \\ \to 2 \text{ ways} \\ \to 1 \text{ ways} \end{matrix}$$

Step-1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step-2 : From next now select 1 place for 1's. Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways = 5 × 4 × 3 × 2 × 1 = 120

79. $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$ is equal to

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$

(4) 2π

Official Ans. by NTA (4)

Ans. (4)

Sol. $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx$

We have $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

Hence $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}\,dx = \frac{48}{2} \times \left[\sin^{-1}\frac{2x}{3}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$$= 24 \times \left[\sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{3}}{4}\right) - \sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{2}}{4}\right)\right]$$

$$= 24 \times \left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= 24 \times \left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= 24 \times \frac{\pi}{12} = 2\pi$$

80. Let A be a 3 × 3 matrix such that $\left|\text{adj}\left(\text{adj}(\text{adj}A)\right)\right| = 12^4$. Then $|A^{-1}\text{adj } A|$ is equal to

(1) $2\sqrt{3}$

(2) $\sqrt{6}$

(3) 12

(4) 1

Official Ans. by NTA (1)

Ans. (1)

Sol. **Given $|\text{adj}(\text{adj}(\text{adj}.A))| = 12^4$**

$\Rightarrow |A|^{(n-1)^3} = 12^4$

Given n = 3

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A|^2 = 12$

$|A| = 2\sqrt{3}$

We are asked

$|A^{-1}.\text{adj}A|$

$= |A^{-1}|.|\text{adj } A|$

$= \frac{1}{|A|}.|A|^{3-1}$

$= |A| = 2\sqrt{3}$

81. The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is

Official Ans. by NTA (432)

Ans. (432)

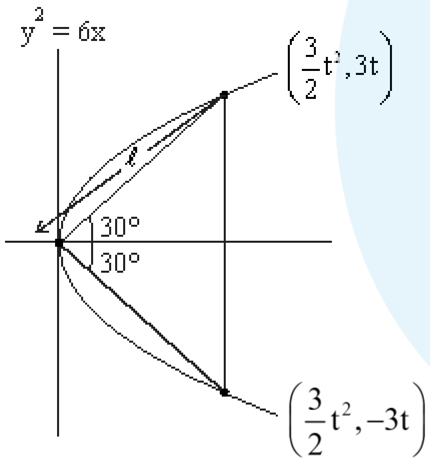
Sol.

Urn A		Urn B		Urn C	
Red	Black	Red	Black	Red	Black
4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$\Rightarrow \lambda = 6$



$$\tan 30° = 3t = \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$$

$$\ell^2 = 18^2 + (6\sqrt{3})^2$$

$= 324 + 108$

$= 432$

82. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8A is equal to

Official Ans. by NTA (36)

Ans. (36)

Sol. $y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

$(y-1)^2 = -(x-1)$

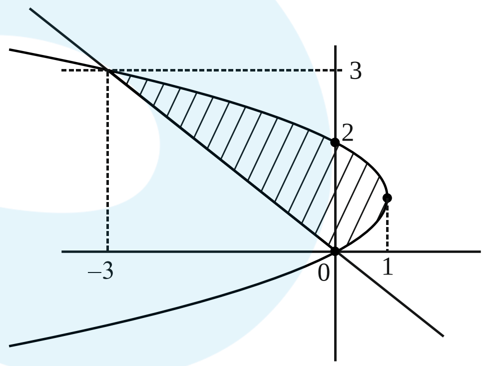
$y = -x$

Points of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$

$x = 0, -3$



$$A = \int_0^3 (-y^2 + 2y + y)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$$

$8A = 36$

83. If $\frac{1^3 + 2^3 + 3^3 + \ldots\ldots \text{upto n terms}}{1\cdot 3 + 2\cdot 5 + 3\cdot 7 + \ldots\ldots \text{upto n terms}} = \frac{9}{5}$, then the value of n is

Official Ans. by NTA (5)

Ans. (5)

Sol. $1^3 + 2^3 + 3^3 ..... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + ... + n$ terms $=$

$\sum_{r=1}^{n} r(2r+1) = \sum_{r=1}^{n}\left(2r^2 + r\right)$

$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{6}(2(2n+1)+3)$

$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$

$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2} \times \frac{(4n+5)}{3}} = \frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$

$\Rightarrow 15n(n+1) = 18(4n+5)$

$\Rightarrow 15n^2 + 15n = 72n + 90$

$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (n-5)(5n+6) = 0$

$\Rightarrow n = \frac{-6}{5}$ or 5

$\Rightarrow n = 5.$

84. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and

$f(x) + \int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\, dt = e,\ \forall x \in \left[0, \frac{\pi}{2}\right].$

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _______.

