

**FINAL JEE-MAIN EXAMINATION – JUNE, 2022**

**(Held On Monday 27<sup>th</sup> June, 2022)**

**TIME : 9 : 00 AM to 12 : 00 PM**

**MATHEMATICS**

**TEST PAPER WITH SOLUTION**

**SECTION-A**

1. The area of the polygon, whose vertices are the non-real roots of the equation  $\bar{z} = iz^2$  is :

(A)  $\frac{3\sqrt{3}}{4}$                       (B)  $\frac{3\sqrt{3}}{2}$

(C)  $\frac{3}{2}$                               (D)  $\frac{3}{4}$

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $\Rightarrow$  Let  $z = x + iy$ ,  $x, y \in \mathbb{R}$

Now  $\bar{z} = iz^2$

then  $x - iy = i(x^2 - y^2 + 2xyi)$

$x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow x = -2xy$  &  $-y = x^2 - y^2$

$\Rightarrow x(1 + 2y) = 0$

$x = 0$  or  $y = -\frac{1}{2}$

Put  $x = 0$  in  $-y = x^2 - y^2$

We get  $y = y^2$

$\Rightarrow y = 0, 1$

Similarly

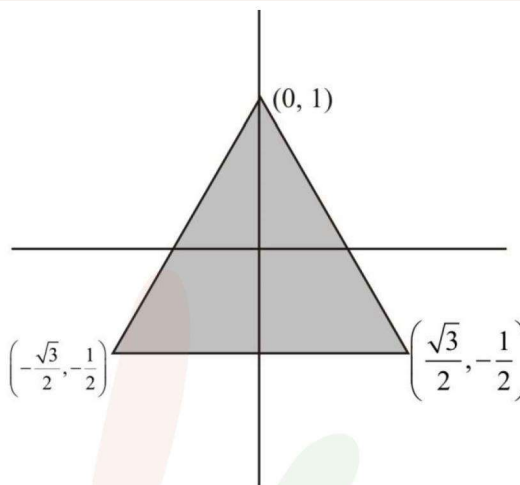
Put  $y = -\frac{1}{2}$  in  $-y = x^2 - y^2$

$\Rightarrow \frac{1}{2} = x^2 - \frac{1}{4}$

$\Rightarrow x^2 = \frac{3}{4}$

$x = \pm \frac{\sqrt{3}}{2}$

$z = \left(0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot (\sqrt{3}) \left(\frac{3}{2}\right) \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

2. Let the system of linear equations  $x + 2y + z = 2$ ,  $\alpha x + 3y - z = \alpha$ ,  $-\alpha x + y + 2z = -\alpha$  be inconsistent.

Then  $\alpha$  is equal to :

(A)  $\frac{5}{2}$                               (B)  $-\frac{5}{2}$

(C)  $\frac{7}{2}$                               (D)  $-\frac{7}{2}$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ -2 & 1 & 2 \end{vmatrix}$

$= (6 + y) - 2((2\alpha - \alpha) + 1(\alpha + 3\alpha))$

$= 7 - 2\alpha + 4\alpha$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix}$$

$$= 14 + 2\alpha$$

$$\alpha = -x_2 = 7$$

$$\Delta_1 \neq 0$$

3. If  $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$ , where a, b, c

are in A.P. and  $|a| < 1, |b| < 1, |c| < 1, abc \neq 0$ , then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C)  $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$  are in A.P.

(D)  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

**Official Ans. by NTA (C)**

**Ans. (C)**

**Sol.**  $x = 1 + a + a^2 = \dots$

$$x = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

$$y = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$

$$z = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$$\Rightarrow 1 - \frac{1}{x}, 1 - \frac{1}{y}, 1 - \frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow -\frac{1}{x}, -\frac{1}{y}, -\frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

4. Let  $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$ , where a, b, c are constants,

represent a circle passing through the point (2, 5).

