

FINAL JEE-MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $x*y = x^2 + y^3$ and $(x*1)*1 = x*(1*1)$.

Then a value of $2 \sin^{-1} \left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2} \right)$ is

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
 (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\because (x * 1) * 1 = x * (1 * 1)$

$$(x^2 + 1) * 1 = x * (2)$$

$$(x^2 + 1)^2 + 1 = x^2 + 8$$

$$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$$

$$x^2 = 2$$

$$\Rightarrow 2 \sin^{-1} \left(\frac{x^4 + x^2 - 2}{x^4 + x^2 + 2} \right) = 2 \sin^{-1} \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{3}$$

2. The sum of all the real roots of the equation

$$(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$$
 is

- (A) $\log_e 3$ (B) $-\log_e 3$
 (C) $\log_e 6$ (D) $-\log_e 6$

Official Ans. by NTA (B)

Ans. (B)

Sol. $(e^{2x} - 4)(6e^{2x} - 3e^x - 2e^x + 1) = 0$

$$(e^{2x} - 4)(3e^x - 1)(2e^x - 1) = 0$$

$$e^{2x} = 4 \text{ or } e^x = \frac{1}{3} \text{ or } e^x = \frac{1}{2}$$

$$\Rightarrow \text{sum of real roots} = \frac{1}{2} \ln 4 + \ln \frac{1}{3} + \ln \frac{1}{2}$$

$$= -\ln 3$$

3. Let the system of linear equations

$$x + y + \alpha z = 2$$

$$3x + y + z = 4$$

$$x + 2z = 1$$

have a unique solution (x^*, y^*, z^*) . If $(\alpha, x^*), (y^*, \alpha)$ and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is :

- (A) 4 (B) 3
 (C) 2 (D) 1

Official Ans. by NTA (C)

Ans. (C)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$$\alpha \neq -3, x = 1, y = 1, z = 0,$$

Now points $(\alpha, 1), (1, \alpha)$ & $(1, -1)$ are collinear

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$$

$$\alpha^2 + \alpha - 1 - \alpha = 0$$

$$\alpha = \pm 1$$

4. Let $x, y > 0$. If $x^3 y^2 = 2^{15}$, then the least value of $3x + 2y$ is

- (A) 30 (B) 32
 (C) 36 (D) 40

Official Ans. by NTA (D)

Ans. (D)

Sol. Using AM \geq GM

$$\frac{x+x+x+y+y}{5} \geq (x^3 \cdot y^2)^{\frac{1}{5}}$$

$$\frac{3x+2y}{5} \geq (2^{15})^{\frac{1}{5}}$$

$$(3x+2y)_{\min} = 40$$

5. Let $f(x) = \begin{cases} \frac{\sin(x - [x])}{x - [x]}, & x \in (-2, -1) \\ \max\{2x, 3[x]\}, & |x| < 1 \\ 1, & \text{otherwise} \end{cases}$

where $[t]$ denotes greatest integer $\leq t$. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is :

(A) (3, 3) (B) (2, 4)

(C) (2, 3) (D) (3, 4)

Official Ans. by NTA (C)

Ans. (C)

Sol. $f(x) = \begin{cases} \frac{\sin(x+2)}{x+2}, & x \in (-2, -1) \\ \max\{2x, 0\}, & x \in (-1, 1) \\ 1, & \text{otherwise} \end{cases}$

$$f(-2^+) = \lim_{h \rightarrow 0} f(-2+h) = \lim_{h \rightarrow 0} \frac{\sinh}{h} = 1$$

f is continuous at $x = -2$

$$f(-1^-) = \lim_{h \rightarrow 0} \frac{\sin(-1-h+2)}{(-1-h+2)} = \sin 1$$

$$f(-1) = f(-1^+) = 0$$

$f(1^+) = 1$ & $f(1^-) = 0 \Rightarrow f$ is not continuous at $x = 1$

f is continuous but not diff. at $x = 0$

$$\Rightarrow f \text{ is discontinuous at } x = -1 \text{ \& } 1 \left. \vphantom{\begin{matrix} \\ \\ \end{matrix}} \right\} \Rightarrow m = 2$$

$$\text{ \& } f \text{ is not diff. at } x = -1, 0 \text{ \& } 1 \left. \vphantom{\begin{matrix} \\ \\ \end{matrix}} \right\} \Rightarrow n = 3$$

6. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x + \cos^6 x)}$$
 is equal to

(A) 2π (B) 0

(C) π (D) $\frac{\pi}{2}$

Official Ans. by NTA (C)

Ans. (C)

Sol. $I = \int_{-\pi/2}^0 \frac{dx}{(1+e^x)(\sin^6 x + \cos^6 x)} + \int_0^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x + \cos^6 x)}$

Put $x = -t$

$$= \int_{\pi/2}^0 \frac{-dt}{(1+e^{-t})(\sin^6 t + \cos^6 t)} + \int_0^{\pi/2} \frac{dx}{(1+e^x)(\sin^6 x + \cos^6 x)}$$

$$= \int_0^{\pi/2} \frac{(e^x + 1) dx}{(1+e^x)(\sin^6 x + \cos^6 x)}$$

$$= \int_0^{\pi/2} \frac{dx}{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)}$$

$$= \int_0^{\pi/2} \frac{(1 + \tan^2 x) \sec^2 x dx}{(\tan^4 x - \tan^2 x + 1)}$$

Put $\tan x = t$

$$= \int_0^{\infty} \frac{(1+t^2) dt}{(t^4 - t^2 + 1)}$$

$$= \int_0^{\infty} \frac{\left(1 + \frac{1}{t^2}\right) dt}{t^2 - 1 + \frac{1}{t^2}} = \int_0^{\infty} \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 1}$$

Put $t - \frac{1}{t} = z$

$$\left(1 + \frac{1}{t^2}\right) dt = dz$$

$$= \int_{-\infty}^{\infty} \frac{dz}{1+z^2} = \left(\tan^{-1} z\right)_{-\infty}^{\infty}$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

7. $\lim_{n \rightarrow \infty} \left(\frac{n^2}{(n^2+1)(n+1)} + \frac{n^2}{(n^2+4)(n+2)} + \frac{n^2}{(n^2+9)(n+3)} + \dots + \frac{n^2}{(n^2+n^2)(n+n)} \right)$

is equal to

- (A) $\frac{\pi}{8} + \frac{1}{4} \log_e 2$ (B) $\frac{\pi}{4} + \frac{1}{8} \log_e 2$
 (C) $\frac{\pi}{4} - \frac{1}{8} \log_e 2$ (D) $\frac{\pi}{8} + \log_e \sqrt{2}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $\lim_{n \rightarrow \infty} \left(\sum_{r=1}^n \frac{n^2}{(n^2+r^2)(n+r)} \right)$

$$= \lim_{n \rightarrow \infty} \left(\sum_{r=1}^n \frac{1}{n \left(1 + \left(\frac{r}{n} \right)^2 \right) \left(1 + \left(\frac{r}{n} \right) \right)} \right)$$

$$= \int_0^1 \frac{dx}{(1+x^2)(1+x)} = \frac{1}{2} \int_0^1 \frac{1-x}{1+x^2} dx + \frac{1}{2} \int_0^1 \frac{1}{1+x} dx$$

$$= \frac{1}{2} \int_0^1 \left(\frac{1}{1+x^2} - \frac{x}{1+x^2} \right) dx + \frac{1}{2} (\ln(1+x)) \Big|_0^1$$

$$= \frac{1}{2} \left[\tan^{-1} x - \frac{1}{2} \ln(1+x^2) \right]_0^1 + \frac{1}{2} \ln 2$$

$$= \frac{1}{2} \left[\frac{\pi}{4} - \frac{1}{2} \ln 2 \right] + \frac{1}{2} \ln 2$$

$$= \frac{\pi}{8} + \frac{1}{4} \ln 2$$

8. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with

- (A) length of latus rectum 3
 (B) length of latus rectum 6
 (C) focus $\left(\frac{4}{3}, 0 \right)$
 (D) focus $\left(0, \frac{3}{4} \right)$

Official Ans. by NTA (A)

Ans. (A)