Official Ans. by NTA (27)

Ans. (27)

Sol. $f(x) + \int_0^x f(t)\sqrt{1-\left(\log_e f(t)\right)^2}\, dt = e$

$\Rightarrow f(0) = e$

$f'(x) + f(x)\sqrt{1-\left(\ln f(x)\right)^2} = 0$

$f(x) = y$

$\frac{dy}{dx} = -y\sqrt{1-(\ln y)^2}$

$\int \frac{dy}{y\sqrt{1-(\ln y)^2}} = -\int dx$

Put $\ln y = t$

$\int \frac{dt}{\sqrt{1-t^2}} = -x + C$

$\sin^{-1} t = -x + C \Rightarrow \sin^{-1}(\ln y) = -x + C$

$\sin^{-1}\left(\ln f(x)\right) = -x + C$

$f(0) = e$

$\Rightarrow \frac{\pi}{2} = C$

$\Rightarrow \sin^{-1}\left(\ln f(x)\right) = -x + \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{-\pi}{6} + \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{3}$

$\Rightarrow \ln f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, we need $\left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27.$

85. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is __________.

Official Ans. by NTA (13)

Ans. (13)

Sol. Given R = {(a, b), (b, c), (b, d)}

In order to make it equivalence relation as per given set, R must be

{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}

There already given so 13 more to be added.

86. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}$, $\vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}$, $\vec{a}\cdot\vec{c}=7$, $2\vec{b}\cdot\vec{c}+43=0$, $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$. Then $|\vec{a}\cdot\vec{b}|$ is equal to

Official Ans. by NTA (8)

Ans. (8)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k},\ \vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k},\ \vec{a}\cdot\vec{c}=7$

$\vec{a}\times\vec{c}-\vec{b}\times\vec{c}=\vec{0},$

$(\vec{a}-\vec{b})\times\vec{c}=0\Rightarrow(\vec{a}-\vec{b})$ is paralleled to $\vec{c}$

$\vec{a}-\vec{b}=\mu\vec{c}$, where μ is a scalar

$-2\hat{i}+7\hat{j}+2\lambda\hat{k}=\mu\cdot\vec{c}$

Now $\vec{a}\cdot\vec{c}=7$ gives $2\lambda^2+12=7\mu$

And $\vec{b}\cdot\vec{c}=-\dfrac{43}{2}$ gives $4\lambda^2+82=43\mu$

$\mu=2$ and $\lambda^2=1$

$|\vec{a}\cdot\vec{b}|=8$

87. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\dfrac{3}{x^2}\right)^n$, $x\neq0$, $n\in N$, be 376. Then the coefficient of x^4 is ________.

Official Ans. by NTA (405)

Ans. (405)

Sol. Given Binomial $\left(x-\dfrac{3}{x^2}\right)^n, x\neq0, n\in N,$

Sum of coefficients of first three terms

${}^nC_0-{}^nC_1\cdot3+{}^nC_2 3^2=376$

$\Rightarrow 3n^2-5n-250=0$

$\Rightarrow (n-10)(3n+25)=0$

$\Rightarrow n=10$

Now general term ${}^{10}C_r x^{10-r}\left(\dfrac{-3}{x^2}\right)^r$

$={}^{10}C_r x^{10-r}(-3)^r\cdot x^{-2r}$

$={}^{10}C_r(-3)^r\cdot x^{10-3r}$

Coefficient of x^4 $\Rightarrow 10-3r=4$

$\Rightarrow r=2$

${}^{10}C_2(-3)^2=405$

88. If the shortest between the lines

$\dfrac{x+\sqrt6}{2}=\dfrac{y-\sqrt6}{3}=\dfrac{z-\sqrt6}{4}$ and

$\dfrac{x-\lambda}{3}=\dfrac{y-2\sqrt6}{4}=\dfrac{z+2\sqrt6}{5}$ is 6, then the square of sum of all possible values of λ is