Then the shortest distance of the point (11, 6) from this circle is :

(A) 10

(B) 8

(C) 7

(D) 5

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.** Let equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x + 2g)}{(2y + 2f)}$$

$$\text{Comparing with } \frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$$

$$\Rightarrow b = 0, a = -2, c = 2$$

$$\Rightarrow -2g = -2 \Rightarrow g = 1 \quad 2f = -2$$

$$f = -1$$

Now circle will be

$$x^2 + y^2 + 2x - 2y + c = 0$$

its passes through (2, 5)

which will give  $c = -23$

so circle will be  $x^2 + y^2 + 2x - 2y - 23 = 0$

centre  $C = (-1, 1)$

and radius 5

Now P is (11, 6)

So minimum distance of P from circle will be

$$= \sqrt{(11+1)^2 + (6-1)^2} - 5$$

$$= 13 - 5$$

$$= 8$$

5. Let a be an integer such that  $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x-3a]}$

exists, where [t] is greatest integer  $\leq t$ . Then a is equal to :

(A) -6

(B) -2

(C) 2

(D) 6

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $\lim_{x \rightarrow 7} \frac{18 - [1-x]}{[x] - 3a}$

L.H.L.  $\lim_{x \rightarrow 7^-} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-6)}{6 - 3a}$$

$$= \frac{24}{6 - 3a}$$

R.H.L.  $\lim_{x \rightarrow 7^+} \frac{18 - [1-x]}{[x] - 3a}$

$$= \frac{18 - (-7)}{7 - 3a}$$

$$= \frac{25}{7 - 3a}$$

Now L.H.L. = R.H.L.

$$\frac{24}{6 - 3a} = \frac{25}{7 - 3a}$$

$$\Rightarrow 168 - 72a = 150 - 75a$$

$$\Rightarrow 18 = -3a$$

$$\Rightarrow a = -6$$

6. The number of distinct real roots of  $x^4 - 4x + 1 = 0$  is :

- (A) 4 (B) 2  
(C) 1 (D) 0

**Official Ans. by NTA (B)**

**Ans. (B)**

**Sol.** Let  $f(x) = x^4 - 4x + 1$

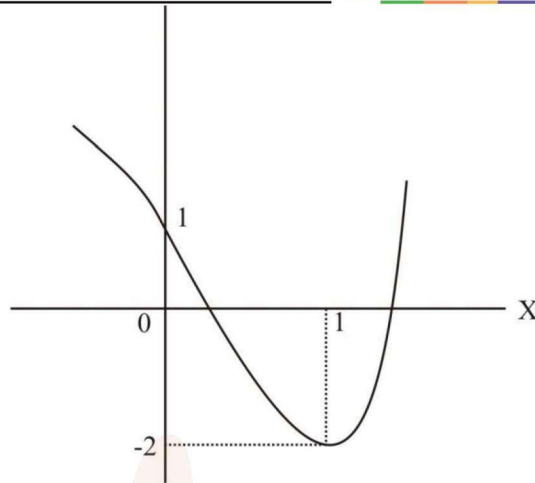
$$f(x) = 4x^3 - 4$$

$$f'(x) = 0 \Rightarrow x = 1$$

$x = 1$  is point of minima.

$$f(1) = -2$$

$$f(0) = 1$$



Hence 2 solutions.

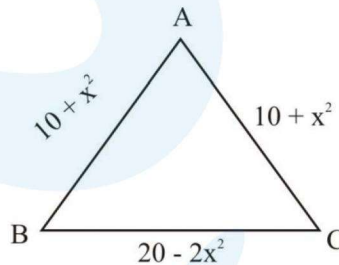
7. The lengths of the sides of a triangle are  $10 + x^2$ ,  $10 + x^2$  and  $20 - 2x^2$ . If for  $x = k$ , the area of the triangle is maximum, then  $3k^2$  is equal to :