Sol. Let Point P(x,y)

$$Y - y = y'(X - x)$$

$$Y = 0 \Rightarrow X = x - \frac{y}{y'}$$

$$Q \left(x - \frac{y}{y'}, 0 \right)$$

Mid Point of PQ lies on y axis

$$x - \frac{y}{y'} + x = 0$$

$$y' = \frac{y}{2x} \Rightarrow 2 \frac{dy}{y} = \frac{dx}{x}$$

$$2 \ln y = \ln x + \ln k$$

$$y^2 = kx$$

It passes through (3, 3) $\Rightarrow k = 3$

$$\text{curve } c \Rightarrow y^2 = 3x$$

$$\text{Length of L.R.} = 3$$

$$\text{Focus} = \left(\frac{3}{4}, 0 \right) \text{ Ans. (A)}$$

9. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$, $a > 2$, having

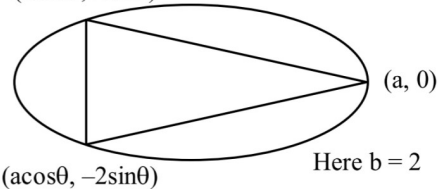
one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is :

- (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{4}$

Official Ans. by NTA (A)

Ans. (A)

Sol. $(a \cos \theta, 2 \sin \theta)$



$$A = \frac{1}{2} a (1 - \cos \theta) (4 \sin \theta)$$

$$A = 2a(1 - \cos\theta) \sin\theta$$

$$\frac{dA}{d\theta} = 2a(\sin^2\theta + \cos\theta - \cos^2\theta)$$

$$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\theta - 2\cos^2\theta = 0$$

$$\cos\theta = 1 \text{ (Reject)}$$

OR

$$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

$$\frac{d^2A}{d\theta^2} = 2a(2\sin^2\theta - \sin\theta)$$

$$\frac{d^2A}{d\theta^2} < 0 \text{ for } \theta = \frac{2\pi}{3}$$

$$\text{Now, } A_{\max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3}$$

$$\boxed{a = 4}$$

$$\text{Now, } e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2} \text{ Ans. (A)}$$

10. Let the area of the triangle with vertices $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ be 4 sq. units. If the point $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear, then β is equal to

- (A) 64 (B) -8
(C) -64 (D) 512

Official Ans. by NTA (C)

Ans. (C)

$$\text{Sol. } \frac{1}{2} \begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$$

$$\alpha = \pm 8$$

Now given points $(8, -8)$, $(-8, 8)$, $(64, \beta)$

OR $(-8, 8)$, $(8, -8)$, $(64, \beta)$

are collinear \Rightarrow Slope = -1.

$$\boxed{\beta = -64} \text{ Ans. (C)}$$

11. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is
(A) 5 (B) 7 (C) 1 (D) 3

Official Ans. by NTA (D)

Ans. (D)

$$\text{Sol. } x^7 - 7x - 2 = 0$$

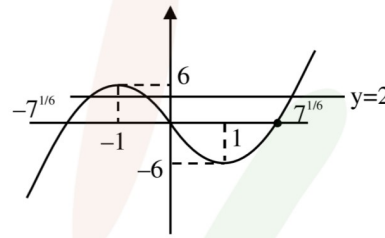
$$x^7 - 7x = 2$$

$$f(x) = x^7 - 7x \text{ (odd) \& } y = 2$$

$$f(x) = x(x^2 - 7^{1/3})(x^4 + x^2 \cdot 7^{1/3} + 7^{2/3})$$

$$f(x) = 7(x^6 - 1) = 7(x^2 - 1)(x^4 + x^2 + 1)$$

$$f(x) = 0 \Rightarrow x = \pm 1$$



$f(x) = 2$ has 3 real distinct solution.

12. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	k	2k	4k	6k	8k

The value of $P(1 < X < 4 \mid X \leq 2)$ is equal to :

- (A) $\frac{4}{7}$ (B) $\frac{2}{3}$
(C) $\frac{3}{7}$ (D) $\frac{4}{5}$

Official Ans. by NTA (A)

Ans. (A)

$$\begin{aligned} \text{Sol. } P\left(\frac{1 < x < 4}{x \leq 2}\right) &= \frac{P(1 < x < 4 \cap x \leq 2)}{P(x \leq 2)} \\ &= \frac{P(1 < x \leq 2)}{P(x \leq 2)} = \frac{P(x = 2)}{P(x \leq 2)} \\ &= \frac{4k}{k + 2k + 4k} = \frac{4}{7} \end{aligned}$$