Official Ans. by NTA (384)

Ans. (384)

Sol. Shortest distance between the lines

$\dfrac{x+\sqrt6}{2}=\dfrac{y-\sqrt6}{3}=\dfrac{z-\sqrt6}{4}$

$\dfrac{x-\lambda}{3}=\dfrac{y-2\sqrt6}{4}=\dfrac{2+2\sqrt6}{5}$ is 6

Vector along line of shortest distance

$=\begin{vmatrix} i & j & k\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}, \Rightarrow -\hat{i}+2\hat{j}-k$ (its magnitude is $\sqrt6$)

Now $\dfrac{1}{\sqrt6}\begin{vmatrix}\sqrt6+\lambda & \sqrt6 & -3\sqrt6\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=\pm6$

$\Rightarrow \lambda=-2\sqrt6, 10\sqrt6$

So, square of sum of these values is 384.

89. Let S = $\{\theta\in[0,2\pi) : \tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0\}$.

Then $\sum_{\theta\in S}\sin^2\left(\theta+\dfrac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (2)

Ans. (2)

Sol. $\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0$

$\tan(\pi\cos\theta)=-\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta)=\tan(-\pi\sin\theta)$

$\pi\cos\theta=n\pi-\pi\sin\theta$

$\sin\theta+\cos\theta=n$ where $n\in I$

possible values are n = 0, 1 and –1 because

$-\sqrt2\le\sin\theta+\cos\theta\le\sqrt2$

Now it gives $\theta\in\left\{0,\dfrac{\pi}{2},\dfrac{3\pi}{4},\dfrac{7\pi}{4},\dfrac{3\pi}{2},\pi\right\}$

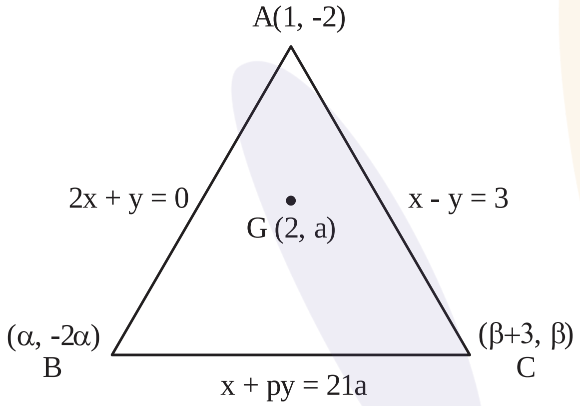
So $\sum_{\theta\in S}\sin^2\left(\theta+\dfrac{\pi}{4}\right)=2(0)+4\left(\dfrac12\right)=2$

90. The equations of the sides AB, BC and CA of a triangle ABC are: $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P (2, a) be the centroid of ΔABC. Then $(BC)^2$ is equal to

Official Ans. by NTA (122)

Ans. (122)

Sol.



Assume $B(\alpha, -2\alpha)$ and $C(\beta + 3, \beta)$

$$\frac{\alpha+\beta+3+1}{3} = 2 \qquad \text{also } \frac{-2\alpha-2+\beta}{3} = a$$

$\Rightarrow \alpha + \beta = 2 \qquad -2\alpha - 2 + \beta = 3a$

$\Rightarrow \beta = 2 - \alpha \qquad -2\alpha - \cancel{2} + \cancel{2} - \alpha = 3a \Rightarrow \alpha = -a$

Now both B and C lies as given line

$\alpha - p \cdot 2\alpha = 21a$

$\alpha(1-2p) = 21a$ ..... (1)

$-\alpha(1-2p) = 21a \Rightarrow p = 11$

$\beta + 3 + p\beta = 21a$

$\beta + 3 + 11\beta = 21a$

$21\alpha + 12\beta + 3 = 0$

Also $\beta = 2 - \alpha$

$21\alpha + 12(2-\alpha) + 3 = 0$

$21\alpha + 24 - 12\alpha + 3 = 0$

$9\alpha + 27 = 0$

$\alpha = -3,\ \beta = 5$

So $BC = \sqrt{122}$ and $(BC)^2 = 122$