- (A) 5 (B) 8  
(C) 10 (D) 12

**Official Ans. by NTA (C)**

**Ans. (C)**

**Sol.**



$$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$$

$$= \frac{a+b+c}{2}$$

$$= 20$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{20(2x^2)(10-x^2)(10-x^2)}$$

$$= 2\sqrt{10} \sqrt{x^2(10-x^2)^2}$$

$$= 2\sqrt{10} |x(10-x^2)|$$

$$= 2\sqrt{10} |10x - x^3|$$

$$S = 10x - x^3$$

$$\frac{ds}{dx} = 10 - 3x^2$$

$$\frac{ds}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$$

$$3x^2 = 10$$

8. If  $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$ ,  $|y| < 2$ , then :

(A)  $x^2y'' + xy' - 25y = 0$

(B)  $x^2y'' - xy' - 25y = 0$

(C)  $x^2y'' - xy' + 25y = 0$

(D)  $x^2y'' + xy' + 25y = 0$

Official Ans. by NTA (D)

Ans. (D)

Sol.  $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$$\cos^{-1}\left(\frac{y}{2}\right) = 5 \log_e\left(\frac{x}{5}\right)$$

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}} \cdot \frac{y'}{2} = 5 \cdot \frac{1}{x} \times \frac{1}{5}$$

$$\Rightarrow \frac{-y'}{\sqrt{4-y^2}} = \frac{5}{x}$$

$$-xy' = 5\sqrt{4-y^2}$$

$$-xy'' - y' = 5 \cdot \frac{1}{2\sqrt{4-y^2}} (-2y y')$$

$$\Rightarrow xy'' + y' = \frac{5y' \cdot y}{\sqrt{4-y^2}}$$

$$xy'' + y' = 5 \cdot \left(\frac{-5}{x}\right) y$$

$$x^2y'' + xy' = -25y$$

9.  $\int \frac{(x^2+1)e^x}{(x+1)^2} dx = f(x)e^x + C$ , Where C is a

constant, then  $\frac{d^3f}{dx^3}$  at  $x = 1$  is equal to :

(A)  $-\frac{3}{4}$  (B)  $\frac{3}{4}$

(C)  $-\frac{3}{2}$  (D)  $\frac{3}{2}$

Official Ans. by NTA (B)

Ans. (B)

Sol.  $\int \left(\frac{x^2+1}{(x+1)^2}\right) e^x \cdot dx$

$$= \int \left(\frac{x^2-1+2}{(x+1)^2}\right) e^x dx$$

$$= \int \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right) e^x dx$$

$$= \int (f(x) + f'(x)) e^x dx$$

$$= f(x) e^x + c$$

Where  $f(x) = \frac{x-1}{x+1}$

$$f'(x) = \frac{2}{(x+1)^2}$$

$$f''(x) = \frac{-4}{(x+1)^3}$$

$$= \frac{12}{(x+1)^4}$$

$$f''(1) = \frac{12}{16}$$

$$= \frac{3}{4}$$

10. The value of the integral  $\int_{-2}^2 \frac{|x^3 + x|}{(e^{x|x|} + 1)} dx$  is equal

to :

- (A)  $5e^2$  (B)  $3e^2$   
 (C) 4 (D) 6

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $f(x) = \frac{|x^3 + x|}{(e^{x|x|} + 1)}$

$$\int_{-2}^2 f(x) dx = \int_0^2 (f(x) + f(-x)) dx$$

$$= \int_0^2 \left( \frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|-x^3 - x|}{(e^{-x|-x|} + 1)} \right) dx$$

$$= \int_0^2 \left( \frac{|x^3 + x|}{(e^{x|x|} + 1)} + \frac{|x^3 + x|}{(e^{-x|x|} + 1)} \right) dx$$

$$= \int_0^2 \left( \frac{x^3 + x}{(e^{x^2} + 1)} + \frac{x^3 + x}{(e^{-x^2} + 1)} \right) dx$$

$$I = \int_0^2 \left( \frac{x^3 + x}{1 + e^{x^2}} + \frac{e^{x^2}(x^3 + x)}{1 + e^{x^2}} \right) dx$$

$$= \int_0^2 (x^3 + x) dx$$

$$= \left[ \frac{x^4}{4} + \frac{x^2}{2} \right]_0^2$$

$$= 4 + 2 = 6$$

11. If  $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$ , then

$y(2)$  is equal to :