13. The number of solutions of the equation

$$\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x, \quad x \in [-3\pi,$$

$3\pi]$ is :

- (A) 8 (B) 5
(C) 6 (D) 7

Official Ans. by NTA (D)

Ans. (D)

Sol. $\cos\left(\frac{\pi}{3} + x\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x$

$$x \in [-3\pi, 3\pi]$$

$$4\left(\cos^2\left(\frac{\pi}{3}\right) - \sin^2 x\right) = \cos^2 2x$$

$$4\left(\frac{1}{4} - \sin^2 x\right) = \cos^2 2x$$

$$1 - 4\sin^2 x = \cos^2 2x$$

$$1 - 2(1 - \cos 2x) = \cos^2 2x$$

$$\text{let } \cos 2x = t$$

$$-1 + 2\cos 2x = \cos^2 2x$$

$$t^2 - 2t + 1 = 0$$

$$(t - 1)^2 = 0$$

$$\boxed{t = 1} \quad \boxed{\cos 2x = 1}$$

$$2x = 2n\pi$$

$$\boxed{x = n\pi}$$

$$n = -3, -2, -1, 0, 1, 2, 3$$

(D) option is correct.

14. If the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda} \quad \text{and} \quad \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$$

is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

- (A) 16 (B) 6
(C) 12 (D) 15

Official Ans. by NTA (A)

Ans. (A)

Sol. SHORTEST distance $\frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$

$$a_1 = (1, 2, 3)$$

$$a_2 = (2, 4, 5)$$

$$\vec{b}_1 = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$$

$$\vec{b}_2 = \hat{i} + 4\hat{j} + 5\hat{k}$$

$$\text{S.D.} = \frac{|((2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|b_1 \times b_2|}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 4\lambda) + \hat{j}(\lambda - 10) + \hat{k}(5)$$

$$= (15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}$$

Now

$$\text{S.D.} = \frac{|(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot [(15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}]|}{\sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}}$$

$$\frac{|15 - 4\lambda + 2\lambda - 20 + 10|}{\sqrt{(15 - 4\lambda)^2 + (\lambda - 10)^2 + 25}} = \frac{1}{\sqrt{3}}$$

square both side

$$3(5 - 2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$$

$$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$$

$$5\lambda^2 - 80\lambda + 275 = 0$$

$$\lambda^2 - 16\lambda + 55 = 0$$

$$(\lambda - 5)(\lambda - 11) = 0$$

$$\Rightarrow \lambda = 5, 11$$

(A) is correct option.

15. Let the points on the plane P be equidistant from

the points $(-4, 2, 1)$ and $(2, -2, 3)$. Then the acute

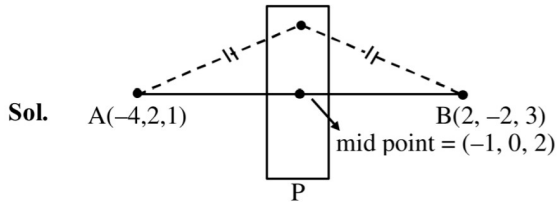
angle between the plane P and the plane $2x + y +$

$3z = 1$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Official Ans. by NTA (C)

Ans. (C)



$$\text{Normal vector} = \overline{AB} = (\overline{OB} - \overline{OA})$$

$$= (6\hat{i} - 4\hat{j} + 2\hat{k})$$

$$\text{or } 2(3\hat{i} - 2\hat{j} + \hat{k})$$

$$P \equiv 3(x + 1) - 2(y) + 1(z - 2) = 0$$

$$P \equiv 3x - 2y + z + 1 = 0$$

$$P' \equiv 2x + y + 3z - 1 = 0$$

$$\text{angle between } P \text{ \& } P' = \left| \frac{\hat{n}_1 \cdot \hat{n}_2}{|\hat{n}_1| |\hat{n}_2|} \right| = \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}} \right)$$

$$\theta = \cos^{-1} \left(\frac{7}{14} \right) = \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$

Option C is correct.