- (A)  $2 + \log_2 3$  (B)  $2 + \log_2 2$   
 (C)  $2 - \log_2 3$  (D)  $2 - \log_2 3$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0,$

$$x, y > 0, y(1) = 1, y(2) = ?$$

$$\frac{dy}{dx} = -\frac{2^x(2^y - 1)}{2^y(2^x - 1)}$$

$$\int \frac{2^y}{2^y - 1} dy = -\int \frac{2^x}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \int \frac{2^y \ln 2}{2^y - 1} dy = -\frac{1}{\ln 2} \int \frac{2^x \ln 2}{2^x - 1} dx$$

$$\frac{1}{\ln 2} \ln|2^y - 1| = \frac{-1}{\ln 2} \ln|2^x - 1| + C$$

$$\text{At } x = 1, y = 1$$

Putting this values in above relation we get  $C = 0$

$$\ln|2^y - 1| + \ln|2^x - 1| = 0$$

$$(2^x - 1)(2^y - 1) = 1$$

$$2^y - 1 = \frac{1}{2^x - 1}$$

$$\text{At } x = 2$$

$$2^y = \frac{1}{3} + 1 = \frac{4}{3}$$

$$y = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 = 2 - \log_2 3$$

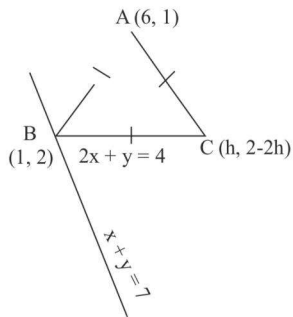
12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is  $2x + y = 4$ . Let the point B lie on the line  $x + 3y = 7$ . If  $(\alpha, \beta)$  is the centroid  $\Delta ABC$ , then  $15(\alpha + \beta)$  is equal to :

- (A) 39 (B) 41  
 (C) 51 (D) 63

**Official Ans. by NTA (C)**

**Ans. (C)**

Sol.



Point B (1, 2)

Now let C be (h, 4 - 2h)

(As C lies on  $2x + y = 4$ )

$\because \Delta$  is isosceles with base BC

$\therefore AB = AC$

$$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$$

$$\sqrt{26} = \sqrt{36+h^2-12h+4h^2+9-12h}$$

$$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$$

$$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$$

$$h = \frac{19}{5} \text{ or } h = 1$$

Thus  $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

$$\text{Centroid} \left( \frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3} \right)$$

$$\left( \frac{35+19}{15}, \frac{15-18}{15} \right)$$

$$\left( \frac{54}{15}, \frac{-3}{15} \right)$$

$$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$$

$$15(\alpha + \beta) = 51$$

13. Let the eccentricity of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b, \text{ be } \frac{1}{4}. \text{ If this ellipse passes}$$

through the point  $\left(-4\sqrt{\frac{2}{5}}, 3\right)$ , then  $a^2 + b^2$  is equal

to :

(A) 29 (B) 31

(C) 32 (D) 34

Official Ans. by NTA (B)

Ans. (B)

Sol.  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{16} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16} a^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$16 \times \frac{2}{5} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{b^2} = 1$$

$$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$$

$$\frac{80}{5a^2} = 1$$

$$16 = a^2$$

$$b^2 = 15$$



14. If two straight lines whose direction cosines are given by the relations  $l + m - n = 0$ ,  $3l^2 + m^2 + cnl = 0$  are parallel, then the positive value of  $c$  is :  
 (A) 6 (B) 4  
 (C) 3 (D) 2

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $l + m - n = 0$   
 $3l^2 + m^2 + cl(1 + m) = 0$   
 $n = 1 + m$   
 $3l^2 + m^2 + cl^2 + clm = 0$   
 $(3 + c)l^2 + clm + m^2 = 0$   
 $(3 + c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \dots\dots(1)$

$\therefore$  lines are parallel.