16. Let \hat{a} and \hat{b} be two unit vectors such that

$$\left| (\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b}) \right| = 2. \text{ If } \theta \in (0, \pi) \text{ is the angle}$$

between \hat{a} and \hat{b} , then among the statements :

$$(S1) : 2|\hat{a} \times \hat{b}| = |\hat{a} - \hat{b}|$$

$$(S2) : \text{The projection of } \hat{a} \text{ on } (\hat{a} + \hat{b}) \text{ is } \frac{1}{2}$$

(A) Only (S1) is true

(B) Only (S2) is true

(C) Both (S1) and (S2) are true

(D) Both (S1) and (S2) are false

Official Ans. by NTA (C)

Ans. (C)

Sol. $\left| (\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b}) \right| = 2, \theta \in (0, \pi)$

$$\left((\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b}) \right) \cdot \left((\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b}) \right) = 4$$

$$|\hat{a} + \hat{b}|^2 + 4|(\hat{a} \times \hat{b})|^2 + 0 = 4$$

Let the angle be θ between \hat{a} and \hat{b}

$$2 + 2\cos\theta + 4\sin^2\theta = 4$$

$$2 + 2\cos\theta - 4\cos^2\theta = 0$$

Let $\cos\theta = t$ then

$$2t^2 - t - 1 = 0$$

$$2t^2 - 2t + t - 1 = 0$$

$$2t(t - 1) + (t - 1) = 0$$

$$(2t + 1)(t - 1) = 0$$

$$t = -\frac{1}{2} \quad \text{or} \quad t = 1$$

$$\cos\theta = -\frac{1}{2} \quad \left| \text{not possible as } \theta \in (0, \pi) \right.$$

$$\theta = \frac{2\pi}{3}$$

Now,

$$S_1 \quad 2|\hat{a} \times \hat{b}| = 2\sin\left(\frac{2\pi}{3}\right)$$

$$|\hat{a} - \hat{b}| = \sqrt{1 + 1 - 2\cos\left(\frac{2\pi}{3}\right)}$$

$$= \sqrt{2 - 2 \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{3}$$

S_1 is correct.

S_2 projection of \hat{a} on $(\hat{a} + \hat{b})$.

$$\frac{\hat{a} \cdot (\hat{a} + \hat{b})}{|\hat{a} + \hat{b}|} = \frac{1 + \cos\left(\frac{2\pi}{3}\right)}{\sqrt{2 + 2\cos\frac{2\pi}{3}}}$$

$$= \frac{1 - \frac{1}{2}}{\sqrt{1}}$$

$$= \frac{1}{2}$$

C Option is true.

17. If $y = \tan^{-1}(\sec x^3 - \tan x^3)$, $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

- (A) $xy'' + 2y' = 0$
- (B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$
- (C) $x^2y'' - 6y + 3\pi = 0$
- (D) $xy'' - 4y' = 0$

Official Ans. by NTA (B)

Ans. (B)

Sol. $y = \tan^{-1}(\sec x^3 - \tan x^3)$

$$= \tan^{-1}\left(\frac{1 - \sin x^3}{\cos x^3}\right)$$

$$= \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} - x^3\right)}{\sin\left(\frac{\pi}{2} - x^3\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right)$$

Since $\frac{\pi}{4} - \frac{x^3}{2} \in \left(-\frac{\pi}{2}, 0\right)$

$$y = \left(\frac{\pi}{4} - \frac{x^3}{2}\right)$$

$$y' = \frac{-3x^2}{2}, y'' = -3x$$

$$4y = \pi - 2x^3$$

$$4y = \pi - 2x^2 \left(\frac{-y''}{3}\right)$$

$$12y = 3\pi + 2x^2y''$$

$$x^2y'' - 6y + \frac{3\pi}{2} = 0$$

18. Consider the following statements :

- A : Rishi is a judge.
- B : Rishi is honest.
- C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

- (A) $B \rightarrow (A \vee C)$
- (B) $(\sim B) \wedge (A \wedge C)$
- (C) $B \rightarrow ((\sim A) \vee (\sim C))$
- (D) $B \rightarrow (A \wedge C)$

Official Ans. by NTA (B)

Ans. (B)

Sol. $\sim((A \wedge C) \rightarrow B)$

$$\sim(\sim(A \wedge C) \vee B)$$

Using De-Morgan's law

$$(A \wedge C) \wedge (\sim B)$$

Option B is correct.