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

$c = 6$  or  $c = -2$

+ve value of  $c = 6$

15. Let  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ . Then the number of vectors  $\vec{b}$  such that  $\vec{b} \times \vec{c} = \vec{a}$  and  $|\vec{b}| \in \{1, 2, \dots, 10\}$  is :  
 (A) 0 (B) 1  
 (C) 2 (D) 3

**Official Ans. by NTA (A)**

**Ans. (A)**

**Sol.**  $\vec{a} = i + j - k$   
 $\vec{c} = 2i - 3j + 2k$   
 $\vec{b} \times \vec{c} = \vec{a}$   
 $|\vec{b}| \in \{1, 2, \dots, 10\}$   
 $\therefore \vec{b} \times \vec{c} = \vec{a}$   
 $\Rightarrow \vec{a}$  is perpendicular to  $\vec{b}$  as well as  $\vec{a}$  is perpendicular to  $\vec{c}$

Now  $\vec{a} \cdot \vec{c} = 2 - 3 - 2 = -3 \neq 0$

This  $\vec{b} \times \vec{c} = \vec{a}$  is not possible.

No. of vectors  $\vec{b} = 0$

16. Five numbers  $x_1, x_2, x_3, x_4, x_5$  are randomly selected from the numbers 1, 2, 3, ..., 18 and are arranged in the increasing order ( $x_1 < x_2 < x_3 < x_4 < x_5$ ). The probability that  $x_2 = 7$  and  $x_4 = 11$  is :

(A)  $\frac{1}{136}$  (B)  $\frac{1}{72}$

(C)  $\frac{1}{68}$  (D)  $\frac{1}{34}$

**Official Ans. by NTA (C)**

**Ans. (C)**

- Sol.** No. of ways to select and arrange  $x_1, x_2, x_3, x_4, x_5$  from 1, 2, 3, ..., 18

$n(s) = {}^{18}C_5$

$x_1 \quad (x_2) \quad x_3 \quad (x_4) \quad x_5$   
 $7 \qquad \qquad \qquad 11$

$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$

$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$

$\frac{1}{17 \times 4} = \frac{1}{68}$

17. Let  $X$  be a random variable having binomial distribution  $B(7, p)$ . If  $P(X = 3) = 5P(X = 4)$ , then the sum of the mean and the variance of  $X$  is :

(A)  $\frac{105}{16}$  (B)  $\frac{7}{16}$

(C)  $\frac{77}{36}$  (D)  $\frac{49}{16}$

**Official Ans. by NTA (C)**

**Ans. (C)**

- Sol.**  $B(7, p)$

$n = 7 \quad p = p$

given

$P(x = 3) = 5P(x = 4)$

$${}^7C_3 \times p^3 (1-p)^4 = 5 \cdot {}^7C_4 p^4 (1-p)^3$$

$$\frac{{}^7C_3}{5 \cdot {}^7C_4} = \frac{p}{1-p}$$

$$1-p = 5p$$

$$6p = 1$$

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

$$n = 7$$

$$\text{Mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{Var} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36}$$

$$= \frac{77}{36}$$

18. The value of  $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$

is equal to :

(A) -1 (B)  $-\frac{1}{2}$

(C)  $-\frac{1}{3}$  (D)  $-\frac{1}{4}$

Official Ans. by NTA (B)

Ans. (B)

Sol.  $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$$

$$= \frac{2 \sin\left(\frac{3\pi}{7}\right)}{2 \sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2 \sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2 \sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19.  $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$  is

equal to :

(A)  $\frac{11\pi}{12}$

(B)  $\frac{17\pi}{12}$

(C)  $\frac{31\pi}{12}$

(D)  $-\frac{3\pi}{4}$

Official Ans. by NTA (A)

Ans. (A)

Sol.  $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$



20. The Boolean expression  $(\sim(p \wedge q)) \vee q$  is equivalent to :

- (A)  $q \rightarrow (p \wedge q)$       (B)  $p \rightarrow q$   
 (C)  $p \rightarrow (p \rightarrow q)$       (D)  $p \rightarrow (p \vee q)$

**Official Ans. by NTA (D)**

**Ans. (D)**

**Sol.**  $(\sim(p \wedge q)) \vee q$

$$= (\sim p \vee \sim q) \vee q$$

$$= \sim p \vee \sim q \vee q$$

$$= \sim p \vee t$$

= this statement is a tautology option D

$p \Rightarrow (p \vee q)$  is also a tautology.

OR

p	q	$P \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$	$P \vee q$	$p \rightarrow (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

**SECTION-B**

1. Let  $f : R \rightarrow R$  be a function defined  $f(x) = \frac{2e^{2x}}{e^{2x} + e}$ .

Then  $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (99)**

**Ans. (99)**

**Sol.**

$$f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2-2x}}{e^{2-2x} + e} = \left[ \frac{e^{2x}}{e^{2x} + e} + \frac{e^2}{e^2 + e^{2x+1}} \right]$$

$$= 2 \left[ \frac{e^{2x-1}}{e^{2x-1} + 1} + \frac{1}{1 + e^{2x-1}} \right] = 2$$

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + \dots + f\left(\frac{99}{100}\right)$$

$$= \left\{ f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right) \right\} + \left\{ f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right) \right\} + \dots + f\left\{ \left(\frac{49}{100}\right) + f\left(\frac{51}{100}\right) \right\} + f\left(\frac{1}{2}\right)$$

$$= (2 + 2 + 2 + \dots + 49 \text{ times}) + \frac{2e}{e + e}$$

$$= 98 + 1 = 99$$

2. If the sum of all the roots of the equation

$$e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0 \text{ is } \log_e P, \text{ then } p \text{ is}$$

equal to \_\_\_\_\_.

**Official Ans. by NTA (45)**

**Ans. (45)**

**Sol.**  $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$  ]

$$(e^x)^3 - 11(e^x)^2 - 45 + \frac{81e^x}{2} = 0$$

$$e^x = t$$

$$2t^3 - 22t^2 + 81t - 90 = 0$$

$$t_1 t_2 t_3 = 45$$

$$e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45$$

$$e^{x_1 + x_2 + x_3} = 45$$

$$\log_e e^{x_1 + x_2 + x_3} = \log_e 45$$

$$x_1 + x_2 + x_3 = \log_e 45$$

$$\log_e P = \log_e 45$$

$$P = 45$$

3. The positive value of the determinant of the matrix

$$A, \text{ whose } Adj(Adj(A)) = \begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix},$$

is \_\_\_\_\_.

Official Ans. by NTA (14)

Ans. (14)

Sol.  $Adj(AdjA) = \begin{bmatrix} 14 & 18 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$$|Adj(AdjA)| = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = 14 \times 14 \times 14 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= (14)^3 [3 - 2(-5) - 1(-1)] = (14)^3 [14] = (14)^4$$

$$|A|^4 = (14)^4 \Rightarrow |A| = 14$$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is \_\_\_\_\_.

Official Ans. by NTA (56)

Ans. (56)

Sol. 16 cubes  $\begin{cases} 11 \text{ Blue} \\ 5 \text{ Red} \end{cases}$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$$

$$x_1, x_6 \geq 0, \quad x_2, x_3, x_4, x_5 \geq 2$$

$$x_2 = t_1 + 2$$

$$x_3 = t_3 + 2$$

$$x_4 = t_4 + 2$$

$$x_5 = t_5 + 2$$

$$x_1, t_2, t_3, t_4, t_5, x_6 \geq 0$$

$$\text{No. of solutions} = {}^{6+3-1}C_3 = {}^8C_3 = 56$$

5. If the coefficient of  $x^{10}$  in the binomial expansion

$$\text{of } \left( \frac{\sqrt{x}}{5^4} + \frac{\sqrt{5}}{x^3} \right)^{60} \text{ is } 5^k l, \text{ where } l, k \in \mathbb{N} \text{ and } l \text{ is co-}$$

prime to 5, then k is equal to \_\_\_\_\_.