19. The slope of normal at any point (x, y) , $x > 0, y > 0$

on the curve $y = y(x)$ is given by $\frac{x^2}{xy - x^2y^2 - 1}$.

If the curve passes through the point $(1, 1)$, then $e.y(e)$ is equal to

(A) $\frac{1 - \tan(1)}{1 + \tan(1)}$ (B) $\tan(1)$

(C) 1 (D) $\frac{1 + \tan(1)}{1 - \tan(1)}$

Official Ans. by NTA (D)

Ans. (D)

Sol. Slope of normal = $\frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$$x^2y^2dx + dx - xydx = x^2dy$$

$$x^2y^2dx + dx = x^2dy + xydx$$

$$x^2y^2dx + dx = x(xdy + ydx)$$

$$x^2y^2dx + dx = xd(xy)$$

$$\frac{dx}{x} = \frac{d(xy)}{1+x^2y^2}$$

$$\ln kx = \tan^{-1}(xy) \dots (i)$$

passes through (1, 1)

$$\ln k = \frac{\pi}{4} \Rightarrow k = e^{\frac{\pi}{4}}$$

equation (i) becomes

$$\frac{\pi}{4} + \ln x = \tan^{-1}(xy)$$

$$xy = \tan\left(\frac{\pi}{4} + \ln x\right)$$

$$xy = \left(\frac{1 + \tan(\ln x)}{1 - \tan(\ln x)}\right) \dots (ii)$$

put $x = e$ in (ii)

$$\therefore ey(e) = \frac{1 + \tan 1}{1 - \tan 1}$$

20. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in \mathbb{R}$. Then $f_{\lambda^*}(1) + f_{\lambda^*}(-1)$ is equal to :

- (A) 36 (B) 48
(C) 64 (D) 72

Official Ans. by NTA (D)

Ans. (D)

Sol. $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$$f'_\lambda(x) = 12\lambda x^2 - 72\lambda x + 36$$

$$f'_\lambda(x) = 12(\lambda x^2 - 6\lambda x + 3) \geq 0$$

$$\therefore \lambda > 0 \text{ \& } D \leq 0$$

$$36\lambda^2 - 4 \times \lambda \times 3 \leq 0$$

$$9\lambda^2 - 3\lambda \leq 0$$

$$3\lambda(3\lambda - 1) \leq 0$$

$$\lambda \in \left[0, \frac{1}{3}\right]$$

$$\therefore \lambda_{\text{largest}} = \frac{1}{3}$$

$$f(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$$

$$\therefore f(1) + f(-1) = 72$$

SECTION-B

1. Let $S = \{z \in \mathbb{C} : |z-3| \leq 1 \text{ and } z(4+3i) + \bar{z}(4-3i) \leq 24\}$.

If $\alpha + i\beta$ is the point in S which is closest to $4i$, then $25(\alpha + \beta)$ is equal to _____.

Official Ans. by NTA (80)

Ans. (80)

Sol. $|z-3| \leq 1$

represent pt. i/s circle of radius 1 & centred at (3, 0)

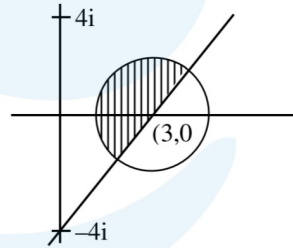
$$z(4+3i) + \bar{z}(4-3i) \leq 24$$

$$(x+iy)(4+3i) + (x-iy)(4-3i) \leq 24$$

$$4x+3xi+4iy-3y+4x-3ix-4iy-3y \leq 24$$

$$8x-6y \leq 24$$

$$4x-3y \leq 12$$



minimum of (0, 4) from circle = $\sqrt{3^2+4^2}-1=4$

will lie along line joining (0, 4) & (3, 0)

\therefore equation line

$$\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12 \dots (i)$$

equation circle $(x-3)^2 + y^2 = 1 \dots (ii)$

$$\left(\frac{12-3y}{4} - 3\right)^2 + y^2 = 1$$

$$\left(\frac{-3y}{4}\right)^2 + y^2 = 1$$

$$\frac{25y^2}{16} = 1 \Rightarrow y = \pm \frac{4}{5}$$

for minimum distance $y = \frac{4}{5}$

$$\therefore x = \frac{12}{5}$$

$$\therefore 25(\alpha + \beta) = 25\left(\frac{4}{5} + \frac{12}{5}\right)$$

$$= 16 \times 5 = 80$$

2. Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, \dots, 100\} \right\}$ and let

$T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

Official Ans. by NTA (100)

Ans. (100)

Sol. $A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -a + ab \\ 0 & b^2 \end{bmatrix}$$

$$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$$

$\therefore b$ must be equal to 1

\therefore In this case A^2 will become identity matrix and a can take any value from 1 to 100

\therefore Total number of common element will be 100.

3. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

Official Ans. by NTA (576)

Ans. (576)

Sol. Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 \rightarrow Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$$\therefore (a + c + e + g) - (b + d + f) = 11x$$

$$a + b + c + d + e + f + g = 31$$

$$\therefore \text{either } a + c + e + g = 21 \text{ or } 10$$

$$\therefore b + d + f = 10 \text{ or } 21$$

Case-1

$$a + c + e + g = 21$$

$$b + d + f = 10$$

$$(b, d, f) \in \{(1, 2, 7) (2, 3, 5) (1, 4, 5)\}$$

$$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$$

$$\therefore \text{Total number in case-1} = (3! \times 3) (4!) = 432$$

Case-2

$$a + c + e + g = 10$$

$$b + d + f = 21$$

$$(a, b, e, g) \in \{1, 2, 3, 4\}$$

$$(b, d, f) \in \{(5, 7, 9)\}$$

$$\therefore \text{Total number in case 2} = 3! \times 4! = 144$$

$$\therefore \text{Total numbers} = 144 + 432 = 576$$

4. The sum of all the elements of the set $\{\alpha \in \{1, 2, \dots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is _____.

Official Ans. by NTA (1633)

Ans. (1633)

Sol. $\text{HCF}(\alpha, 24) = 1$

$$\text{Now, } 24 = 2^2 \cdot 3$$

$\rightarrow \alpha$ is not the multiple of 2 or 3

Sum of values of α

$$= S(U) - \{S(\text{multiple of } 2) + S(\text{multiple of } 3)$$

$$- S(\text{multiple of } 6)\}$$

$$= (1 + 2 + 3 + \dots + 100) - (2 + 4 + 6 + \dots + 100) - (3$$

$$+ 6 + \dots + 99) + (6 + 12 + \dots + 96)$$

$$= \frac{100 \times 101}{2} - 50 \times 51 - \frac{33}{2} \times (3 + 99) + \frac{16}{2} (6 + 96)$$

$$= 5050 - 2550 - 1683 + 816 = 1633 \text{ Ans.}$$

5. The remainder on dividing $1 + 3 + 3^2 + 3^3 + \dots + 3^{2021}$ by 50 is _____.

Official Ans. by NTA (4)

Ans. (4)

Sol.
$$\frac{1 \cdot (3^{2022} - 1)}{2} = \frac{9^{1011} - 1}{2}$$

$$= \frac{(10 - 1)^{1011} - 1}{2}$$

$$= \frac{100\lambda + 10110 - 1 - 1}{2}$$

$$= 50\lambda + \frac{10108}{2}$$

$$= 50\lambda + 5054$$

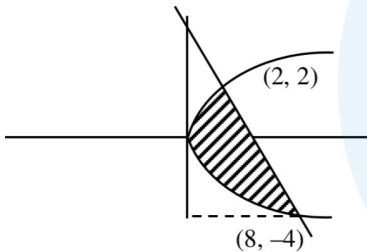
$$= 50\lambda + 50 \times 101 + 4$$
 Rem (50) = 4.

6. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _____.