Official Ans. by NTA (5)

Ans. (5)

Sol.  $\left( \frac{\sqrt{x}}{5^{1/4}} + \frac{\sqrt{5}}{x^{1/3}} \right)^{60}$

$$T_{r+1} = {}^{60}C_r \left( \frac{x^{1/2}}{5^{1/4}} \right)^{60-r} \left( \frac{5^{1/2}}{x^{1/3}} \right)^r$$

$$= {}^{60}C_r 5^{\frac{3r-60}{4}} \cdot x^{\frac{180-5r}{6}}$$

$$\frac{180-5r}{6} = 10 \Rightarrow r = 24$$

$$\text{Coeff. of } x^{10} = {}^{60}C_{24} 5^3 = \frac{60}{24 \cdot 36} 5^3$$

$$\text{Powers of 5 in } = {}^{60}C_{24} \cdot 5^3 = \frac{5^{14}}{5^4 \times 5^8} \times 5^3 = 5^5$$

6. Let

$$A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}. \text{ If } 27 \text{ (Area } A_1) = 5$$

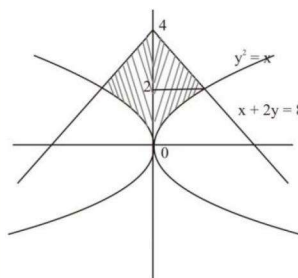
(Area  $A_2$ ), then k is equal to :

Official Ans. by NTA (6)

Ans. (6)

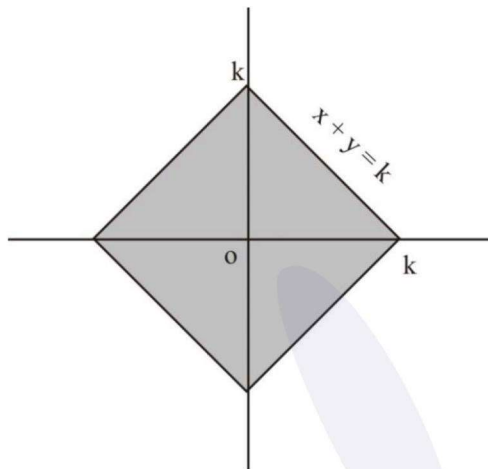
Sol.  $A_1 = \{(x, y) : |x| \leq y^2, |x| + 2y \leq 8\} \text{ and}$

$$A_2 = \{(x, y) : |x| + |y| \leq k\}.$$



$$\text{area}(A_1) = 2 \left[ \int_0^2 y^2 dy + \int_2^4 (8-2y) dy \right]$$

$$= 2 \left[ \left( \frac{y^3}{3} \right)_0^2 + (8y - y^2)_2^4 \right]$$



$$\text{area}(A_1) = 2 \times \frac{20}{3} = \frac{40}{3}$$

$$\text{Area}(A_2) = 4 \times \frac{1}{2} k^2$$

$$\text{Area}(A_2) = 2k^2$$

Now

$$27(\text{Area } A_1) = 5(\text{Area } A_2)$$

$$9 \times 4 = k^2$$

$$k = 6$$

7. If the sum of the first ten terms of the series

$$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots \text{ is } \frac{m}{n}, \text{ where}$$

m and n are co-prime numbers, then m + n is equal to \_\_\_\_\_.