Official Ans. by NTA (18)

Ans. (18)

Sol. $x = 4 - y$
 $y^2 = 2(4 - y)$
 $y^2 = 8 - 2y$
 $y^2 + 2y - 8 = 0$
 $y = -4, y = 2$
 $x = 8, x = 2$



$$\int_{-4}^2 \left[(4 - y) - \frac{y^2}{2} \right] dy$$

$$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6} \right]_{-4}^2$$

$$= 8 - 2 - \frac{8}{6} + 16 + \frac{16}{2} - \frac{64}{6}$$

$$= 22 + 8 - \frac{72}{6}$$

$$= 30 - 12 = 18$$

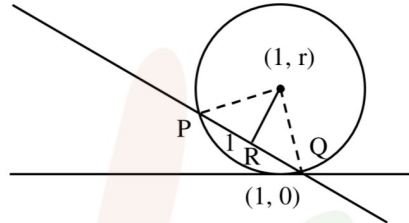
7. Let a circle $C : (x - h)^2 + (y - k)^2 = r^2, k > 0$, touch the x-axis at $(1, 0)$. If the line $x + y = 0$ intersects the

circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

Official Ans. by NTA (7)

Ans. (7)

Sol. $k = r$
 $h = 1$
 $OP = r, PR = 1$
 $OR = \left| \frac{r+1}{\sqrt{2}} \right|$



$$r^2 = 1 + \frac{(r+1)^2}{2}$$

$$2r^2 = 2 + r^2 + 1 + 2r$$

$$r^2 - 2r - 3 = 0$$

$$(r - 3)(r + 1) = 0$$

$$\boxed{r = 3}, -1$$

$$h + k + r = 1 + 3 + 3 = 7$$

8. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to _____.

Official Ans. by NTA (479)

Ans. (479)

Sol. $A = \{1, 2, 3, 4\} : P(A) = \frac{3}{4} \rightarrow$ Correct
 $B = \{5, 6, 7, 8, 9, 10\} : P(B) = \frac{1}{4}$ Correct
 8 Correct Ans.:

$$(4, 4): {}^4C_4 \left(\frac{3}{4}\right)^4 \cdot {}^6C_4 \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^2$$

$$(3, 5): {}^4C_3 \left(\frac{3}{4}\right)^3 \cdot \left(\frac{1}{4}\right)^1 \cdot {}^6C_5 \left(\frac{1}{4}\right)^5 \cdot \left(\frac{3}{4}\right)$$

$$(2, 6): {}^4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2 \cdot {}^6C_6 \left(\frac{1}{4}\right)^6$$

$$\text{Total} = \frac{1}{4^{10}} [3^4 \times 15 \times 3^2 + 4 \times 3^3 \times 6 \times 3 + 6 \times 3^2]$$

$$= \frac{27}{4^{10}} [2.7 \times 15 + 72 + 2]$$

$$\Rightarrow K = 479$$

9. Let the hyperbola H : $\frac{x^2}{a^2} - y^2 = 1$ and the ellipse

E : $3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12(e_H^2 + e_E^2)$ is equal to _____.

Official Ans. by NTA (42)

Ans. (42)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{1} = 1$ $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$$e_H = \sqrt{1 + \frac{1}{a^2}} \quad e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

$$\ell.R. = \frac{2}{a} \quad \ell.R. = \frac{2 \times 3}{2} = 3$$

$$\frac{2}{a} = 3$$

$$\boxed{a = \frac{2}{3}}$$

$$e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

$$12(e_H^2 + e_E^2) = 12\left(\frac{13}{4} + \frac{1}{4}\right)$$

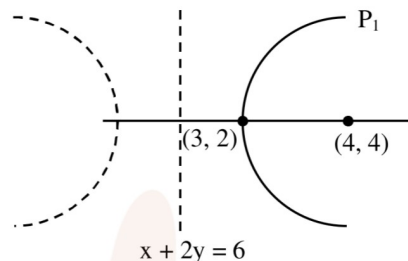
$$= \frac{12 \times 14}{4} = 42$$

10. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y = \underline{\hspace{2cm}}$.

Official Ans. by NTA (10)

Ans. (10)

Sol.



P_1 : Directorix :

$$x + 2y = k$$

$$x + 2y - k = 0$$

$$\left| \frac{3 + 4 - k}{\sqrt{5}} \right| = \sqrt{5}$$

$$|7 - k| = 5$$

$$7 - k = 5 \quad 7 - k = -5$$

$$\boxed{k = 2}$$

$$\boxed{k = 12}$$

Accepted

Rejected

Passes through

focus

$$\left. \begin{array}{l} D_1 = x + 2y = 2 \\ \ell = x + 2y = 6 \\ D_2 = x + 2y = C \end{array} \right\} \Rightarrow d \Rightarrow \boxed{c = 10}$$