**Official Ans. by NTA (276)**

**Ans. (276)**

**Sol.**  $\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \dots$

$$T_n = \frac{n}{4n^4 + 1}$$

$$= \frac{n}{(2n^2 + 1)^2 - (2n)^2} = \frac{n}{(2n^2 + 2n + 1)(2n^2 - 2n + 1)}$$

$$= \frac{1}{4} \left[ \frac{1}{2n^2 - 2n + 1} - \frac{1}{2n^2 + 2n + 1} \right]$$

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4} \left[ \frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \dots + \frac{1}{200 + 20 + 1} \right]$$

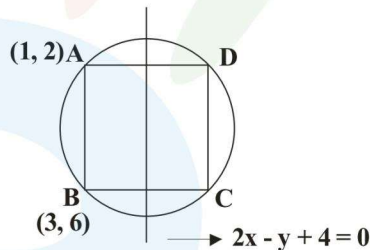
$$= \frac{1}{4} \left[ 1 - \frac{1}{221} \right] = \frac{1}{4} \times \frac{220}{221} = \frac{55}{221} = \frac{m}{n}$$

$$m + n = 55 + 221 = 276$$

8. A rectangle R with end points of the one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is  $2x - y + 4 = 0$ , then the area of R is \_\_\_\_\_.

**Official Ans. by NTA (16)**

**Ans. (16)**



**Sol.**

Eq. of line AB

$$y = 2x$$

Slope of AB = 2

Slope of given diameter = 2

So the diameter is parallel to AB

Distance between diameter and line AB

$$= \left( \frac{4}{\sqrt{2^2 + 12}} \right) = \frac{4}{\sqrt{5}}$$

$$\text{Thus } BC = 2 \times \frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$$

$$AB = \sqrt{(1-3)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

$$\text{Area} = AB \times BC = \frac{8}{\sqrt{5}} \times 2\sqrt{5} = 16 \text{ Ans.}$$

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola  $y^2 = 2x$  and touches the parabola  $y = \left(x - \frac{1}{4}\right)^2 + \alpha$ , where  $\alpha > 0$ .

Then  $(4\alpha - 8)^2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (63)**

**Ans. (63)**

**Sol.** Vertex and focus of parabola  $y^2 = 2x$

are V (0, 0) and  $S\left(\frac{1}{2}, 0\right)$  resp.

Let equation of circle be

$$(x - h)^2 + (y - k)^2 = 4$$

$\therefore$  Circle passes through (0, 0)

$$\Rightarrow h^2 + k^2 = 4 \dots\dots(1)$$

$\therefore$  Circle passes through  $\left(\frac{1}{2}, 0\right)$

$$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \dots\dots(2)$$

On solving (1) and (2)

$$4 - h = \frac{15}{4}$$

$$h = 4 - \frac{15}{4} = \frac{1}{4}$$

$$k = +\frac{\sqrt{63}}{4}$$

$k = -\frac{\sqrt{63}}{4}$  is rejected as circle with centre

$\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$  can't touch given parabola.

Equation of circle is

$$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$$

From figure

$$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$$

$$4\alpha - 8 = \sqrt{63}$$

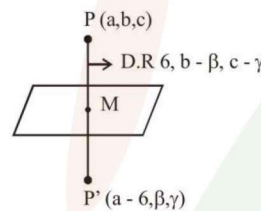
$$(4\alpha - 8)^2 = 63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane  $3x - 4y + 12z + 19 = 0$  be (a - 6,  $\beta$ ,  $\gamma$ ). If  $a + b + c = 5$ , then  $7\beta - 9\gamma$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (137)**

**Ans. (137)**

**Sol.**



$$M = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$$

Since M lies on  $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \dots\dots(1)$$

Since  $PP'$  is parallel to normal of the plane then

$$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$$

$$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$$

$$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$$

$$\Rightarrow a = -\beta - \gamma - 11$$

Now putting these values in (1) we get

$$6(-\beta - \gamma - 11) - 4(\beta - 8) + 12(\gamma + 24) - 4\beta + 12\gamma + 20 = 0$$

$$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$$